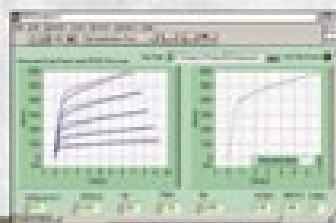
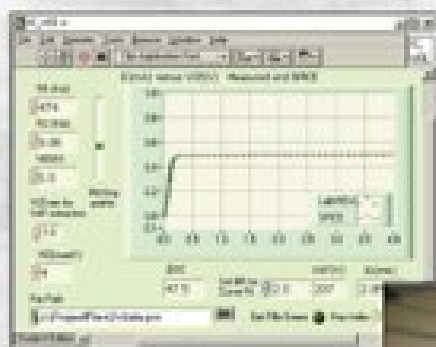




VIRTUAL INSTRUMENTATION SERIES



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Analog Electronics with LabVIEW®

By [Kenneth L. Ashley](#)

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Publisher: **Prentice Hall**

Pub Date: **October 04, 2002**

Print ISBN-10: **0-13-047065-1**

Print ISBN-13: **978-0-13-047065-2**

Pages: **432**

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Library of Congress Cataloging-in-Publication Data

Ashley, Kenneth L.

Analog electronics with LabVIEW / Kenneth L. Ashley.

p. cm. — (National Instruments virtual instrumentation series)

Includes bibliographical references and index.

ISBN 0-13-047065-1 (pbk. : alk. paper)

1. Electronics. 2. Electronic circuits—Computer-aided design. 3. LabVIEW. I. Title. II. Series.

TK7816 .A84 2002

621.381--dc21

2002072656

Editorial/production supervision: Patti Guerrieri

Cover design director: Jerry Votta

Cover designer: Nina Scuderi

Manufacturing manager: Alexis R. Heydt-Long

Publisher: Bernard Goodwin

Editorial assistant: Michelle Vincenti

Marketing manager: Dan DePasquale

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Publishing as Prentice Hall PTR

Upper Saddle River, NJ 07458

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Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

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Preface

This book presents a study of analog electronics as a stand-alone course or as a course to be augmented by one of the many complete undergraduate textbooks on the subject. Theory and closely coupled laboratory projects, which are based entirely on computer-based data acquisition, follow in a sequential format. All analytical device characterization formulations are based exactly on SPICE.

In addition to traditional curricula in electrical engineering and electronics technology, the course is suitable for the practicing engineer in industry. For the engineer with a general undergraduate electronics background, for example, the course of study can provide an upgrade in basic analog electronics. Under these or similar circumstances, it can be taken as self-paced or with minimum supervision.

Two course sequences are possible, depending on the emphasis desired:

- For a course that stresses MOSFET characterization and circuits, beginning with [Unit 1](#) and following the sequence is recommended. A brief review of relevant circuit analysis and the most rudimentary basics of electronics are presented initially, with associated projects. The projects include an introduction to LabVIEW programming along with the measurements of basic circuits. The programming aspects are directly relevant to the thrust of the course; they emphasize the measurement of analog electronics circuits. The student is thus provided with a basic understanding of LabVIEW concepts used throughout the projects.
- If, on the other hand, interest is directed more toward LabVIEW and computer data acquisition, device characterization, and circuit simulation, the appropriate beginning sequence is Units A through C. The associated projects are [Project A](#), [Projects B](#), [Project C1](#), and [Project C2](#). [Project A](#) is a programming and measurement exercise that emphasizes and explores the use of LabVIEW DAQ software, the discrete nature of analog-to-digital and digital-to-analog conversions, LabVIEW-based voltmeters with autoranging, ac voltmeters, and simultaneous sending and receiving of waveforms initiated with a function generator. This is followed with projects on transistors and transistor circuits, which are based on the bipolar junction transistor. Although the BJT is losing ground as the most important transistor in electronics (compared to the MOSFET), its inherently more complex behavior provides for a rich array of circuit simulation formulations and design challenges. The projects include the mix of NPN and PNP devices in a single amplifier. The transistors recommended are the complementary pair NTE 186 (2N6288) and NTE 187 (2N62xx). The transistors are rated at 3 A and are therefore almost indestructible. At the much lower current levels of the projects, device heating is negligible, which is important, as all measurements assume that the circuit is at room temperature. Also, highlevel model effects are avoided, whereas low-level effects abound.

With both approaches, all the measurement LabVIEW programs are provided. Many of the extraordinary features provided by LabVIEW are included in the programs. The programs therefore may serve additionally as a tutorial in advanced aspects of LabVIEW. The basics of operational amplifiers and their applications are treated in two units and two projects.

The book format consists of one or more units of background material for each laboratory project. A given set of theoretical units and the associated project have a related Mathcad problems file (Problemxx.mcd) and Mathcad exercise file (ExerciseXX.mcd), relating to the theory and project, respectively. The files are also in a pdf format (ProblemXX.pdf, ExerciseXX.pdf). A Mathcad file (ProjectXX.mcd) for evaluating the results of the projects is included with each project. Accompanying each Mathcad project file are SPICE simulator files based on PSPICE. The SPICE models for the simulations use, in each case, the parameters for the devices obtained in laboratory projects. Since the Mathcad projects use the exact SPICE formulations, the results from Mathcad and SPICE are identical in the case of the use of basic simulation levels.

Samples of all of the projects have been completed and are included. These provide for either demonstrations or simulated results without actually running the programs with circuits. The measured data are stored in LabVIEW graphics and can be extracted to obtain data files in the same manner as actually making the measurements. In some cases, the simultaneous taking of data, plotting and curve fitting is simulated. Units 13 and 14 are theoretical only but each has Mathcad problems on the topic of these respective units.

Special features of the lab experience are as follows:

- The lab projects are based entirely on computer data acquisition using LabVIEW and a National Instruments data acquisition card (DAQ) in the computer for interfacing with the circuit board.
- Each device category has an associated project for evaluating SPICE parameters in which device model parameters are obtained. Subsequent amplifier projects use the parameters in performance assessment.
- No external instrumentation is required. The function generator, voltmeters, and oscilloscopes are virtual and provided by LabVIEW and a DAQ card in the computer. The projects on the current-mirror load common-source amplifier and the operational amplifier require an external power supply.
- Circuits are constructed on a special circuit board. The board is connected to the computer DAQ card through a National Instruments shielded 68-pin cable. The circuit board allows expedient, error-free construction of the circuits, as connector strips for the respective output and input channels and ground are available directly on the board.

Topics included in this course treat many of the most relevant aspects of basic modern analog electronics without straying into peripheral areas. The course essentially streamlines the study of analog electronics. There is not a unit on, for example, feedback per se, but most basic types of feedback are addressed at some point. The role that the device plays in frequency response is omitted. This is consistent with the fact that to a large extent, the intention is that theory and measurements can be connected.

Students of electrical engineering or electronics engineering of today have a vast array of subjects to attempt to master; it is not reasonable to expect them to labor through a classical extensive study of the subject of analog electronics, although some basic knowledge should be required. Specialization can come at a later stage, if desired.

As mentioned, many LabVIEW features are utilized in the projects. To some extent, the goal of demonstrating the extensive array of the capabilities of LabVIEW influences the design of the various projects. This includes sending voltages (including waveforms), receiving voltages (including autoscaling), scanning, graphics, reading data files, writing data files, computations such as extraction of harmonic content of a signal, assembling data in a composite form, along with a host of array manipulation processes and data curve fitting.

References

CMOS analog circuits including applications (advanced):

Allen P., and R. Holberg. CMOS Analog Circuit Design, 1st Ed. Holt, Reinhart and Winston, New York, 1986.

Allen P., and R. Holberg. CMOS Analog Circuit Design, 2nd Ed. Oxford University Press, New York, 2002.

Extensive coverage of analog circuits, which includes a comprehensive discussion of feedback and frequency response (advanced):

Gray, P., P. Hurst, S. Lewis, and R. Meyer. Analysis and Design of Analog Integrated Circuits, 4th Ed. Wiley, New York, 2001.

CMOS analog circuits (with some BJT circuits) with extensive coverage of applications (advanced):

Johns D., and K. Martin. Analog Integrated Circuit Design. Wiley, New York, 1997.

Presentation of the physical and empirical association between semiconductor devices and their models, MOSFETs and BJTs:

Massobrio G., and P. Antognetti. Semiconductor Device Modeling with SPICE. McGraw-Hill, New York, 1993.

General textbook on electronics (basic):

Millman J., and A. Grabel. Microelectronics, 2nd Ed. McGraw-Hill, New York, 1987.

Physical description of semiconductor devices:

Muller R., and T. Kamins. Device Electronics for Integrated Circuits, 2nd Ed. Wiley, New York, 1986.

General textbook on electronics (basic):

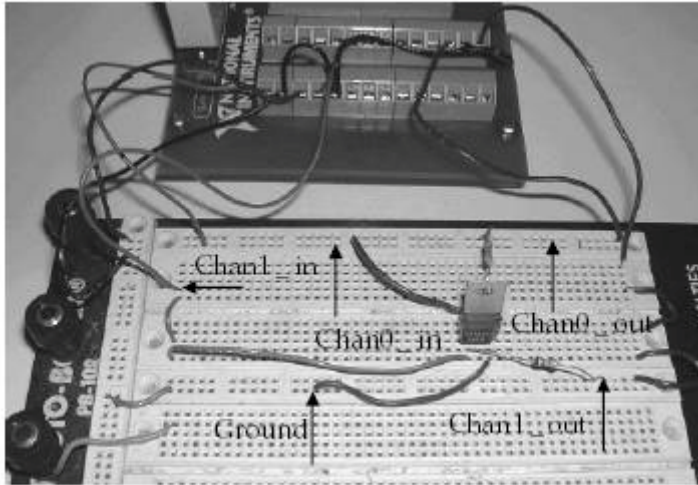
Sedra, A.S., and K.C. Smith. Microelectronic Circuits, 4th Ed. Oxford University Press, Oxford, 1998.

General treatment of analog circuits including applications (basic to advanced):

Soclof, S. Design and Applications of Analog Integrated Circuits, Prentice Hall, Upper Saddle River, N.J., 1991.

Hardware and Software Requirements

Circuit connections to the DAQ require a cable and a facility for connecting to individual pins. An efficient system is based on a National Instruments Connector Block (CB-68LP) and a basic circuit board as shown here.



Connections to the circuit board from the connector block are made one time. The two resistors of the circuit are connected to output channels 0 and 1, respectively. Thus, for example, Chan0_out, as noted, is dedicated to the top strip on the circuit board. The bottom top strip is associated with Chan0_in, and so forth.

All of the project LabVIEW files are programmed to be consistent with the plus bus (rail), Chan0_out, and the minus bus (rail), Chan1_out. Therefore, it is intuitively helpful to have the output channels physically connected in this fashion.

The project examples included with the book were conducted on a special circuit box that connects directly to the shielded 68-pin connector. This bypasses the connector block. A shielded cable is strongly recommended in any event. Many of the projects involve the measurement of relatively low voltage signals.

In addition, the lab projects included in the book require the following (or equivalent):

- Pentium PC (or equivalent).
- National Instruments DAQ PCI-MIO-16E-4.
- LabVIEW 6.0i Student Edition or LabVIEW 6.0i or later version.
- Mathcad Professional 2001 or later version.
- National Instruments Shielded 68-pin Cable.

Semiconductor Devices and Components (Recommended)

6-Transistor (3-gate) CMOS Array – CD4007^[*]

CMOS Opamp – SGS-Thomson TS271^[**]

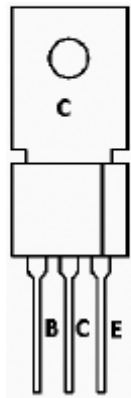
NPN - Medium-Power NPN BJT – NTE186^[***]

PNP - Medium-Power PNP BJT – NTE187^[****]

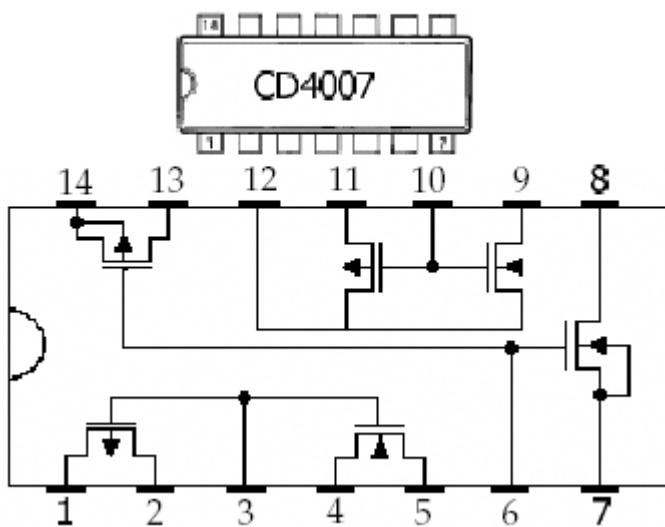
Capacitors

Resistors

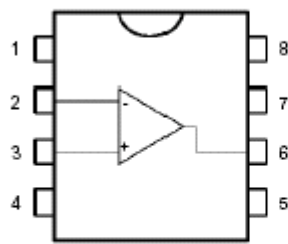
NTE186
NPN



CD4007



TS271 (STMicroelectronics)



- 1 - Offset Null 1
- 2 - Inverting Input
- 3 - Non-inverting Input
- 4 - V_{CC-}
- 5 - Offset Null 2
- 6 - Output
- 7 - V_{CC+}
- 8 - I_{SET}

Connector Block Pins (AT-MIO-E or PCI-E Series)

Chan0_out	Pin 21	Chan0_in	Pin 68	Gnd – Pin 34
Chan1_out	Pin 22	Chan1_in	Pin 33	Gnd – Pin 66
Output Channel Gnd	Pin 55	Chan2_in	Pin 65 - plus	Pin 31 - minus
Input Channel Gnd	Pin 67	Input and output grounds are connected.		
+5 V Supply Voltage	Pin 14			

[*] The CD4007 chip contains three CMOS inverters or three PMOS and three NMOS transistors. Since they are inverters, NMOS and PMOS pairs have Hardware and Software Requirements internally connected gates. However, this does not prevent having a sufficient number of the individual transistors in the analog laboratory projects.

[**] The TS271 is chosen as it has simple external resistor biasing. Thus, students can gain an intuitive feel for the relation between the characteristics of the CMOS opamp and bias current with straightforward exchange of bias resistors. In the case of a group of students, for example, each student can select a different bias current, such that all of the results can be assembled to plot the opamp characteristics, such as gain and frequency response versus bias current. In addition, the circuitry of the opamp is straightforward and may be understood within the scope of the book. Extensive experience in our laboratory with devices has demonstrated that this opamp can withstand considerable abuse without failing even though it is a MOSFET chip. It is however, strongly advised that the power supply never be turned on until the power-supply pins, input pins and output pin are connected in the circuit.

[***] The NTE186 is a rugged npn BJT that is investigated at current levels well below the normal operating range. Heating of the device is thus minimized and for the

measurements, it can be assumed to be at room temperature. Also, various high-level injection effects, which render the basic SPICE parameter set invalid, are avoided.

[****] Complementary paired with the NTE186.

LabVIEW VI Libraries and Project and Problem Folders and Files

Each project has a folder, which contains the LabVIEW library plus any related Mathcad files for that project. Mathcad files include those for the exercises and results analysis (project files). The project folder also has circuit-simulator subfolders for Schematics and Capture.

A LabVIEW VI library is included for each project. These are LabVIEW files with extension llb. The LabVIEW files within a library have extension vi. A given project library will contain most of the LabVIEW virtual instruments for that project. The additional VIs are in the User.lib folder, which is in the LabVIEW application folder. The User.lib folder contains all the LabVIEW libraries and other LabVIEW files that are not included in the individual project libraries. The folders are Read_Rite, Dat_File, FunctGen, and Subvi.

Each problem folder has a set of problems associated with the unit with the same number. Each problem set has a pdf file (Word), a Mathcad solutions file, a pdf version on the Mathcad file and a circuit-simulator subfolder.

There are also pdf files for the composite of the problems (WordProb.pdf), Mathcad problem-solution files (MathcadProb.pdf), project exercises (MathcadExer.pdf), project Schematics exercises (SchematicsExer.pdf), and project Capture exercise (CaptureExer.pdf).

The procedure for installation of the libraries from the CD onto the computer is described in the Readme files.

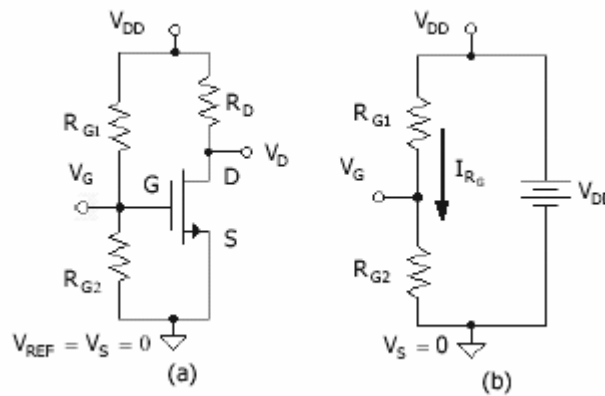
Unit 1. Elementary Circuit Analysis for Analog Electronics

In this unit, we present a basic review of segments of circuit analysis which recur repeatedly in electronic circuits. A firm grasp on these is essential to developing an understanding of the analysis and design of basic electronic circuits. A transistor is included in the circuits to show a correlation between circuit analysis and electronics. Only steady-state circuit situations are considered here. This includes dc and sinusoidal. Some transient analysis is considered in connection with operational amplifier applications with capacitors.

1.1. Resistor Voltage Divider and MOSFET DC Gate Voltage

[Figure 1.1](#)(a) shows a basic NMOS amplifier stage. This is the dc (or bias) portion of the circuit, which excludes the signal part. The terminals of the transistor are designed G (gate), D (drain) and S (source). The design calls for a dc voltage V_G , with respect to the zero reference voltage, which is obtained by dividing the supply voltage V_{DD} between bias resistors R_{G1} and R_{G2} . Since the gate terminal has zero current, the voltage, V_G , at the gate can be assessed with the resistor network separated from the circuit as in [Fig. 1.1](#)(b). The goal is to relate the node voltage V_G to the values of R_{G1} and R_{G2} and V_{DD} . The result is the basic resistor voltage-divider relation.

Figure 1.1. (a) Dc circuit for the basic NMOS amplifier. (b) Circuit for determining the gate voltage, V_G .



Note that since V_{DD} is given with respect to the reference zero volts, the V_{DD} designation at the top node is equivalent to the supply voltage, also referred to as V_{DD} . The current I_{R_G} is

Equation 1.1

$$I_{R_G} = \frac{V_{DD}}{R_{G1} + R_{G2}}$$

The voltage across the resistor R_{G2} is V_G (since V_G is with respect to the zero reference) and this is

Equation 1.2

$$V_G = V_{GS} = V_{R_{G2}} = I_G R_{G2} = \frac{R_{G2}}{R_{G1} + R_{G2}} V_{DD}$$

It can be concluded that the gate voltage is the value of R_{G2} divided by the sum of the two gate-bias resistors.

1.2. Output Circuit and DC Drain Voltage

For the dc circuit in [Fig. 1.1](#), the drain voltage is determined from

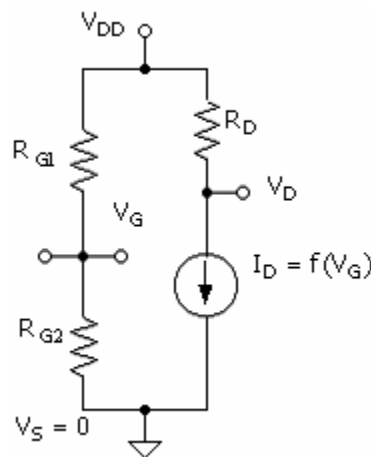
Equation 1.3

As illustrated in [Fig. 1.2](#), for the purpose of a solution to (1.3), the transistor can be replaced by a current source as shown in [Fig. 1.2](#). Drain current I_D is a function of V_G ; that is, $I_D = f(V_G)$. Thus, in a design, the value of V_G determines the value of V_D . I_D is related to V_G according to

Equation 1.4

$$V_D = V_{DD} - I_D R_D$$

Figure 1.2. Circuit for illustrating the determination of the drain voltage, V_D .

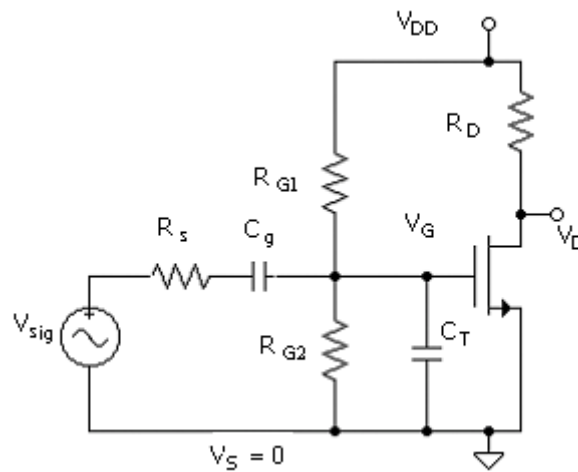


This relation and parameters V_{th0} and k_n are discussed in [Unit 2](#).

1.3. Frequency Response of the Amplifier Stage

Capacitance associated with amplifiers may cause the output to fall off at low and high frequencies. This effect is referred to as the *frequency response* of the amplifier. A generalization of possible capacitance is shown in the circuit of Fig. 1.3. Capacitor C_g is an external capacitance, which is included to attach a sine-wave signal source, consisting of V_{sig} (e.g., sine-wave peak) and R_s , without interrupting the dc bias circuitry. Similarly, there could be an output capacitance, which couples the signal output voltage to an external load resistor. Capacitor C_T is associated with the internal capacitance of the transistor. It may be regarded as an equivalent effective capacitance that represents all of the capacitance of the transistor.

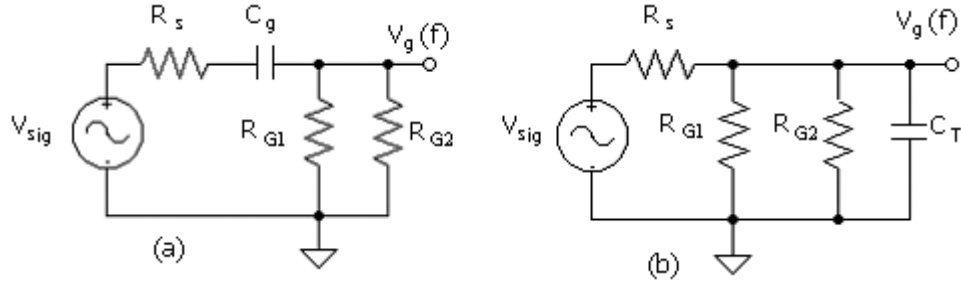
Figure 1.3. Amplifier including possible circuit capacitance.



Generally, the frequency range over which a given capacitor is effective is much different for the two capacitors. Capacitor C_g affects the output at low frequencies, while the effect of C_T is realized at the high end of the spectrum. Thus, their effects can be considered separately if, as assumed in the following, the high and low ends of the response function are widely separated in frequency, that is, by several orders of magnitude.

Figure 1.4 shows the signal circuits for the two cases of low (a) and high (b) frequencies. As discussed in Unit 2, the signal circuit is formulated from the complete circuit by setting all dc voltages to zero. This includes, for this amplifier, the power supply and dc voltage across the capacitor C_g . Note that the transistor plays no apparent role in the frequency response in the equivalent circuit. It is, of course, critically important in dictating the value of C_T .

Figure 1.4. Circuits for low (a) and high (b) frequencies.



The two circuits, (a) and (b), are technically *high-pass* and *low-pass* circuits, respectively. In combination, they have a *midband* range, which is the normal range of frequency for operation of the amplifier. As mentioned above, if the midband separates the low and high portions by a sufficient range of frequency, the effects may be considered separately, as suggested in Fig. 1.4.

The response function is obtained by considering the frequency dependence of the node voltage $V_g(f)$ for the constant-magnitude sine-wave source voltage, V_{sig} . (Since the only voltages under consideration in the circuits of Fig. 1.4 are those associated with signals, lowercase subscript is used. This is discussed further in Unit 2.)

The frequency response is first considered for the low end of the spectrum and involves C_g only, as in the circuit of Fig. 1.4(a). We can utilize the voltage-divider relation obtained above as (1.2). For this case this is

Equation 1.5

$$V_g(f) = \frac{R_G}{R_G + R_s + \frac{1}{j2\pi f C_g}} V_{sig} = \frac{R_G}{R_G + R_s} \frac{1}{1 + \frac{1}{j2\pi f (R_G + R_s) C_g}} V_{sig}$$

where $R_G = R_{G1} \parallel R_{G2}$.

Using the definitions

Equation 1.6

$$V_{go} = \frac{R_G}{R_G + R_s} V_{sig}$$

and

Equation 1.7

$$f_{lo} = \frac{1}{2\pi(R_G + R_s)C_g}$$

the result is condensable to

Equation 1.8

$$V_g(f) = V_{go} \frac{1}{1 - \frac{jf_{lo}}{f}}$$

The magnitude is

Equation 1.9

$$|V_g(f)| = V_{go} \frac{1}{\sqrt{1 + \left(\frac{f_{lo}}{f}\right)^2}}$$

At $f = f_{lo}$, $V_g(f) = V_{go} / \sqrt{1+1}$. This, by definition, is the response magnitude for the 3-dB frequency, f_{3dB} , for the low-frequency end of the response function. That is, in general, f_{3dB} is the frequency at which the response falls to $1/\sqrt{2}$ (for decreasing frequency) from the maximum, asymptotic value. Thus, for the simple case here of one capacitor, $f_{3dB} = f_{lo}$.

The equation for the response function associated with C_T is similar to (1.8) and is

Equation 1.10

$$V_g(f) = V_{go} \frac{1}{1 + \frac{jf}{f_{hi}}}$$

where

Equation 1.11

$$f_{hi} = 1 / 2\pi (R_S // R_G) C_T$$

The frequency f_{3dB} for this case is just f_{hi} . The frequency response of circuits of the type of Fig. 1.4 is measured in Project 1. In the design of the project circuits, capacitors and resistors are selected to give widely different f_{lo} and f_{hi} values.

1.4. Summary of Equations

$V_G = V_{GS} = V_{R_{G2}} = I_G R_{G2} = \frac{R_{G2}}{R_{G1} + R_{G2}} V_{DD}$	Resistor-circuit voltage divider.
$I_D = k_n (V_G - V_{t_{no}})^2$	Drain current and gate voltage relation.
$V_g(f) = V_{go} \frac{1}{1 - \frac{jf}{f_{lo}}}$	Low-frequency frequency-response function.
$V_g(f) = V_{go} \frac{1}{1 + \frac{jf}{f_{hi}}}$	High-frequency frequency-response function.
$V_{go} = \frac{R_G}{R_G + R_s} V_{sig}$	Midband magnitude of the signal gate voltage.

1.5. Exercises and Projects

Project Mathcad Exercise01.mcd - Project01.mcd
Files

Laboratory Project 1 [Basic Circuit Analysis for Electronic Circuits and Programming Exercises](#)

P1.1 [Resistor Voltage-Divider Measurements](#)

P1.2 [Resistor Voltage Divider with Current Measurement](#)

P1.3 [Resistor Voltage Divider with Resistor Measurement](#)

P1.4 [Resistor Voltage Divider with a Sine-Wave Source Voltage](#)

P1.5 [Frequency Response of a Resistor-Capacitor Circuit](#)

Unit 2. Transistors and Voltage Amplification

Radio transmitters and receivers have existed since before the end of the nineteenth century. A practical form of wireless telegraph, attributed to G. Marconi, appeared in 1895, and successful transmission across the Atlantic Ocean was achieved in 1901. However, in the early part of the twentieth century, systems were limited by the lack of a means of voltage amplification. The appearance of a voltage amplification device, the vacuum tube, dramatically improved the concept, as microvolt signals could be boosted for receiving and transmitting.

In the middle of the twentieth century, the transistor appeared. The idea of transistors based on a sandwich of pn junctions (BJT) and a field-effect transistor based on pn junctions (JFET) and on a metal – oxide – semiconductor (MOS) structure (basically, a capacitor) were all understood at the time. However, pn-junction devices became a practical realization much sooner than the MOS structure, due to fabrication complications in producing the MOS device as well as perhaps a perceived lack of need. The JFET served as an interim field-effect transistor until the MOS technology evolved. It provided for a transistor with very high input resistance and was used extensively as the input transistors for BJT opamps.

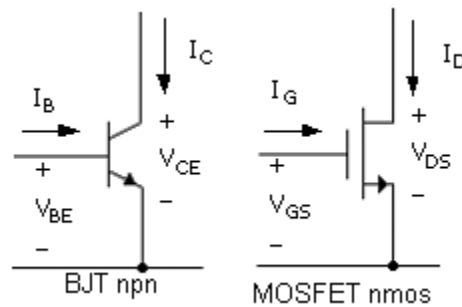
A textbook on radio, *Elements of Radio*, published in 1948 (Marcus and Marcus, 1948), makes no mention of transistors. A 1958 text, (Millman, 1958), *Vacuum-Tube and Semiconductor Electronics*, gives equal weight to vacuum tubes and BJTs in electronic circuits but makes no mention of the field-effect transistor. Slightly later (Nanavati, 1963), in *An Introduction to Semiconductor Electronics*, as the title suggests, vacuum tubes are dropped completely and the only reference to a field-effect transistor is in one section of the last chapter and this refers to a junction field-effect transistor. In 1965, in his textbook *Analysis and Design of Electronic Circuits*, Chirlian devotes a small portion of the book to vacuum tubes, but most of the emphasis is on circuits based on the BJT (Chirlian, 1965). No mention is made of the field-effect-transistor. An example of a book in which BJTs and field-effect transistors of both types were finally given balanced treatment was published in 1979 (Millman, 1979). Textbooks tend to lag the industry a bit, and during the 1970s, MOSFET circuits were emerging rapidly, driven by the simultaneous development of integrated circuits. The four editions of a text on analog circuits by Gray and Meyer, (1977, 1984, 1993) and Gray, et al. (2001) serve well as a series through which we observe a transition from mostly BJT to, in the last two editions, more-or-less equal treatment of BJT and MOSFET devices. A recent textbook on the subject of analog integrated circuits (Johns and Martin, 1997) takes the approach that such circuits are now totally dominated by MOSFETS but includes some BJT applications. BiCMOS, a combination of MOSFET and BJT devices on the same integrated circuit, is growing in popularity as more ways of taking advantage of the superior properties of the two transistor types are developed.

Since the earliest transistors, there has been persistent competition between BJT and MOS transistors. It has been, to a large extent (along with many other considerations), a matter of power consumption versus speed; the BJT has been faster but is associated with high power consumption. The MOSFET has gradually taken over as the most important transistor, with increased emphasis on integrated circuits and improved speeds.

2.1. BJT and MOSFET Schematic Symbols, Terminal Voltages, and Branch Currents

The BJT can be either a pnp or an npn. The MOSFET similarly can be a pmos or an nmos. The equivalents are npn and nmos and pnp and pmos. The following discussion is based on the npn and nmos, as shown in [Fig. 2.1](#). (All polarities and current directions are reversed for the pnp and pmos. This provides for important versatility in applications.)

Figure 2.1. BJT npn and MOSFET nmos transistors. The terminal configurations are designated common emitter and common source.



The BJT terminals are designated collector, base, and emitter while those of the MOSFET are drain, gate, and source. The terminal configurations in [Fig. 2.1](#) are, for the BJT, the common emitter, and for the MOSFET, the common source, in amplifier-stage parlance. This suggests that both the input (left side) and output (right side) are referred to the common terminal. For example, for the BJT, the input terminal voltage is V_{BE} and the output terminal voltage is V_{CE} . Similarly, for the MOSFET, we have V_{GS} and V_{DS} . Note that in the convention of subscripts in electronics, the first subscript is assigned positive. This matches the assignments in the diagram, and the plus and minus signs are superfluous.

Note also the convention for symbols for all currents and voltage.

Total voltage and current: v_{XY} , i_X

Dc, bias, quiescent, or operating point: V_{XY} , I_X

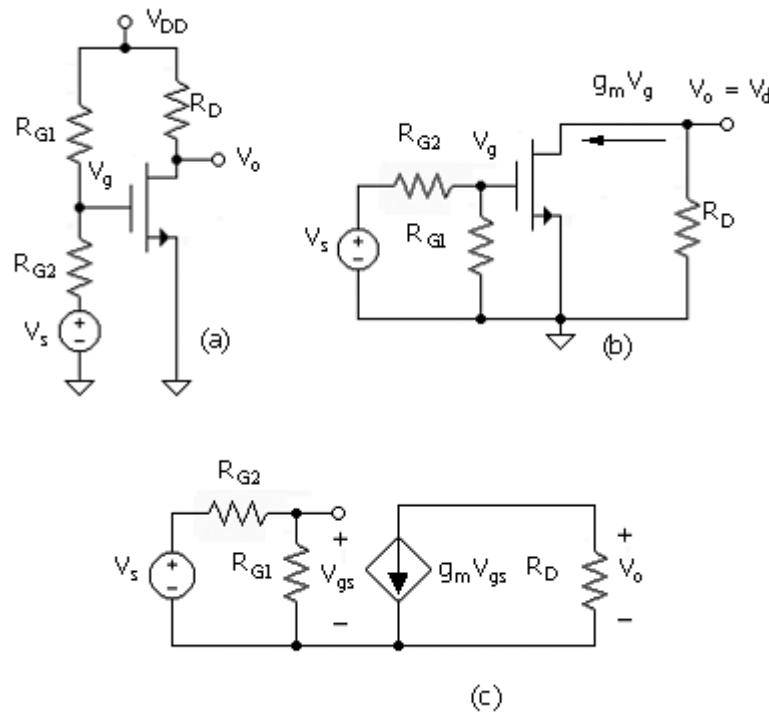
Signal or ac (RMS, peak): V_{xy} , I_x

General instantaneous signal: v_x , i_x

The voltage and current symbols in [Fig. 2.4](#) are therefore for dc. For a voltage, a single subscript means that this terminal (or node) voltage is referred to the common terminal. For example, in the npn case above, $V_{CE} = V_C$.

Figure 2.4. Basic NMOS amplifier with resistor gate biasing and input signal V_s . (a) Complete circuit. (b) Signal (or ac or incremental) circuit. The signal circuit

is obtained by setting the power supply (dc) node to zero volts. (c) Linear signal circuit replaces the linear schematic representation.



The input terminals v_{BE} and v_{GS} are the control terminals; that is, they control the output currents i_C and i_D . In both cases, the terminal pairs possess extremely nonsymmetrical voltage – current behavior. With the polarities as shown, the currents flow readily, whereas with the opposite polarities, the output currents are cut off or are, for most purposes, essentially zero.

The basic (simplified) general relations between the currents and voltages are:

Equation 2.1

$$\text{BJT: } i_C = I_S e^{v_{BE}/V_T}$$

Equation 2.2

$$\text{NMOS: } i_D = k_n (v_{GS} - V_{tn})^2$$

I_S , V_T , k_n , V_{tn} , and V_T are device model parameters or physical constants.

In linear circuit applications, for example, as amplifier stages, the transistors are provided with a circuit configuration that sets up dc, or bias, currents and terminal voltages (sometimes referred to as the Q-point, for quiescent, or in SPICE, the operating point). In the amplifier application, a signal voltage is applied to the input, that is, superimposed on the dc magnitude, which must be much smaller than the dc voltage if the signal input-output relation is to be linear. This is apparent from (2.1) and (2.2), which are nonlinear relations. All of the currents and terminal voltages will change in response to the input signal, and all of these incremental changes must be small compared to any of the dc currents or voltages, in order for the linear relationships to be valid.

In circuit applications, both types of transistors are operated in all three possible terminal configurations. This provides for a wide variety of amplifier-stage characteristics, including gain and input and output impedance.

2.2. Fundamentals of Signal Amplification: The Linear Circuit

The most fundamental property of a useful electronic voltage amplification device is that it possess a transconductance that leads to the possibility of voltage gain.

Transconductance is defined as the ratio of the signal (ac, incremental) current out, $i_{out} \equiv \delta i_{OUT}$, and the applied input signal voltage, $v_{in} \equiv \delta v_{IN}$. That is, transconductance g_m is

Equation 2.3

$$g_m = \frac{\delta i_{OUT}}{\delta v_{IN}} = \frac{di_{OUT}}{dv_{IN}} = \frac{i_{out}}{v_{in}}$$

For the BJT, $i_{OUT} \equiv i_C$ and $v_{IN} \equiv v_{BE}$, while for the NMOS, $i_{OUT} \equiv i_D$ and $v_{IN} \equiv v_{GS}$. Thus, (2.1) and (2.2) can be used for the BJT and MOSFET, respectively, to obtain an expression for g_m . The results are

Equation 2.4

$$\text{BJT: } g_m = \frac{I_C}{V_{be}} = \frac{I_C}{V_T}$$

and

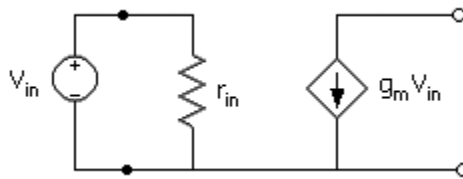
Equation 2.5

$$\text{MOSFET: } g_m = \frac{I_d}{V_{GS}} = \frac{2 I_D}{V_{GS} - V_{tn}}$$

I_C and I_D are the dc (bias) currents of the transistors, so for comparison they can be made equal. At room temperature, the thermal voltage is $V_T = 26$ m. For the MOSFET, V_{GS} is the gate – source bias voltage and V_{tn} is the transistor threshold voltage. The difference, as in the denominator of the transconductance expression, could typically be about $V_{GS} - V_{tn} = 500$ mV.

The expression (2.3) suggests the linear model given in Fig. 2.2. Included in the model is an input resistance, r_{in} , which accounts for the fact that there can be an incremental current flowing into the input terminal for an increment of input voltage. The model applies in general to amplifying devices, including the vacuum tube (VT), BJT, JFET, and MOSFET. There exists a wide range of magnitude of transconductance and input resistance between the devices. The input resistance, though, affects only the loading of the input signal source; otherwise, the relation of (2.3) applies in all cases, and the transconductance is the key to the gain for a given device type. The input resistance is essentially infinite for the vacuum tube and the MOSFET (common source) but can be as low as a few ohms in some configurations for the BJT (e.g., common base).

Figure 2.2. Basic linear model of a voltage amplification device. Model parameters are g_m and r_{in} .



It is interesting to compare the transconductance of the BJT and MOSFET along with the vacuum tube. We will make a comparison at $I_D = I_C = 10$ mA (suitable for a vacuum tube) even though transistors would not usually be operated at such high currents, especially in an integrated circuit. Consulting a source of information for a triode 6SN7 (perhaps one of the most common tubes of all time), one deduces from a graphical analysis the plate characteristics that, for example, $g_m(\text{VT}) = 3$ mA/V. From (2.4) and (2.5), we obtain $g_m(\text{BJT}) = 385$ mA/V and $g_m(\text{MOSFET}) = 40$ mA/V with $V_{GS} - V_{tn} = 500$ mV. The BJT is decidedly superior in this respect, and this is one of the factors contributing to the sustained life of the transistor in industry. That is, the BJT amplifier stage can potentially have a much higher voltage gain. The vacuum tube is clearly inferior to both transistors

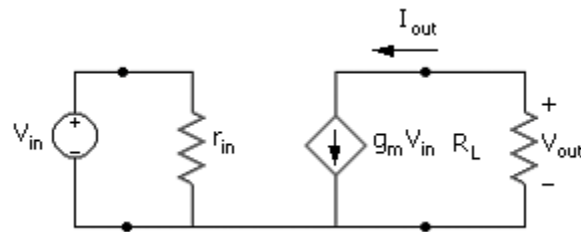
and points to the reason for the need for so many amplification stages in some VT amplifiers.

The output voltage of amplifiers based on any of the devices will depend on the value of the load resistance, R_L , which is added to the circuit of [Fig. 2.2](#) in [Fig. 2.3](#). Note that, in general, R_L is not necessarily an actual resistor but could be an effective resistance, as dictated by the amplifier circuitry that is connected to the output of a given stage, combined with a bias resistor. The output voltage induced across R_L will be

Equation 2.6

$$V_{out} = -I_{out}R_L = -g_m V_{in}R_L$$

Figure 2.3. Basic linear model of a voltage amplification device with load R_L connected.



The minus sign is a result of the current flowing up through R_L . The signal voltage gain is the incremental output voltage divided by the incremental input voltage such that the gain can readily be obtained from [\(2.6\)](#) as

Equation 2.7

$$a_v = \frac{V_{out}}{V_{in}} = -g_m R_L$$

Thus, the gain is directly related to the parameter g_m for a given transistor. In general, a_v can be positive or negative, depending on the terminal configuration. For example, the common base (BJT) and common gate (MOSFET) are positive (noninverting) gain amplifiers.

2.3. Basic NMOS Common-Source Amplifier

An example of the application of the transconductance relation for the transistor is the basic circuit in [Fig. 2.4](#). Setting dc voltages (in this case, V_{DD}) equal to zero in [Fig. 2.4\(a\)](#) leads to the signal (or ac) circuit [[Fig. 2.4\(b\)](#)]. This follows from the fact that the signal circuit involves only incremental variables (changes) and V_{DD} is a constant.

The schematic symbol for the transistor in the signal circuit associates the output current with the input voltage according to the linear relation of (2.3). For linear circuit analysis, the linear equivalent circuit of [Fig. 2.2](#) ([Fig. 2.4\(c\)](#)) replaces the linear schematic-symbol representation [[Fig. 2.4\(b\)](#)]. For the MOSFET, r_{in} is infinite and therefore omitted.

The overall gain from the signal source to the output is $a_v = V_o/V_s$, which is

Equation 2.8

$$a_v = \frac{V_o}{V_s} = \frac{V_g}{V_s} \frac{V_o}{V_g} = -\frac{R_{G1}}{R_{G1} + R_{G2}} g_m R_D$$

where V_o/V_g is (2.7) and V_g/V_s is provided by the simple resistor-divider relation given in (1.2).

2.4. Transistor Output Resistance and Limiting Gain

The linear-equivalent circuit of [Fig. 2.2](#) includes an idealization in that the output is a pure current source. In real transistors, the collector (BJT) or drain (MOSFET) current increases with increasing V_{CE} or V_{DS} . This is accounted for by including an output resistance, r_{out} , in the linear model, as added to the circuit in [Fig. 2.5](#). For the BJT and MOSFET, respectively, the value of r_{out} is

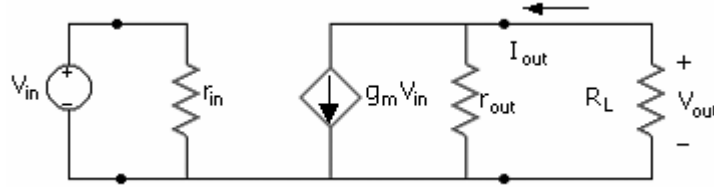
Equation 2.9

$$r_{out} = r_{ce} = \frac{\partial v_{CE}}{\partial i_C} = \frac{V_A}{I_C}$$

Equation 2.10

$$r_{out} = r_{ds} = \frac{\partial v_{DS}}{\partial i_D} = \frac{V_A}{I_D}$$

Figure 2.5. Basic linear circuit with transistor linear model. Circuit includes signal source voltage and load R_L . Transistor model now includes output resistance r_{out} .



where V_A is the characterizing transistor parameter. Note that this voltage dependence is not included in (2.1) and (2.2); these equations are consistent with the simplified circuit model of Fig. 2.2. Similarly, the voltage dependence will alter g_m from the simple forms of (2.4) and (2.5). This is discussed in Unit 4.

The actual gain, with a load R_L , which includes the output resistance, can be obtained from modification of (2.7) to include r_{out} in parallel with R_L as in Fig. 2.5. The result is

Equation 2.11

$$a_v = -g_m \frac{R_L r_{out}}{R_L + r_{out}}$$

The parameter V_A of both transistors can typically be about 100 V. (In MOSFETs, the parameter is usually referred to as λ , which is the reciprocal, $\lambda = 1/V_A$.) A useful comparison between the devices is the maximum limiting gain of the common-emitter and common-source amplifier voltage gains, which applies for the case of $R_L \rightarrow \infty$. The gain in this case is

Equation 2.12

$$a_{vlim} = -g_m r_{out}$$

Using (2.4), (2.5), (2.9), and (2.10), we obtain for the limiting gain:

Equation 2.13

$$\text{BJT: } a_{v\text{lim}} = -\frac{V_A}{V_T}$$

Equation 2.14

$$\text{MOSFET: } a_{v\text{lim}} = -\frac{2V_A}{V_{GS} - V_{tn}}$$

Using sample numbers from above, the comparison gives $a_{v\text{lim}}(\text{BJT}) \approx -4000$ and $a_{v\text{lim}}(\text{MOSFET}) \approx -400$. The vacuum tube, type 6SN7, has a typical output resistance $r_{\text{out}} \equiv r_p \approx 7\text{K}\Omega$ (p for plate), which leads to a limiting gain magnitude of about 21. (This is referred to as the μ of the tube.) You have to respect the amplifier designers of the vacuum-tube era when considering what was accomplished despite the limitations of these amplifying devices.

In modern integrated circuits, it is possible to implement load circuits, which have an effective $R_L \gg r_{\text{out}}$ such that the limiting gain can be achieved. This is particularly important in MOSFET amplifiers to make up for the relatively low value of g_m .

2.5. Summary of Equations

$i_C = I_S e^{v_{BE}/V_T}, g_m = \frac{I_C}{V_T}$	BJT transfer function and transconductance relation.
$i_D = k_n (v_{GS} - V_{tn})^2, g_m = \frac{2I_D}{V_{GS} - V_{tn}}$	MOSFET transfer function and transconductance relation.
$a_v = \frac{V_{\text{out}}}{V_{\text{in}}} = -g_m R_L$	General voltage-gain relation from the transistor input.
$a_v = -\frac{R_{G1}}{R_{G1} + R_{G2}} g_m R_D$	Gain relation from the signal source.
$r_{\text{out}} = \frac{V_A}{I_C}$	General relation for the transistor output resistance.
$a_v = -g_m \frac{R_L r_{\text{out}}}{R_L + r_{\text{out}}}$	Voltage gain, including the effect of the transistor output resistance.

2.6. Exercises and Projects

Project Mathcad Exercise02.mcd - Project02.mcd
Files

Laboratory Project 2 [Basic NMOS Common-Source Amplifier with Programming Exercises](#)

P2.1 [NMOS Common-Source Circuit with Drain Current Measurement](#)

P2.2 [NMOS Common-Source Amplifier with Resistor Gate Bias Circuit](#)

P2.3 [Amplifier with Signal and Gain Measurement](#)

2.7. References to the Electronics Book Sequence

Chirlian, P. M. Analysis and Design of Electronic Circuits. McGraw-Hill, New York, 1965.

Gray, P. R. , and R. G. Meyer . Analysis and Design of Analog Integrated Circuits, 1st, 2nd, and 3rd eds. Wiley, New York, 1977, 1984, and 1993.

Gray, P. R. , P. J. Hurst , S. H. Lewis , and R. G. Meyer . Analysis and Design of Analog Integrated Circuits, 4th ed. Wiley, New York, 2001.

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Marcus, A. , and W. Marcus . Elements of Radio, 2nd ed., Prentice-Hall, New York, 1948.

Millman, J. Vacuum-tube and Semiconductor Electronics. McGraw-Hill, New York, 1958.

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Nanavati, R. P. An Introduction to Semiconductor Electronics. McGraw-Hill, New York, 1963.

Unit 3. Characterization of MOS Transistors for Circuit Simulation

In this unit, the basic (Level 1 SPICE) dc MOSFET characteristic equations are introduced. The amplifier exercises and projects use the results for design and analysis. Circuit solutions are compared with measured results from the circuit to make an assessment of the degree to which the transistor models for the MOSFET represent actual device behavior. The parameters for this unit are presented in [Table 3.1](#). Note that in the

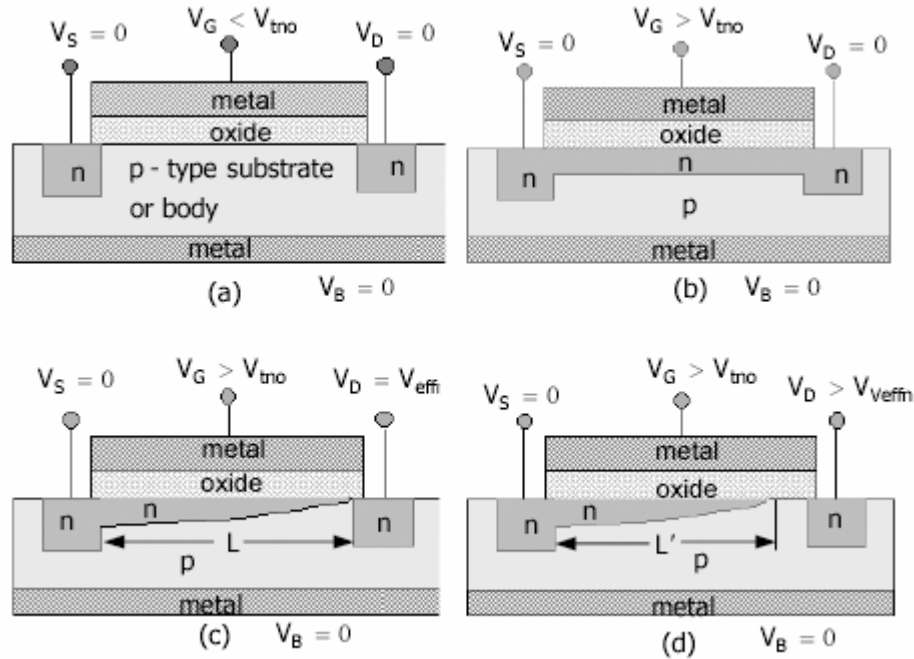
case of KP, we can only measure K and would be able to extract KP only if gate width W and length L were known.

TABLE 3.1		
SPICE Name	Math symbol	Description
VTO	$V_{t_{no}}, V_{t_{po}}$	Zero-bias threshold voltage.
KP	$k = \frac{KP}{2} \frac{W}{L}$	Transconductance parameter.
Gamma	γ_n, γ_p	Body-effect parameter.
Phi	$2\Phi_F$	Surface inversion potential.
Lambda	λ_n, λ_p	Channel length modulation.

3.1. Physical Description of the MOSFET

A diagrammatic NMOS is shown in [Fig. 3.1](#). The device consists of a three-layer structure of metal–oxide–semiconductor (MOS). A two-terminal MOS structure (connections to metal and semiconductor) is essentially a parallel-plate capacitor. In the same manner as for a normal capacitor, when a positive gate voltage, V_G , is applied with respect to the p-type body (for NMOS) [i.e., with respect to the metal contact on the underside of the p-type semiconductor body (or substrate)], negative charges are induced under the oxide layer in the semiconductor. When V_G (with respect to the semiconductor body) exceeds the threshold voltage, $V_{t_{no}}$, a channel of free-carrier electrons forms under the oxide; that is, the onset of the channel occurs when $V_G = V_{t_{no}}$. The substrate is n type for the PMOS and the channel is made up of free-carrier holes.

Figure 3.1. MOS transistor consisting of a metal – oxide – semiconductor layered structure (plus a metal body contact on the bottom). A positive gate voltage, $V_G > V_{t_{no}}$, induces a conducting channel under the oxide, which connects the two n regions, source and drain. All voltages are with respect to V_B , that is, the body (substrate) of the transistor. (a) No channel; (b) uniform channel; (c) channel is just pinched off at the drain end of the channel; (d) channel length is reduced due to drain pn-junction depletion region extending out along the channel.



An n-channel MOSFET device is then completed by fabricating n regions, source and gate, for contacting the channel on both ends of the channel. For $V_G < V_{tno}$ [Fig. 3.1(a)] there is no channel under the oxide, and the two n regions are isolated pn junctions. When $V_G > V_{tno}$ and source voltage, V_S , and drain voltage, V_D , are both zero (all with respect to the body) [Fig. 3.1(b)] a uniform-thickness n-type channel exists along the length of the oxide layer and the source and drain regions are connected by the channel. Thus, the channel is a voltage-controlled resistor where the two ends of the resistor are at the source and drain and the control voltage is applied at the gate.

In electronic circuit applications, the terminal voltages are referred to the source; gate and drain voltages are designated as V_{GS} and V_{DS} (NMOS). In analog circuits, $V_{GS} > V_{tno}$ (in order for a channel to exist), V_{DS} is positive, and a drain current flows through the channel and out by way of the source (and the gate current is zero). On the drain end of the channel, the voltage across the oxide layer is $V_{GD} = V_{GS} - V_{DS}$. The channel at the drain end just shuts off when $V_{GD} = V_{tno}$. $V_{DSsat} = V_{GS} - V_{tno}$ [Fig. 3.1(c)] is defined for this condition as the saturation voltage. The transistor is referred to as in the linear (or triode) region or active region for $V_{DS} < V_{DSsat}$ and $V_{DS} > V_{DSsat}$, respectively.

For $V_{DS} > V_{DSsat}$ (active mode of operation), the channel length decreases from L to L' as the reverse-biased depletion region of the drain pn junction increases along the channel (along the oxide – semiconductor interface) [Fig. 3.1(d)]. The increment $V_{DS} - V_{DSsat}$ drops across the depletion region of the drain pn junction. In long-channel devices, the reduction of channel length is relatively small compared to the channel length. In this case, the length is roughly a constant and the channel resistance, for a given V_{GS} , is independent of V_{DS} .

From a circuit point of view, for $V_{DS} \gg V_{DSsat}$, by Ohm's law,

Equation 3.1

$$I_D \approx \frac{V_{DSsat}}{R_{chan}(V_{GS})}$$

where $R_{chan}(V_{GS})$ is the resistance of the channel and is a function of V_{GS} . Assuming that $L' \approx L$, for a given V_{GS} , $R_{chan}(V_{GS})$, and thus I_D , is approximately a constant for $V_{DS} \geq V_{DSsat}$. Thus, the drain, in circuit terms, appears like a current source. In many modern MOSFET devices, this is only marginally valid. In the following, the definition $V_{effn} \equiv V_{DSsat} = V_{GS} - V_{tno}$ will be used. (The subscript is an abbreviation for effective.) The PMOS has a counterpart, $V_{effp} \equiv V_{SDsat} = V_{SG} - V_{tpo}$. V_{effp} is a frequently recurring term in device and circuit analytical formulations.

3.2. Output and Transfer Characteristics of the MOSFET

The equations used in the following to characterize the MOSFET transistor are from the SPICE Level 1 model. SPICE also has more detailed models in Level 2 and Level 3. These can be specified when running SPICE. However, the number of new model parameters, in general, in circuit simulation is practically boundless. Level 1 is chosen here as it is the most intuitive, that is, the most suitable for an introductory discussion of device behavior. Some new models, for example, which focus on frequency response at very high frequencies, can include pages of equations. In addition, Level 1 is suitable and adequate for many examples of circuit simulation.

The basic common-source NMOS circuit configuration is repeated in [Fig. 3.2](#). Here it serves as a basis for discussing the dc SPICE parameters of the MOSFET transistor. In the example, $V_{DS} = V_{DD}$. The output characteristic is a plot of I_D versus V_{DS} for $V_{GS} = \text{const}$. A representative example is shown in [Fig. 3.3](#). As mentioned, the low-voltage region is referred to as the linear region, triode region, or presaturation region. Outside this region for higher voltages is the active (saturation) region. This is referred to here as the active region to avoid confusion with the fact that the nomenclature is just the opposite in the case of the BJT; that is, the low-voltage region is called the saturation region. As discussed in [Unit 3.1](#), the linear and active regions are delineated by $V_{effn} \equiv V_{DSsat} = V_{GS} - V_{tno}$.

Figure 3.2. Common-source circuit configuration for discussion of the dc model parameters of the NMOS transistor. The three-terminal transistor symbol implies that the body and source are connected.

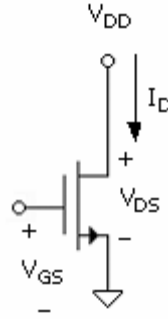
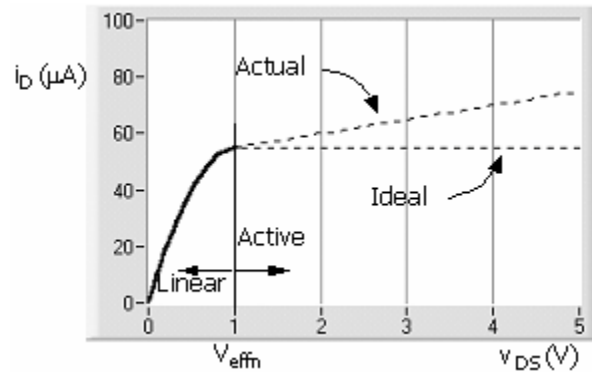


Figure 3.3. Mathcad-generated output characteristic for the NMOS transistor. The plot illustrates the linear and active regions. The linear region is also called the triode region or presaturation region. Current is in microamperes and $V_{effn} = 0.8$ V. Also plotted is the ideal characteristic with zero slope in the active region.



The output-characteristic equation in the linear region corresponds to V_{DS} ranging from the condition of [Fig. 3.1\(b\)](#) to that of [Fig. 3.1\(c\)](#). As V_{DS} increases from zero, the channel begins to close off at the drain end (i.e., the channel becomes progressively more wedge shaped). The result is an increase in the resistance of the channel as a function of V_{DS} , and therefore a sublinear current – voltage relation develops.

When $V_{GS} > V_{tno}$, the electron charge in the channel can be related to the gate voltage by $Q_{chan} = C_{ox}(V_{GS} - V_{tno})$ (per unit area of MOSFET looking down at the gate), where C_{ox} is the parallel-plate capacitance (per unit area) formed by the MOS structure. This provides a simple linear relation between the gate voltage and the charge in the channel.

The conductivity in the channel is $\sigma_{chan} = \mu_n Q_{chan}/t_{chan}$, where μ_n is the mobility of the electrons in the channel and t_{chan} is the thickness of the channel into the semiconductor. Thus, in the case of a uniform channel (i.e., for $V_{DS} \rightarrow 0$), the channel conductance is

Equation 3.2

$$G_{\text{chan}} = \sigma_{\text{chan}} \frac{t_{\text{chan}} W}{L} = \mu_n Q_{\text{chan}} \frac{W}{L} = \mu_n C_{\text{ox}} \frac{W}{L} (V_{\text{GS}} - V_{\text{tno}}) = 2k_n V_{\text{effn}}$$

where

Equation 3.3

$$k_n = \frac{KP_n}{2} \frac{W}{L}$$

and where $KP_n = \mu_n C_{\text{ox}}$ is the SPICE transconductance parameter (the n subscript is the equation symbolic notation for the NMOS; the parameter in the device model is just KP), W is the physical gate width, and L , again, is the channel length. Parameter KP_n is related to the electron mobility in the channel and the oxide thickness. Therefore, it is very specific to a given MOSFET device.

As V_{DS} increases, but is less than V_{effn} [transition from [Fig. 3.1\(b\)](#) to [3.1\(c\)](#)], the wedge-shaped effect on the channel is reflected functionally in the channel conductance relation as

Equation 3.4

$$G_{\text{chan}} = 2k_n \left(V_{\text{effn}} - \frac{V_{\text{DS}}}{2} \right)$$

This leads to an output characteristic equation for the linear region, which is

Equation 3.5

$$I_D = G_{\text{chan}} V_{\text{DS}} = 2k_n \left(V_{\text{effn}} V_{\text{DS}} - \frac{V_{\text{DS}}^2}{2} \right)$$

The derivation leading to (3.4) and (3.5) is given in [Unit 3.4](#). The linear-region relation, (3.5), is applicable for V_{DS} up to $V_{\text{DS}} = V_{\text{effn}}$, which is the boundary of the linear and active regions. The active-region equation is then obtained by substituting into (3.5), $V_{\text{DS}} = V_{\text{effn}}$, giving

Equation 3.6

$$I_D = k_n V_{effn}^2$$

This active-region current corresponds to the zero-slope ideal curve in [Fig. 3.3](#). As discussed in [Unit 3.1](#), the drain current is not actually constant in the active region (in the same manner as for a BJT), due to the fact that the physical length of the channel is reduced for increasing V_{DS} beyond $V_{DS} = V_{effn}$. The reduction in the channel length has the effect of slightly reducing the resistance of the channel. This is taken into account using the fact that $k_n \propto 1/L$, from [\(3.3\)](#), where L is the effective physical length between the source and drain regions. For $V_{DS} > V_{effn}$, a reduced length $L' = L(1 - \lambda_n V_{DS})$ is defined which leads to a new effective k'_n ,

Equation 3.7

$$k'_n \approx k_n (1 + \lambda_n V_{DS})$$

where λ_n is the SPICE channel-length modulation parameter (Lambda). Substituting k'_n for k_n in [\(3.6\)](#) produces

Equation 3.8

$$I_D = k_n V_{effn}^2 (1 + \lambda_n V_{DS})$$

(Note: A preferred form would be $I_D = k_n V_{effn}^2 [1 + \lambda_n (V_{DS} - V_{effn})]$ because the channel-length effect only begins for $V_{DS} > V_{effn}$ and k_n could be defined properly for effective length L at $V_{DS} = V_{effn}$. Level 1 SPICE uses [\(3.8\)](#).)

Level 1 SPICE also applies this channel-length reduction factor to the equation in the linear region, [\(3.5\)](#). To match the linear-region equation to the active-region equation, [\(3.5\)](#) becomes, at the edge of the active – linear regions,

Equation 3.9

$$I_{DSat} = k_n V_{effn}^2 (1 + \lambda_n V_{effn})$$

and, in general

Equation 3.10

$$I_D = 2k_n (1 + \lambda_n V_{DS}) \left(V_{eff} V_{DS} - \frac{V_{DS}^2}{2} \right)$$

[Again, the fact that the channel length is not reduced with the transistor in the linear region would suggest the use of (3.9) throughout the linear region. Level 1 SPICE uses (3.8) and (3.10).]

In general, V_{tn} is a function of the source-body voltage, V_{SB} . We assume for the moment that $V_{SB} = 0$. This applies, for example, to the common-source circuit in Fig. 3.2, since the body will always be at zero volts, and the source in this case is grounded as well. For this case, $V_{tn} = V_{tn0}$, as used above. In laboratory projects we measure the output characteristic from which parameter λ_n can be obtained. This is based on (3.8). The $I - V$ slope in the active region is

Equation 3.11

$$\text{slope} = \partial I_D / \partial V_{DS} = k_n V_{eff}^2 \lambda_n$$

From the data measured, a straight-line curve fit determines the slope and the zero V_{DS} intercept (I_D at $V_{DS} = 0$). These are used in (3.11) to obtain λ_n from $\lambda_n = \text{slope}/\text{intercept}$. The intercept is the extension of the active region of Fig. 3.3 (dashed line) to the I_D axis.

When applying the equations of this development to the PMOS, the voltage between the gate and source is defined as positive with respect to the source, that is, V_{SG} . To be consistent, the threshold voltage for the PMOS, V_{tp} , is also positive. In the SPICE model, however, the threshold voltage is assigned negative because positive is taken for both types of devices with respect to the gate (V_{GS} is negative for the PMOS), and the threshold voltage for the PMOS is negative.

The transfer characteristic is obtained by holding V_{DS} constant and varying V_{GS} . In the MOSFET parameter-determination experiments of Projects 3 and 4, we plot V_{GS} versus $\sqrt{I_D}$ for the transistor biased into the active region. The equation is

Equation 3.12

$$V_{GS} = \sqrt{\frac{I_D}{k_n}} + V_{tn0}$$

where k'_n is (3.7)

$$k'_n \approx k_n (1 + \lambda_n V_{DS})$$

The slope in (3.12) is $1/\sqrt{k'_n}$ and the zero $\sqrt{I_D}$ intercept is expected to be V_{tn0} . LabVIEW obtains the slope and intercept from a straight-line fit to the data. The measured transfer characteristic thus yields the two parameters k'_n and V_{tn0} .

In [Project 4](#), parameter λ_n is obtained from finding k'_n at two different V_{DS} values. This is based on

Equation 3.13

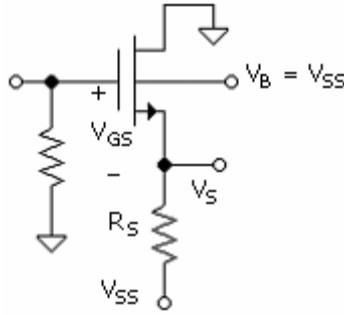
$$\frac{k'_{n10}}{k'_{n3}} = \frac{1 + \lambda_n (10 \text{ V})}{1 + \lambda_n (3 \text{ V})}$$

where the k'_n values are measured and λ_n is the only unknown.

3.3. Body Effect and Threshold Voltage

In [Fig. 3.4](#) is shown an example of a circuit in which the body and source cannot be at the same voltage. We now use the four-terminal symbol for the NMOS, which includes the body contact. In most applications, the body would be tied to the lowest potential in the circuit (NMOS), in this case, V_{SS} (e.g., $V_{SS} = -5 \text{ V}$). But by the nature of the circuit, the source voltage is $V_S = V_{SS} + I_D R_S$, such that the source-body voltage is $V_{SB} = I_D R_S$.

Figure 3.4. NMOS transistor circuit with a resistor, R_S , in the source branch.
With the body attached to V_{SS} , $V_{SB} = I_D R_S$.



In MOSFET devices, the threshold voltage depends on V_{SB} and in SPICE is modeled according to (NMOS)

Equation 3.14

$$V_{tn} = V_{tn0} + \gamma_n \left(\sqrt{V_{SB} + 2\phi_F} - \sqrt{2\phi_F} \right)$$

SPICE parameters contained in the equation are V_{tn0} (VTO), γ_n (Gamma), and $2\phi_F$ (Phi) (Table 3.1). Typically, $\gamma_n \approx 0.5 \text{ V}^{1/2}$ and $2\phi_F \approx 0.6 \text{ V}$. Therefore, for example, for $V_{SB} = 5 \text{ V}$, the body effect adds 0.8 V to V_{tn0} .

In the case of the CMOS array ICs of our lab projects (CD4007), the body effect for the PMOS is significantly less pronounced than for the NMOS ($\gamma_p < \gamma_n$), but the parameter for the channel-length-modulation effect is much smaller for the NMOS than for the PMOS ($\lambda_n < \lambda_p$). The combination suggests that the chip is a p-well configuration; that is, the NMOS devices are fabricated in "wells" of p-type semiconductor that are fabricated into an n-type substrate. The PMOS devices reside directly in the n-type substrate material. As far as our measurements are concerned, the extremes in parameters are an advantage, as we are interested in observing the effects of the various parameters.

In Project 4, a number for γ_n is obtained by measuring V_{tn} as a function of V_{SB} . The results are plotted as V_{tn} versus $X \left(\sqrt{V_{SB} + 2\phi_F} - \sqrt{2\phi_F} \right)$. LabVIEW calculates the X variable. The data plotted should give a straight line with slope γ_n . The effectiveness of SPICE modeling for representing the behavior of the transistor in a circuit is investigated in Project 4. The transfer characteristic, V_{GS} versus I_D , is measured for a circuit of the type shown in Fig. 3.4, where the circuit has $V_{SB} = I_D R_S$. In the project, V_{SS} is swept over a range of values to produce a range of I_D of about a decade. From (3.14), the threshold voltage characteristic that includes the body effect is

Equation 3.15

$$V_{tn} = V_{tn0} + \gamma_n \left(\sqrt{I_D R_S + 2\phi_F} - \sqrt{2\phi_F} \right)$$

The input circuit loop equation ([Fig. 3.4](#)) is

Equation 3.16

$$-V_{SS} = I_D R_S + V_{GS}$$

Including the body effect, V_{GS} is now [from ([3.12](#))]

Equation 3.17

$$V_{GS} = \sqrt{\frac{I_D}{k_n (1 + \lambda_n V_{GS})}} + V_{tn}$$

where, for this special case, $V_{DS} = V_{GS}$ ($V_D = 0$, $V_S = 0$).

A solution is obtainable through combining ([3.14](#)), ([3.16](#)), and ([3.17](#)) for I_D and V_{GS} . These equations contain every parameter from this discussion of MOSFET SPICE parameters along with R_S . In a project Mathcad file, Project04.mcd, a solution is obtained to provide a comparison with the measured V_{GS} versus I_D for the circuit. The Mathcad iteration formulation is, from ([3.16](#)),

Equation 3.18

$$I_D(V_{SS}, V_{GS}) = \frac{|V_{SS}| - V_{GS}}{R_S}$$

and, from ([3.15](#)) and ([3.17](#)),

Equation 3.19

$$V_{GS} = \sqrt{\frac{I_D(V_{GS}, V_{SS})}{k_n (1 + \lambda_n V_{GS})}} + V_{tn0} + \gamma_n \left(\sqrt{I_D(V_{GS}, V_{SS}) R_S + 2\phi_F} - \sqrt{2\phi_F} \right)$$

I_D and V_{GS} are found for the range of V_{SS} corresponding to that used in the measurement, which is $2.5 < |V_{SS}| < 10$ V.

3.4. Derivation of the Linear-Region Current – Voltage Relation

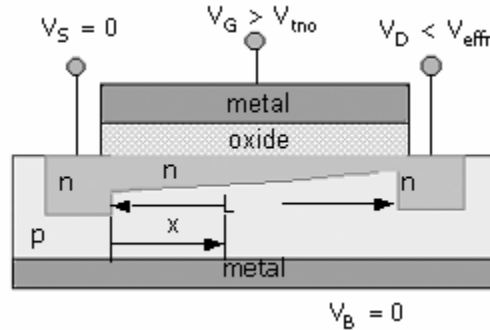
The voltage along the channel is defined as $V_c(x)$ (Fig. 3.5), with the range 0 at $x = 0$ (source) to $V_c(L) = V_D$ (drain). The device is in the linear-region mode, that is, $V_D < V_{effn}$. The charge in the channel at x is

Equation 3.20

$$Q_{chan} = C_{ox} [V_{GS} - V_c(x) - V_{tno}]$$

where the charge at the source is $Q_{chan} = C_{ox}(V_{GS} - V_{tno})$, as in the case of (3.2).

Figure 3.5. Diagrammatic NMOS transistor biased into the linear region.



A generalization of (3.2) for the incremental conductance, $dG_{chan}(x)$, at x over a length dx is

Equation 3.21

$$dG_{chan}(x) = \sigma_{chan} \frac{t_{chan} W}{dx} = \mu_n Q_{chan}(x) \frac{W}{dx} = 2k_n \frac{L}{dx} (V_{GS} - V_c(x) - V_{tno})$$

The voltage drop across the length dx , for a drain current I_D , is

Equation 3.22

$$dV_c(x) = dR_{chan}(x)I_D$$

where $R_{chan}(x) = 1/G_{chan}(x)$. Using (3.21), the incremental voltage across dx is

Equation 3.23

$$dV_c(x) = \frac{dx}{k_n L [V_{GS} - V_c(x) - V_{tno}]} I_D$$

Rearranging the equation and integrating gives

Equation 3.24

$$k_n \int_0^{V_{DS}} [V_{GS} - V_c(x) - V_{tno}] dV_c(x) = \frac{I_D}{L} \int_0^L dx = I_D$$

which leads to the result (3.5), repeated here

$$I_D = 2k_n \left[(V_{GS} - V_{tno}) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

3.5. Summary of Equations

Equations for NMOS. For PMOS, reverse the order of subscripts and define I_D out of the drain.

$I_D = 2k_n (1 + \lambda_n V_{DS}) \left(V_{effn} V_{DS} - \frac{V_{DS}^2}{2} \right)$	Current – voltage relation for the linear region.
$Q_{chan} = C_{ox} [V_{GS} - V_c(x) - V_{tno}]$ $k_n = \frac{K P_n}{2} \frac{W}{L}$ $V_{effn} = V_{GS} - V_{tno}$	Current – voltage relation for the active region.
$V_{tn} = V_{tno} + \gamma_n \left(\sqrt{V_{SB} + 2\phi_F} - \sqrt{2\phi_F} \right)$	Threshold-voltage dependence on source-body voltage.

3.6. Exercises and Projects

Project Mathcad Files	Exercise03.mcd - Project03.mcd - Exercise04.mcd, Project04.mcd
Laboratory Project 3	Characterization of the PMOS Transistor for Circuit Simulation
P3.4	Low-Voltage Linear Region of the Output Characteristic
P3.5	PMOS Parameters from the Transfer Characteristic
P3.6	PMOS Lambda from the Transfer Characteristic
P3.7	PMOS Output Characteristic
P3.8	PMOS Lambda
Laboratory Project 4	Characterization of the NMOS Transistor for Circuit Simulation
P4.4	NMOS Parameters from the Transfer Characteristic
P4.5	NMOS Lambda from the Transfer Characteristic
P4.6	NMOS Gamma SubVI
P4.7	NMOS Gamma
P4.8	NMOS Circuit with Body Effect

Unit 4. Signal Conductance Parameters for Circuit Simulation

The basic low-frequency linear model for a MOS transistor has three conductance parameters: the transconductance parameter, g_m , the body-effect parameter, g_{mb} , and the output conductance parameter g_{ds} . They are the proportionality constants between incremental variables of current and voltage. For the linear model to be valid, the increment must be small compared to the dc (bias) value of the variable. To qualify as small, the increment must be sufficiently small, in each case, as to avoid unacceptable degrees of nonlinearity in the variable relationships. The conditions are explored in [Unit 5](#) in connection with linearity of an amplifier gain function.

In the following, the three conductance parameters are explored, and in each case, an expression for obtaining the circuit value is developed. The discussion is based on a standard amplifier stage to provide for an association with electronic circuits.

4.1. Amplifier Circuit and Signal Equivalent Circuits

To serve as an illustration of the utility of the parameters, the discussion of the linear model and g parameters will be accompanied by a signal-performance evaluation of an NMOS transistor in the most general amplifier configuration ([Fig. 4.1](#)). The circuit includes drain and source resistors, and input is at the gate terminal. As shown in [Fig.](#)

4.1(b), output can be at the drain (common-source amplifier) or source (source-follower amplifier).

Figure 4.1. (a) Ideal NMOS in a basic common-source amplifier circuit (output, V_{ocs}). Dc supply nodes of Fig. 4.1(a) are set to zero volts to obtain the signal circuits of Fig. 4.1(b) and (c). An alternative output is V_{oef} [shown in (b)], which is the source-follower amplifier stage. Voltage variable V_g is the input for both cases.

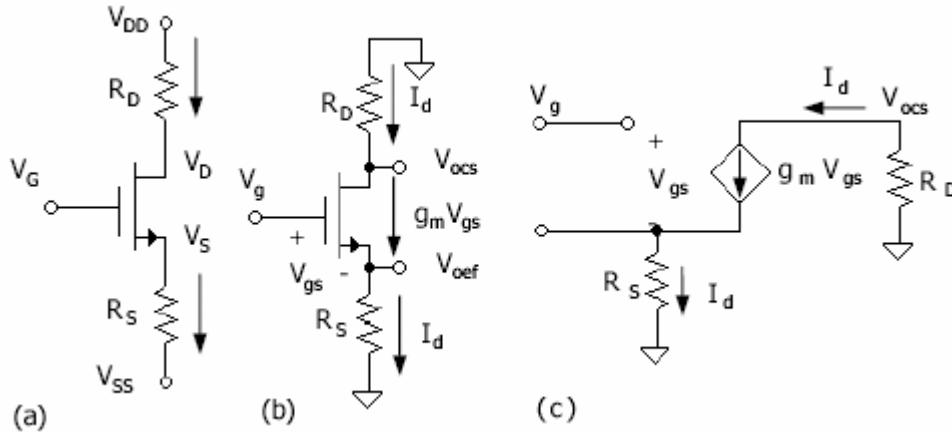


Figure 4.1 shows the dc (bias) circuit (a) and signal circuit (b). Replacing all dc nodes with signal ground and replacing the dc variables with signal variables as in Fig. 4.1 produces the signal circuit. It will be assumed that the schematic symbol for the transistor in signal circuit (b) is equivalent to the ideal, intrinsic linear model of the transistor. The total drain current of the model is $I_d = g_m V_{gs}$, as, for example, in the basic circuit of Fig. 2.4. The linear equivalent model is that of Fig. 4.1(c). The symbolic transistor in Fig. 4.1(b) is more intuitively representative in terms of the overall circuit perspective than that of Fig. 4.1(c). For this reason, the Fig. 4.1(b) version is chosen for use in all of the following discussions of MOSFET circuits. All other details of the transistor model, as discussed in this unit, will be added externally.

Signal $V_g = V_i$ is applied to the gate (input) and, in response, a signal voltage, V_o , appears at the drain (or source). We would like to analyze the signal performance in terms of voltage gain, $a_v = V_o/V_i = V_d/V_g$ (or $a_v = V_s/V_g$), of the circuit based on a linear (small-signal) analysis. In any case, the voltage gain is $a_v = G_m R_x$, where G_m is the circuit transconductance (as opposed to the transistor transconductance) and $x = D$ (common source) or $x = S$ (source follower). Thus, the goal will be to obtain a relation for G_m for a given linear model of the transistor. Circuit transconductance is determined in the following for models with the various parameters included.

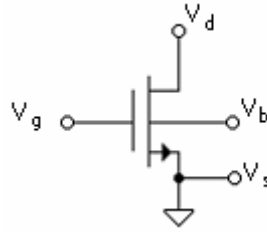
4.2. Transistor Variable Incremental Relationships

As illustrated previously diagrammatically, for example, in [Fig. 3.2](#), the MOSFET is a four-terminal device. The four-terminal version of the schematic symbol is repeated here in [Fig. 4.2](#). The terminals again are the source, drain, gate, and body. The drain current and the three terminal-pair voltages are all interdependent such that $i_D = f(v_{DS}, v_{GS}, v_{SB})$. Use of the three-terminal schematic symbol for the transistor, as in [Fig. 4.1](#), conveys the assumption that the body and source are connected. For an applied incremental V_{gs} , for example, there will be, in response, incremental drain current I_d and incremental voltages V_{ds} and V_{sb} . The linear model is based on relating the current to the three voltages. This is

Equation 4.1

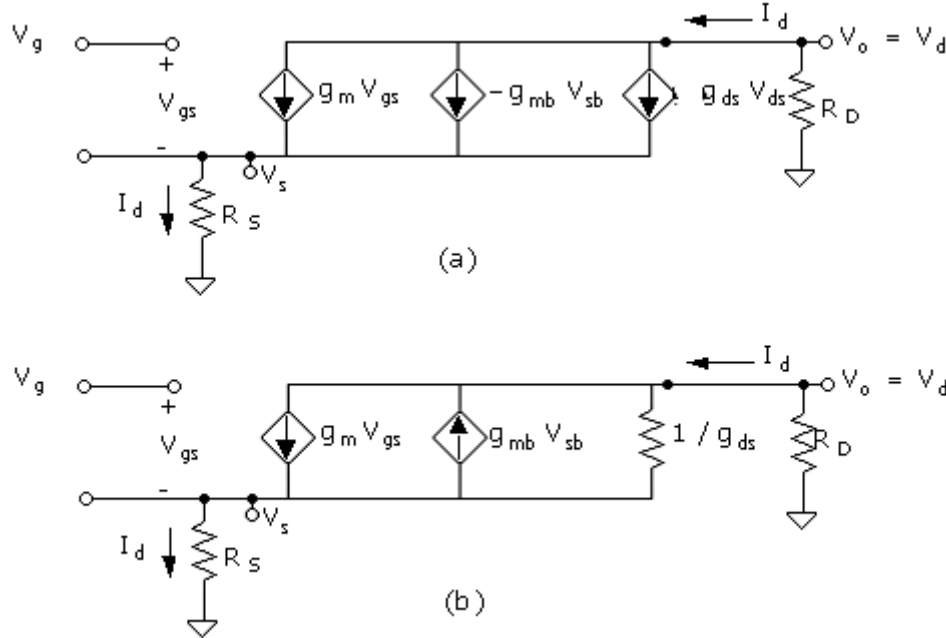
$$I_d = \frac{\partial i_D}{\partial v_{DS}} v_{ds} + \frac{\partial i_D}{\partial v_{GS}} v_{gs} + \frac{\partial i_D}{\partial v_{SB}} v_{sb} = g_m v_{gs} + g_{ds} v_{gs} + g_{mb} v_{sb}$$

Figure 4.2. Four-terminal NMOS schematic symbol in a common source configuration.



The linear-model representation is shown in [Fig. 4.3](#). [Figure 4.3\(a\)](#) shows a current-source version. The body-effect parameter, g_{mb} , is defined as positive. The minus sign is required as the partial derivative in (4.1) is negative. In [Fig. 4.3\(b\)](#) the body-effect current source is reversed to eliminate the minus sign, and the current source associated with g_{ds} is replaced with a resistance. The latter is possible as the voltage-dependent current source is between the same nodes as the voltage.

Figure 4.3. (a) Linear model that includes all contributions to the signal drain current, I_d , as given in (4.1). The body-effect parameter, g_{mb} , is a positive number such that current from the current source is in the direction opposite the arrow. (b) Current source of body effect is reversed to eliminate the minus sign, and a resistor replaces the g_{ds} current source.



In the following units, using the detailed functions (3.8) and (3.14), which relate the four variables, expressions, as used in SPICE, will be obtained for the three proportionality constants: transconductance parameter, g_m , output conductance parameter, g_{ds} , and body-effect transconductance parameter, g_{mb} . The results will be used to obtain numerical results for the circuit transconductance, G_m , for various cases.

4.3. Transconductance Parameter

The transconductance parameter, g_m , was introduced in Unit 2 in the treatment on the rudimentary electronic amplifier; it is the proportionality constant of the linear relationship between the output (responding) current and the input (control) voltage [(2.3)]. For the MOSFET, NMOS, or PMOS, $I_d = g_m V_{gs}$, where I_d is into the drain for both transistor types. An ideal transistor can be modeled with this alone. A simple model, which includes no other components, would often be adequate for making estimates of circuit performance.

To obtain an expression for g_m as a function of the general form $i_D = f(v_{GS}, v_{DS}, v_{SB})$ [e.g., (3.8)], we use the definition [from (4.1)]

Equation 4.2

$$g_m = \left. \frac{\partial i_D}{\partial v_{GS}} \right|_{v_{DS}=v_{SB}=0}$$

Using (3.8) to express i_D , the resulting relation for g_m is

Equation 4.3

$$g_m = \frac{\partial \left[k_n (v_{GS} - V_{tn0})^2 (1 + \lambda_n V_{DS}) \right]}{\partial v_{GS}} = 2k_n (V_{GS} - V_{tn0}) (1 + \lambda_n V_{DS}) = 2k'_n V_{effn}$$

where $k'_n = k_n (1 + \lambda_n V_{DS})$ [(3.7)] and $V_{effn} = V_{GS} - V_{tn0}$. Note that the use of V_{DS} is consistent with the partial derivative taken with respect to v_{GS} , that is, $V_{ds} = 0$. Also, the use of V_{tn0} implies that $v_{SB} = 0$. In general, V_{SB} could be nonzero, although in the definition of g_m , V_{sb} must be zero. For the case of nonzero V_{SB} (bias), one substitutes for V_{tn0} a constant $V_{tn}(V_{SB})$ in the g_m expression.

Alternative forms for the g_m expression can be obtained from (3.8), which is, solving for V_{effn} ,

Equation 4.4

$$V_{effn} = \sqrt{\frac{I_D}{k'_n}}$$

Using (3.8), (4.3), and (4.4), g_m takes on altogether three forms:

Equation 4.5

$$g_m = 2k'_n V_{effn} = 2\sqrt{k'_n I_D} = 2 \frac{I_D}{V_{effn}}$$

Usually, in initial design, k_n replaces k'_n to eliminate the V_{DS} dependence without a serious penalty in accuracy.

Using the simple linear transistor model, an expression for the circuit transconductance, G_m , for the circuit of Fig. 4.1 will now be obtained. The input loop equation for an applied gate signal voltage, V_g , is

Equation 4.6

$$V_g = V_{gs} + I_d R_S$$

which is, with $V_{gs} = I_d/g_m$,

Equation 4.7

$$V_g = \frac{I_d}{g_m} + I_d R_S = \frac{1 + g_m R_S}{g_m} I_d$$

and

Equation 4.8

$$G_m \equiv \frac{I_d}{V_g} = \frac{g_m}{1 + g_m R_S} = \frac{g_m}{1 + \frac{2I_D}{V_{effn}} R_S}$$

The far right-hand side uses (4.5). For example, for a 1-V drop across R_S and $V_{effn} = 0.5$ V, $G_m = g_m/5$. Note that the ratio of the signal voltage drop across R_S and signal voltage V_{gs} is $g_m R_S : 1$. The G_m concept is utilized routinely in MOSFET circuits (and BJT circuits), which gives the effective reduced transconductance, referred to V_g , in the presence of the source resistor.

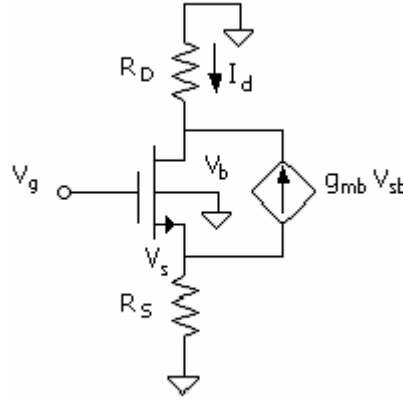
4.4. Body-Effect Transconductance Parameter

For circuits in which the signal V_{sb} is nonzero, there will be an additional component of I_d , $g_{mb} V_{sb}$. An example is the circuit of [Fig. 4.1](#) but with body terminal connected to ground. This feature is added to the circuit as shown in [Fig. 4.4](#). The proportionality constant for this case, g_{mb} , is the body-effect transconductance. It is defined as

Equation 4.9

$$g_{mb} = \left. \frac{\partial i_D}{\partial v_{sb}} \right|_{V_{gs}=V_{ds}=0}$$

Figure 4.4. Signal circuit with the addition of a current source due to the body effect. In this example, $V_b = 0$ V and $V_{sb} = I_d R_S$.



which is [with (3.8) and (3.14) for i_D and V_{tn}]

Equation 4.10

$$g_{mb} = \frac{\partial i_D}{\partial v_{tn}} \frac{\partial v_{tn}}{\partial v_{SB}} = \left| -g_m \frac{\gamma_n}{2\sqrt{V_{SB} + 2\phi}} \right|$$

The minus sign is consistent with a current source in the opposite direction from that of Fig. 4.4 (as shown in Fig. 4.3) as i_D is defined as positive into the drain. The preference is to turn the current source around as in Figs. 4.3 and 4.4 and use positive g_{mb} . The result for g_{mb} is a factor, η , times g_m , that is

Equation 4.11

$$g_{mb} = \frac{\gamma_n}{2\sqrt{V_{SB} + 2\phi_F}} g_m = \eta g_m$$

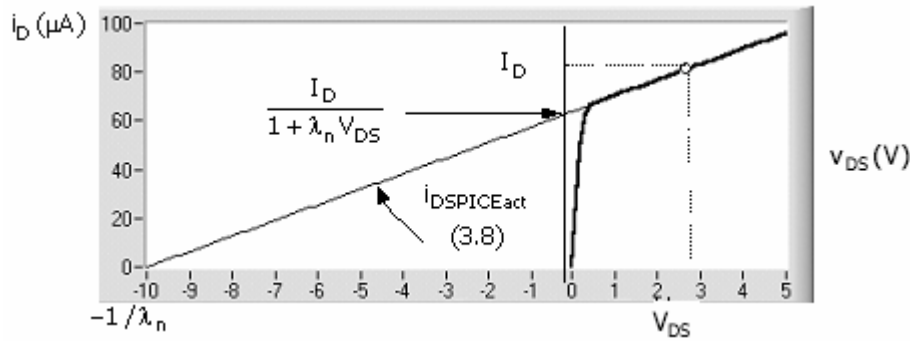
For $\gamma_n = 0.5 \text{ V}^{1/2}$, $V_{SB} = 5 \text{ V}$, and $2\phi_F = 0.7 \text{ V}$, $g_{mb} = 0.1 g_m$ ($\eta = 0.1$). Note that g_{mb} is not zero even with the source connected to the body [i.e., with $V_{SB} = 0$ in (4.11)]. However, signal V_{sb} is zero in such a case, such that g_{mb} does not have to be taken into account. In general, even with $V_{SB} \approx 0$ it is possible for $V_{sb} \neq 0$, in which case, g_{mb} must be included

in the model (e.g., in [Project 8](#) on the study of the source-follower stage, at the low end of the bias current scan).

4.5. Output Conductance Parameter

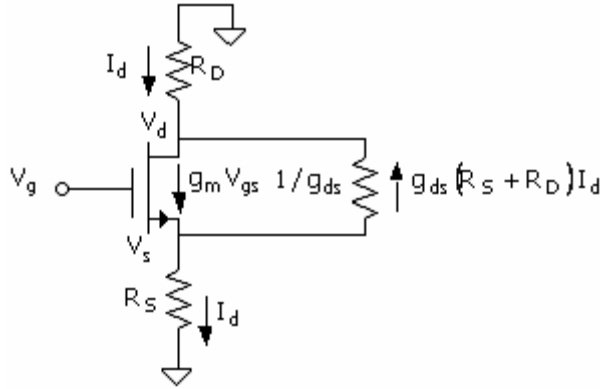
The output conductance accounts for the finite slope of the output characteristic, an example of which is shown in [Fig. 4.5](#). The plot for the SPICE formulation [(3.8)] of the active region is also shown (applicable to the device for $V_{DS} > V_{effn}$). Both plots are for $V_{GS} = \text{constant}$ (i.e., $V_{gs} = 0$ and $g_m V_{gs} = 0$).

Figure 4.5. Output characteristics illustrating nonzero active-region slope. Also shown is the SPICE formulation for the active region. For the plots, $\lambda_n = 1/10 \text{ V}$. A possible bias point with I_D and V_{DS} is included.



In the ideal case, for any v_{DS} in the active region, the current is the same for a given V_{GS} , and the drain current is simply $I_d = g_m V_{gs}$. The real case, though, obviously possesses a current dependence on v_{DS} . The linear model treats g_m as a constant (calculated at the bias values) and includes the effect of the nonzero slope with the output conductance, g_{ds} , such that $I_d = g_m V_{gs} + g_{ds} V_{ds}$ (without or neglecting the body effect). The circuit shown in [Fig. 4.6](#) includes the g_{ds} component.

Figure 4.6. Signal circuit that includes addition of the output conductance. The equivalent resistance has magnitude $1/g_{ds}$.



An expression for the output conductance is obtained from the definition

Equation 4.12

$$g_{ds} = \left. \frac{\partial i_D}{\partial v_{GS}} \right|_{v_{DS}=v_{SB}=0}$$

Again using (3.8), the result is

Equation 4.13

$$g_{ds} = \frac{\partial [k_n V_{eff}^2 (1 + \lambda_n V_{DS})]}{\partial v_{DS}} = \lambda_n k_n V_{eff}^2 = \frac{\lambda_n I_D}{1 + \lambda_n V_{DS}} \approx \lambda_n I_D$$

where $V_{eff} = V_{GS} - V_{th}$ is constant and where I_D is the dc (bias) current. The last term (on the right-hand side) is the form that is generally used in practice for an initial design, as it does not require a value for V_{DS} . Note that g_{ds} is not a conductance in the physical sense but has the correct dimensions and behaves in the circuit like a conductance.

We will now obtain the circuit transconductance for the case where the effect of g_{ds} is included. Upon application of an input signal, V_g , a signal drain current, I_d , will flow in the output circuit. This causes a signal voltage to appear across R_S and R_D , which is equal to the voltage to V_{ds} . That is,

Equation 4.14

$$V_{ds} = (R_S + R_D)I_d$$

The associated current through the output resistance is thus

Equation 4.15

$$I_{g_{ds}} = g_{ds}V_{ds} = g_{ds} (R_S + R_D) I_d$$

The effect is to reduce the current through R_S and R_D and thus reduce the circuit transconductance of the common-source amplifier stage. The output current with the current of (4.15) subtracted from the basic $g_m V_{gs}$ is

Equation 4.16

$$I_d = g_m V_{gs} - g_{ds} (R_S + R_D) I_d$$

which is, when solved for V_{gs} ,

Equation 4.17

$$V_{gs} = \frac{1 + g_m (R_S + R_D)}{g_m} I_d$$

Again using (4.6), which is $V_g = V_{gs} + I_d R_S$, the new circuit transconductance is

Equation 4.18

$$G_m = \frac{g_m}{1 + g_{ds} (R_S + R_D) + g_m R_S} = \frac{g_m}{1 + \lambda_n I_D (R_S + R_D) + \frac{2I_D}{V_{effn}} R_S}$$

With a 1 - V drop across R_S and a 5 - V drop across R_D , and with $\lambda_n = 1/50$ V, the new G_m is $g_m/5.12$, compared with $g_m/5$ when neglecting g_{ds} [(4.8)].

In general, the complete circuit includes, in addition, the body-effect transconductance current source of [Figs. 4.3](#) and [4.4](#). The omission of this current source in [Fig. 4.6](#) implies that $V_{sb} = 0$ because the source and body are connected. This connection is possible to implement in special cases such as in some of our MOSFET lab projects where only one transistor on the chip is used or for the case of a differential stage where the source of two transistors is at the same node. It is also possible to eliminate the body-effect current source by bypassing the source resistor with a bypass capacitor. The capacitor places the source at signal ground. However, in this case, the dc threshold voltage is still affected by $V_{SB} = I_D R_S$.

In [Unit 8](#), the circuit transconductance equivalent to [\(4.8\)](#) and [\(4.18\)](#), but which includes the body effect [[\(8.49\)](#)], is given as

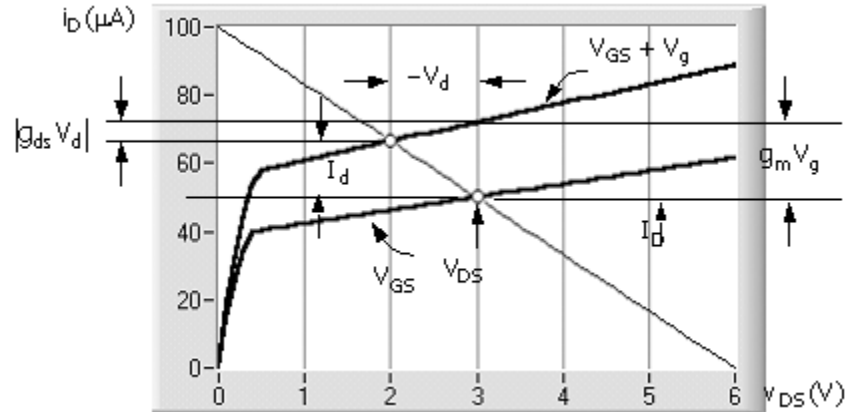
$$G_m = \frac{g_m}{(R_D + R_S)g_{ds} + 1 + g_m(1 + \eta)R_S}$$

where η is defined in [\(4.11\)](#). For $g_{ds} = 0$ and $\eta = 0$, [\(4.19\)](#) reduces to [\(4.8\)](#). Note that including the body effect will in general have more effect on G_m than including g_{ds} , as η can be on the order of 0.2. In all cases where we can calculate the circuit transconductance, G_m , the magnitude of the voltage gain is obtainable from $-G_m R_D$ (common-source stage) and $G_m R_S$ (source-follower stage).

4.6. Graphical Perspective of Output Characteristics and the Load Line

The transistor output characteristics from [Unit 3](#) and the variable increments along with their linear relationships are illustrated in [Fig. 4.7](#). This would be applicable, for example, to the amplifier of [Fig. 2.4](#). (As in that circuit, no body effect is included.) The circuit is biased with drain – source voltage V_{DS} and drain current I_D . A positive signal V_g is applied to the gate terminal. In response, there appears drain signal voltage $-V_d$, due to the rise in drain current. Signal voltages are with respect to the source or ground.

Figure 4.7. Transistor output characteristics without and with input signal, V_g . A solution for the drain current and drain – source voltage in both cases is the intersection between the respective characteristics and the load line of the amplifier circuit.



The two output characteristic curves correspond to bias V_{GS} only and with gate voltage $V_{GS} + V_g$. In both cases, the solution to the drain current and voltage is the intersection between the transistor characteristic curve and the load line, which is a plot of the output circuit loop equation. This is (with reference, for example, to [Fig. 2.4](#))

Equation 4.19

$$i_D = \frac{V_{DD}}{R_D} - \frac{V_{DS}}{R_D}$$

The solution is always constrained to this straight-line equation. The solution for v_{DS} with and without signal is based on [\(4.19\)](#) and [\(3.8\)](#), which is

$$i_D = k_n (v_{GS} - V_{th0})^2 (1 + \lambda_n v_{DS})$$

The combined contributions to I_d associated with the two g parameters is

Equation 4.20

$$I_d = g_m V_g - |g_{ds} V_d|$$

By the nature of the load-line function, the two terms will always have opposite signs; when V_g is negative, V_d will be positive.

4.7. Summary of Equations

$g_m = 2k'_n V_{effn} = 2\sqrt{k'_n I_D} = 2 \frac{I_D}{V_{effn}}$ <p>where</p> $k'_n = k_n (1 + \lambda_n V_{DS})$	Transconductance.
$g_m = 2k_n V_{effn} = 2\sqrt{k_n I_D} = 2 \frac{I_D}{V_{effn}}$ <p>where</p> $V_{effn} = \sqrt{\frac{I_D}{k_n}}$	Approximate transconductance.
$G_m \equiv \frac{I_d}{V_g} = \frac{g_m}{1 + g_m R_S}$	Amplifier circuit transconductance with a source resistance.
$g_{mb} = \frac{\gamma_n}{2\sqrt{V_{SB} + 2\phi_F}} g_m$	Body-effect transconductance.
$g_{ds} = \frac{\lambda_n I_D}{1 + \lambda_n V_{DS}}$	Output conductance.
$g_{ds} = \lambda_n I_D$	Approximate output conductance.
$G_m = \frac{g_m}{1 + g_{ds}(R_S + R_D) + g_m R_S}$	Amplifier circuit transconductance with g_{ds} included.
Subscripts are for NMOS. All equations are the same for PMOS with "p" subscript substitution and subscript-order reversal for bias-voltage variables.	

Unit 5. Common-Source Amplifier Stage

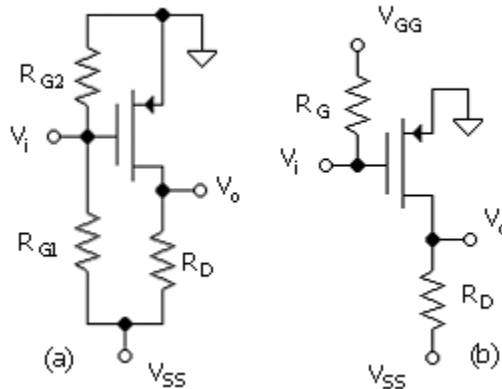
Two types of common-source amplifiers will be investigated in lab projects. One is with the source grounded and the other is with a current-source bias (dual power supply). In [Units 5.1](#) and [5.2](#) we discuss various aspects of the common-source stage with grounded source, in [Unit 5.3](#) we take up circuit-linearity considerations, and in [Unit 5.4](#) we cover the basics of the dual-power-supply amplifier. Both amplifiers are based on the PMOS, as in the projects. The first two units are mostly a review of the basic amplifier as presented in previous units, to reinforce the basic concepts. The PMOS replaces the NMOS ([Units 2](#) and [4](#)) in this unit, to provide familiarity with the opposite polarity in bias considerations and to illustrate that the linear model applies in the same manner for both transistor types.

5.1. DC (Bias) Circuit

Dc circuits for the grounded-source amplifier are shown in [Fig. 5.1](#) (PMOS). The circuit in (a) is based on a single power supply, and the gate bias is obtained with a resistor voltage-divider network. The circuit in (b) is for a laboratory project amplifier. Both V_{GG} and V_{SS} are negative, since the source is at ground. There is no voltage drop across R_G since there is negligible gate current. R_G is necessary only to prevent shorting the input signal, V_i . The bias current I_D for a given applied V_{SG} will respond according to (3.8), which is

$$I_D = k_p (V_{SG} - V_{tp0})^2 (1 + \lambda_p V_{SD})$$

Figure 5.1. Basic PMOS common-source amplifiers. Single-power-supply amplifier (a) and laboratory amplifier (b) with $V_{SG} (= V_{GG})$ and V_{SS} controlled by DAQ output channels. Note that either end of the circuit of (a) can be at ground.

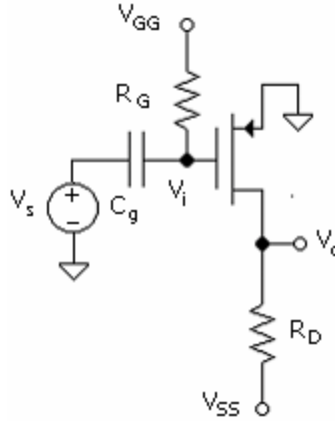


The two circuits are equivalent, as V_{GG} and R_G of [Fig. 5.1b](#) are the Thévenin equivalent of the bias network of the [Fig. 5.1\(a\)](#). In the project on the amplifier, they are actually a voltage and a resistor. This is not a bias-stable circuit, as a slight change in V_{SG} or the transistor parameters can result in a significant change in I_D . The dual-power-supply circuit of [Unit 5.4](#) is considerably better in this respect.

5.2. Amplifier Voltage Gain

This dc (bias) circuit becomes an amplifier now simply by adding a signal source at the gate as in [Fig. 5.2](#). This requires a coupling capacitor, as shown here in the complete circuit, to prevent disturbing the bias upon connecting the input signal to the circuit.

Figure 5.2. A signal source is connected to the gate through a coupling capacitor. The capacitor is necessary to isolate the dc circuit from the signal source.



In the amplifier of [Project 5](#), the signal will be superimposed on the bias voltage at the node of V_{GG} . This can be facilitated with LabVIEW and the DAQ. A capacitor, as in an actual amplifier, is therefore not required. The requirement for having LabVIEW control over both V_{GG} and V_{SS} , and the limitation of two output channels, dictates this configuration.

In [Project 5](#) we measure the gain as a function of bias current, I_D . For a SPICE comparison, we need an expression for the gain. For the ideal case, which neglects the output conductance, g_{ds} , the output current is related to the input voltage by [\(4.1\)](#), which is

$$I_d = g_m V_{gs} = g_m V_i$$

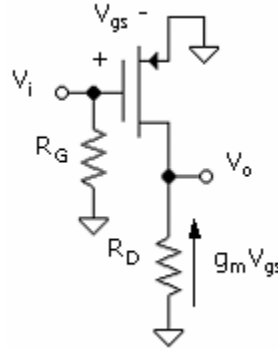
The output signal voltage is, in general,

Equation 5.1

$$V_{ds} = V_o = -I_d R_D$$

The convention used here for subscript order for signal (linear) variables is common to the NMOS and PMOS. This is consistent with the fact that the linear model does not distinguish between the two types. Thus, for example, the dc terminal voltage for a PMOS is V_{SG} , but the signal equivalent is V_{gs} ([Fig. 5.3](#)) and the signal input voltage is positive at the input terminal (common-source, gate input). For the PMOS, i_D is defined as positive out of the drain, but the signal output current is into the drain (as in the NMOS). We note that a positive V_{gs} ($V_{gs} = -V_{sg}$) corresponds to a decrease in the total gate – source voltage, v_{SG} , which is consistent with a decrease of i_D and positive I_d .

Figure 5.3. Signal-equivalent version of the amplifier stage. Dc nodes are set to zero volts (circuit reference). The reactance of C_g is assumed to be zero.



Thus, the negative sign in (5.1) is consistent with the flow of current I_d up through the resistor (Fig. 5.3) for positive $V_i = V_{gs}$. The common-source stage is an inverting amplifier and has an inherent 180° phase shift. From (4.1) and (5.1), the gain is

Equation 5.2

$$a_v = \frac{V_o}{V_i} = -g_m R_D$$

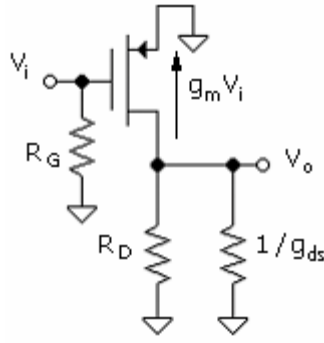
where both $V_i = V_{gs}$ and $V_o = V_{ds}$ are with respect to ground or the source terminal for the common-source stage.

If the output resistance, $1/g_{ds}$, cannot be neglected (which is the case for the project on PMOS amplifiers), the transistor current, $g_m V_i$, is shared between the output resistance and R_D . The portion that flows through R_D is (Fig. 5.4)

Equation 5.3

$$I_{R_D} = g_m V_i \frac{1}{1 + g_{ds} R_D}$$

Figure 5.4. Common-source amplifier stage signal circuit, with all dc nodes set to zero volts. The transistor model includes output resistance $1/g_{ds}$, which appears directly in parallel with R_D with the source grounded.



Note again that the signal schematic transistor represents a current source with value $g_m V_i$, as established in connection with [Fig. 4.1](#). The additional feature of the transistor model is included with the addition of $1/g_{ds}$. This resistance is actually part of the transistor and is between the drain and source of the transistor, but the circuit as given is equivalent, as the source is at ground. Since the output voltage is $V_o = -I_{RD} R_D$, the new gain result is

Equation 5.4

$$a_v = -g_m \frac{R_D}{1 + g_{ds} R_D}$$

Note that this form evolves from ideal transistor current, $g_m V_{gs}$, flowing through the parallel combination of the output resistance and R_D .

To facilitate an intuitive grasp of the magnitude of the effect of g_{ds} , we use the expression for g_{ds} ([4.13](#)) in ([5.4](#)), to obtain

Equation 5.5

$$a_v = -g_m \frac{R_D}{1 + \lambda_p I_D R_D}$$

Note that $I_D R_D$ is the voltage drop across R_D . For example, for a -10-V power supply, we choose $I_D R_D \approx 5\text{ V}$. A measurement of λ_p for our devices will show that $\lambda_p \approx 1/20\text{ V}$, which results in $\lambda_p I_D R_D \approx 1/4$. Thus, the effect of g_{ds} ($= \lambda_p I_D$) for this case is significant.

Finally, we can get an overall current dependence for a_v with the elimination of g_m , using ([4.5](#)) with $k'_p = k_{p'} \approx k_p$, which results in

Equation 5.6

$$a_{vO} = -2\sqrt{k_p I_D} \frac{R_D}{1 + \lambda_p I_D R_D}$$

Using an alternative form for $g_m (= 2I_D/V_{effp})$, also (4.5), the gain expression is

Equation 5.7

$$a_v = -2 \frac{I_D}{V_{effp}} \frac{R_D}{1 + \lambda_p I_D R_D}$$

where

$$V_{effp} = \sqrt{\frac{I_D}{k_p (1 + \lambda_p V_{SD})}} \approx \sqrt{\frac{I_D}{k_p}}$$

For simplicity, approximate forms of (4.5) and (4.13) of g_m and g_{ds} are used here, which are independent of V_{SD} . For reference, the "exact" and approximate forms of (4.5) and (4.13), respectively, are repeated here:

$$g_m = 2\sqrt{k_p (1 + \lambda_p V_{SD}) I_D} \approx 2\sqrt{k_p I_D}$$

and

$$g_{ds} = \lambda_p \frac{I_D}{1 + \lambda_p V_{SD}} \approx \lambda_p I_D$$

The "exact" equations of g_m and g_{ds} are used in conjunction with the amplifier projects to compare the computed gain with the measured gain plotted against I_D . This is done in both LabVIEW and Mathcad. Parameters k_p and V_{tp0} (to get V_{effp}) will be extracted from the measured dc data, and λ_p will be used as an adjustable parameter to fit the SPICE and measured gain data.

5.3. Linearity of the Gain of the Common-Source Amplifier

The connection between I_d and V_{gs} is linear provided that V_{gs} is small enough, as considered in the following units. Use of the linear relations also assumes that the output signal remains in the active region (i.e., neither in the linear region nor near cutoff). This is discussed below. NMOS subscripts are used. The results are the same for the PMOS, with a "p" subscript substituted for "n" and the subscript order reversed for all bias-voltage variables.

5.3.1. Nonlinearity Referred to the Input

The general equation again is (3.8)

$$i_D = k'_n (v_{GS} - V_{tno})^2$$

Then using $I_d = i_D - I_D$ and $v_{GS} = V_{GS} + V_{gs}$, the equation for the incremental drain current becomes

Equation 5.8

$$I_d = k'_n [2(V_{GS} - V_{tno})V_{gs} + V_{gs}^2]$$

which leads to a nonlinear (variable) transconductance, g'_m , given by

Equation 5.9

$$g'_m = \frac{I_d}{V_{gs}} = \frac{k'_n [2V_{effn}V_{gs} + V_{gs}^2]}{V_{gs}} = g_m \left(1 \pm \frac{|V_{gs}|}{2V_{effn}} \right)$$

Therefore, the condition for linearity is that $V_{gs} \ll 2V_{eff}$, with $V_{effn} = V_{GS} - V_{tno}$ and using

With this condition not satisfied, an output signal is distorted. However, for the purpose of measuring the amplifier gain, our signal voltmeter will take the average of the positive and negative peaks, which is

Equation 5.10

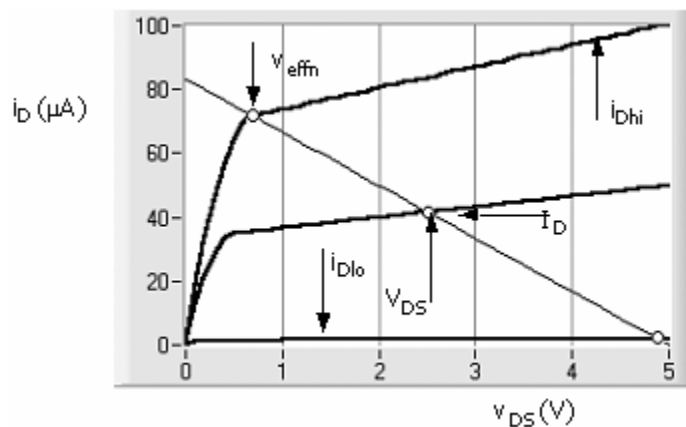
$$I_{davg} = \frac{k'_n (2V_{eff}V_{gs} + V_{gs}^2) + k'_n (2V_{eff}V_{gs} - V_{gs}^2)}{2}$$

In the parabolic relationship, the squared terms cancel entirely. In general, though, the output signal contains harmonic content (distortion) when V_{gs} is too large compared to V_{effn} .

5.3.2. Nonlinearity Referred to the Output

The discussion above of limits imposed on V_{gs} assumes that the transistor remains in the active mode. To clarify this point, reference is made to the output characteristics of [Fig. 5.5](#). The graph has plots of the output characteristic for three values of v_{GS} in addition to the load line. The characteristic plot in the midrange is for no signal. Operating point variables are $V_{DS} \approx 2.5$ V and $I_D \approx 40$ μ A. With a large, positive V_{gs} , the characteristic moves up to the high-level plot (i_{Dhi}) and the opposite occurs for a large but negative V_{gs} (i_{Dlo}). The high-level plot is shown for when the transistor is about to move out of the active region and into the linear region. Attempts to force v_{DS} to lower values will create considerable distortion in the output signal voltage. The lower curve suggests that the positive output signal is on the verge of being cut off (clipped) for an additional increase in the negative-input signal voltage.

Figure 5.5. Common-source amplifier stage output characteristics. Output characteristics are from top to bottom, large high-current signal swing, i_{Dhi} , dc bias, I_D , low-current signal swing, i_{Dlo} . Also shown is the load line. The current – voltage circuit solution is always the intersection between a given characteristic and the load line.



According to the discussion above, the negative signal output voltage is limited to

Equation 5.11

$$V_{ds\text{minus}} = V_{DS} - V_{\text{effn}}$$

Technically, V_{effn} is from the high-current signal state, but for simplicity, a reasonable estimate can be made from the dc case; that is, $V_{\text{effn}} = V_{GS} - V_{\text{tno}}$. The positive signal limit is

Equation 5.12

$$V_{ds\text{plus}} = V_{DD} - V_{DS} = I_D R_D$$

The actual output-signal limit is dictated by the smaller of the two for a symmetrical periodic signal such as a sine-wave. In the example shown in [Fig. 5.5](#), $V_{\text{effn}} \approx 0.5$ V, $V_{DS} \approx 2.5$ V, and $V_{DD} = 5$ V. The plus and minus signal-voltage limits are about 2.5 V and 2.0 V, respectively. Depending on the dc bias, the limit could be dictated by one or the other. In the amplifier projects, the gain will typically be measured over a range of dc bias current for a fixed resistor. This means that for the low-current end of the scan, the signal will be limited by the magnitude of $I_D R_D$ and, by design, the plus and minus swings will be made to be about equal at the highest dc current.

Distortion associated with the nonlinear $I_d - V_{gs}$ relation and that due to signal limits at the output may be taking place simultaneously. This is seen from the gain expression ([5.7](#)) ($g_{ds} = 0$)

$$|a_v| \approx 2 \frac{V_{R_D}}{V_{\text{effp}}}$$

where $a_v \equiv V_{ds}/V_{gs}$ and where the approximation is for the case of neglecting the λ_n factor. Thus, for a given V_{ds} , V_{gs} is

Equation 5.13

$$V_{gs} = \frac{V_{\text{effp}}}{2V_{R_D}} V_{ds}$$

If, for example, V_{ds} is pushed to the positive output-signal limit, then $V_{ds} = V_{RD}$. According to (5.13), $V_{gs} = V_{effp}/2$, and V_{gs} exceeds the condition for a linear $I_d - V_{gs}$ relation as given in (5.9),

$$g'_m = \frac{I_d}{V_{gs}} = g_m \left(1 \pm \frac{|V_{gs}|}{2V_{effm}} \right)$$

5.4. Current-Source Common-Source Amplifier: Common-Source Amplifier with a Source Resistor

The bias circuit of the current-source bias amplifier, shown in Fig. 5.6, has a dual power supply. One advantage of this is that the input is at zero dc volts such that the signal can be connected directly without interfering with the bias. The dc circuit equation for setting up the bias is

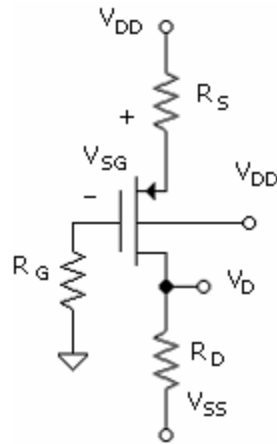
Equation 5.14

$$I_D = \frac{V_{DD} - V_{SG}}{R_S}$$

where $(k'_p \approx k_p) V_{SG} \approx \sqrt{I_D/k_p} + V_{tp}$.

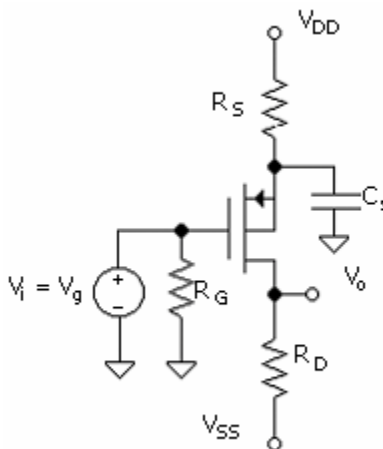
This circuit is more bias stable than the grounded source amplifier, as slight changes in V_{SG} (due to device parameter variations or temperature) are usually small compared to V_{DD} . Note that V_{tp} is used in lieu of V_{tp0} as $V_{BS} \neq 0$. The chip (CD4007) used in the projects is a p-well device (as noted in Unit 3), with the NMOS transistors in the well. The well is connected to V_{SS} , while the body of the chip is connected, as in Fig. 5.6, to V_{DD} . The pn junction formed by the well and the bulk is thus reverse-biased with a voltage $|V_{SS}| + V_{DD}$.

Figure 5.6. Dc circuit of the dual-power-supply common-source amplifier. The gate is at ground potential, allowing the signal to be connected directly to the gate. R_G is necessary only to prevent shorting out the input signal.



In the amplifier projects, however, we have the latitude to connect the body and source as there is only one transistor in the circuit and the body can float along with the source. Thus we can assume that $V_{tp} = V_{tp0}$. As shown in [Fig. 5.7](#), the signal circuit requires the addition of a bypass capacitor, C_s . This places the source at signal ground provided that the capacitor is large enough. The criterion for this is discussed in [Unit 6](#). The voltage-gain equation is the same as in the amplifier, with the source actually grounded.

Figure 5.7. Amplifier circuit with a bypass capacitor attached between the source and ground to tie the source to signal ground. Signal input is attached directly to the gate. Body and source are connected internally in the project chip for the transistor used in the amplifier.



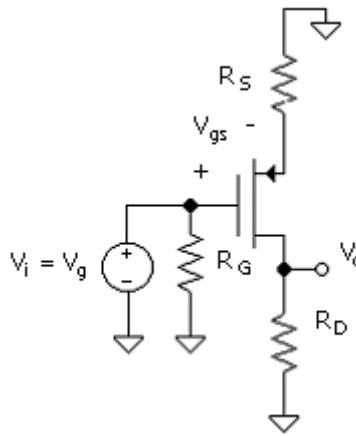
Without the bypass capacitor, R_S is in the signal circuit and a fraction of the applied signal voltage at the gate is dropped across the resistor. The signal circuit for this case is

shown in [Fig. 5.8](#). The circuit transconductance of the amplifier with R_S was discussed initially in [Unit 4](#). This is reviewed in the following.

An applied input signal, $V_i = V_g$, divides between the gate – source terminals and the source resistor according to [\[\(4.6\)\]](#)

$$V_g = V_{gs} + I_d R_S$$

Figure 5.8. Signal circuit for dual-power supply common-source amplifier. Input signal voltage, V_i , is divided between V_{gs} , the control voltage, and the source resistor according to the ratio 1: $g_m R_S$.



When combining this with $I_d = g_m V_{gs}$, we obtain [\[\(4.7\)\]](#)

$$V_g = V_{gs} + g_m V_{gs} R_S = (1 + g_m R_S) V_{gs} = (1 + g_m R_S) \frac{I_d}{g_m}$$

The circuit transconductance, G_m , is then [\[\(4.8\)\]](#)

$$G_m = \frac{I_d}{V_g} = \frac{g_m}{1 + g_m R_S}$$

The gain for this case is thus (neglecting g_{ds})

Equation 5.15

$$a_v = \frac{V_d}{V_g} = -G_m R_D = -\frac{g_m R_D}{1 + g_m R_S}$$

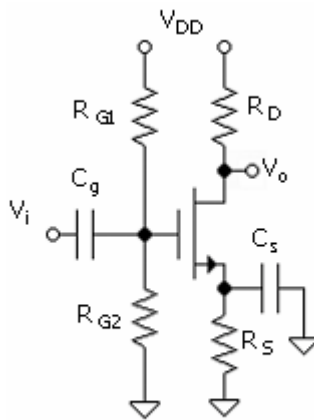
In one of the amplifier projects, $R_S = R_D$, and the gain without the bypass capacitor is actually less than unity.

5.5. Design of a Basic Common-Source Amplifier

Unlike in the laboratory environment, an actual practical common-source amplifier would have a single power supply for the base and collector circuit bias. Also, the circuit design requires a tolerance to a wide range of parameter variation, including that due to temperature change. In this unit, the design process for a possible common-source amplifier is discussed. Emphasis is on dc bias stability, that is, on tolerance to device parameter and circuit component variations.

The common-source amplifier to be designed is shown in [Fig. 5.9](#). Source resistor, R_S , is included for bias (and gain) stabilization. The goal is for the circuit to function properly for any NMOS transistor, which has device parameters k_n and V_{tn0} that fall into a wide range of values, as is normally expected. Tolerance to component variation, such as resistor values, could also be built into the design.

Figure 5.9. NMOS common-source amplifier with R_S for bias and gain stabilization. Gate bias is provided by a voltage-divider network consisting of R_{G1} and R_{G2} . The body and source terminals are connected.



Gate voltage V_G is provided by the voltage divider, consisting of resistors R_{G1} and R_{G2} . Since there is no gate current, the gate bias voltage is [(1.2)]

$$V_G = \frac{R_{G2}}{R_{G2} + R_{G1}} V_{DD}$$

Voltage V_G is thus relatively stable and can be considered constant. Once V_G has been established, the drain current will be dictated by

Equation 5.16

$$I_D = \frac{V_G - V_{GS}}{R_S}$$

Since the gate – source voltage is given by

Equation 5.17

$$V_{GS} = \sqrt{\frac{I_D}{k_n}} + V_{tno}$$

the drain current, I_D , may be expressed in terms of the device parameters as

Equation 5.18

$$I_D = \frac{V_G - \sqrt{\frac{I_D}{k_n}} - V_{tno}}{R_S}$$

This result reveals the dependence of I_D on the magnitudes of k_n and V_{tno} . (Again, for simplicity, as in the amplifier projects, we will assume that the body and source are connected such that $V_{tn} = V_{tno}$.)

Bias current I_D is assumed to be a given. The initial design then is conducted for the NMOS nominal, average values for k_n and V_{tno} . Any combination of V_G and R_S that satisfies (5.18) will provide the design I_D . Specific values for V_G and R_S will be dictated by stability requirements. Suppose that k_n is expected to fall within $k_{no} \pm \delta k_n$ and V_{tno} within $V_{tnoo} \pm \delta V_{tno}$, where k_{no} and V_{tnoo} are the nominal values of the original design. Assume that the design bias current associated with k_{no} and V_{tnoo} is I_{Do} . At the extremes for the parameters, the low and high currents will be

Equation 5.19

$$I_{Dlo,Dhi} = \frac{V_G - \sqrt{\frac{I_D}{k_{no} \mp \delta k_n}} - (V_{tno0} \pm \delta V_{tno})}{R_S}$$

Resistor R_S , for the given V_G and design drain current, is

Equation 5.20

$$R_S = \frac{V_G - V_{GS0}}{I_{D0}}$$

V_{GS0} is obtained from (5.17), using the nominal parameter values. The low and high current limits tend to converge on V_G/R_S as V_G becomes large. That is, in the limit, V_G dominates the voltages in the numerator of (5.19), thus rendering the expression insensitive to the minor contributions from changes in k_n and V_{tno} .

An important design consideration is drain – source voltage, V_{DS} , as this dictates the output signal range. This is calculated from

Equation 5.21

$$V_{DS} = V_{DD} - I_D(R_D + R_S)$$

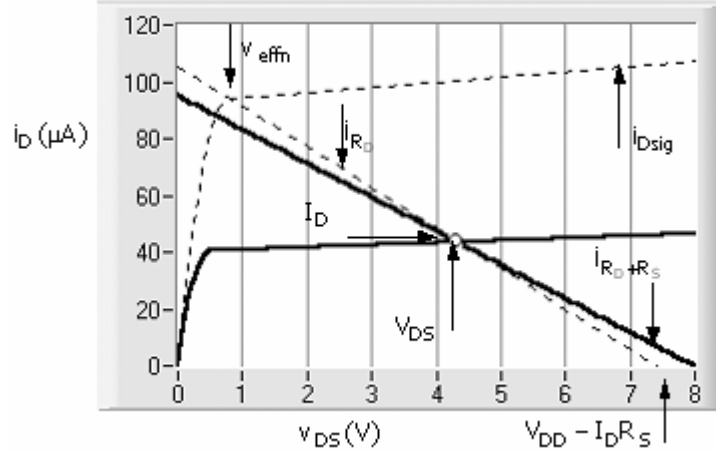
In the design of the amplifier, drain resistor R_D is normally selected for equal positive and negative peak-signal maximums. This configuration is illustrated in Fig. 5.10, which shows the output characteristic of the transistor in the circuit. The signal is limited by $V_{DD} - V_{RS}$ and $V_{effno} = V_{GS0} - V_{tno}$ at the high and low ends of the voltage range, respectively. Therefore, nominal bias should be set at

Equation 5.22

$$V_{DS0} = \frac{V_{DD} - I_{D0}R_S + V_{effno}}{2}$$

Figure 5.10. Output characteristic of the transistor of the amplifier with bias V_{DS} set approximately according to (5.22). The signal is restricted within the range

$V_{DD} - V_{RS}$ and approximately v_{effn} . The characteristic curves are for no signal (solid plot) and for the signal at a maximum (dashed plot), as limited by the transistor going into the inactive (linear) region. The load lines are dc (solid line) and signal (ac, dashed line).



For simplicity, it is assumed that $v_{effn} \approx V_{effno}$. V_{effno} , V_{DDo} , and I_{Do} are the bias values at the nominal parameter values. The bias drain voltage is V_{DSo} plus the drop across R_S , that is,

Equation 5.23

$$V_{Do} = V_{DSo} + I_{Do}R_S$$

Knowing V_{Do} then provides for the calculation of R_D from

Equation 5.24

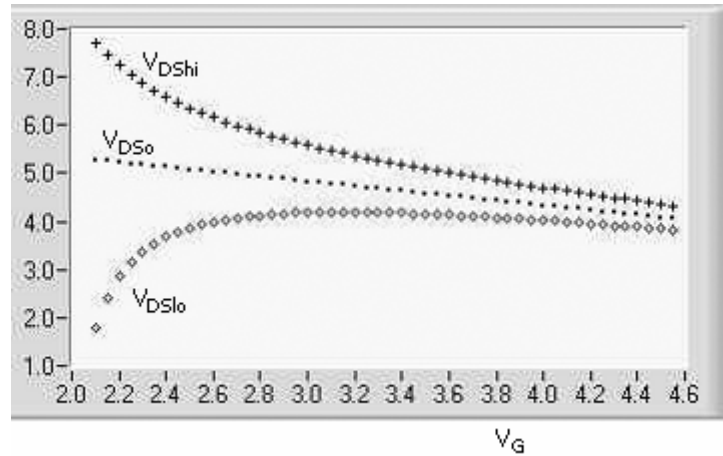
$$R_D = \frac{V_{DD} - V_{Do}}{I_{Do}}$$

where V_{Do} and I_{Do} are for the initial design with k_{no} and V_{tno} .

An optimization design sequence plots the limits for a range of V_G and for specified δV_{tno} and δk_n . An example is shown in [Fig. 5.11](#). The plot of V_{DSo} corresponds to the nominal k_{no} and V_{tno} . The curve slopes downward as $I_S R_S$ increases for increasing V_G at constant nominal bias current, I_{Do} . V_{DShi} is for the combination of δV_{tno} and δk_n , which gives the

maximum positive deviation from the nominal, and V_{DSlo} is the opposite. The example of [Fig. 5.11](#) is for $V_{DD} = 10$ V and design bias current of $I_{D0} = 100$ μ A and nominal parameters $k_{no} = 300$ μ A/V², $V_{tnoo} = 1.5$ V, $\delta V_{tno} = 0.1$ V, and $\delta k_n = 100$ μ A/V². Experience with the CMOS chip of our amplifier project ([Project 7](#)) indicates that these are representative.

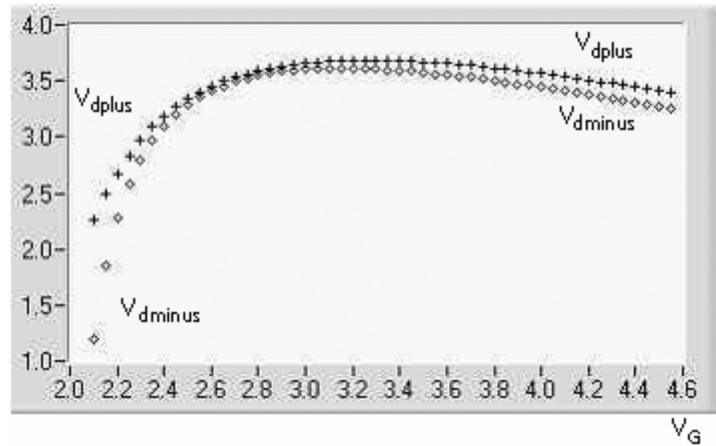
Figure 5.11. Computed high and low range of V_{DS} as a function of gate-bias voltage V_G . The computation is with $k_{no} = 300$ μ A/V², $V_{tnoo} = 1.5$ V, $\delta V_{tno} = 0.1$ V, and $\delta k_n = 100$ μ A/V².



[Figure 5.12](#) shows plots of the computed positive and negative signal-peak limits. Due to the increasing V_{RS} with increasing V_G , the signal range decreases, as shown by the plots. Thus, the signal-peak limits have a maximum, as is evident in the graph. The design of the amplifier uses V_G at the maximum of the lower curve. The value of V_G is consistent

with the maximum V_{DSlo} in the plot of [Fig. 5.11](#). In the example, $V_G = 3$ V.

Figure 5.12. Computed maximums for negative and positive output voltage signal peaks as a function of V_G : V_{dplus} , positive maximum; V_{dminus} , negative maximum.



Once V_G is determined, the selection of R_{G1} is made from (1.2), which is

$$V_G = \frac{R_{G2}}{R_{G2} + R_{G1}} V_{DD} = \frac{R_G}{R_{G1}} V_{DD}$$

where R_G is the parallel combination

$$R_G = \frac{R_{G2} R_{G1}}{R_{G2} + R_{G1}}$$

R_G can be selected somewhat arbitrarily but could be dictated by the coupling capacitor, C_G , requirement. Associated with the coupling capacitor is the 3 - dB frequency (6.2), which is

$$f_{3dB} = \frac{1}{2\pi R_G C_G}$$

R_{G2} is then calculated from

$$R_{G2} = \frac{R_{G1} R_G}{R_{G1} - R_G}$$

The gain equations for the circuit of [Fig. 5.9](#), with and without a bypass capacitor, are (5.2) and (5.15), respectively. These are

$$a_v = -g_m R_D$$

and

$$a_v = -\frac{g_m R_D}{1 + g_m R_S}$$

In the design procedure outlined in this unit, emphasis is on stability and the gain falls out. This would typically be the case for this type of amplifier. We note that due to the characteristically small g_m of MOSFETs, the voltage gain is relatively small. Gain can be improved considerably through the use a current-source load, as in the amplifier of [Unit 10](#).

5.6. Summary of Equations

$a_v = -g_m R_D$	Common-source amplifier-stage voltage gain.
$a_v = -g_m \frac{R_D}{1 + \lambda_p I_D R_D}$	Common-source amplifier gain including output resistance, PMOS. (Same for NMOS with λ_n .)
$g'_m = g_m \left(1 \pm \frac{ V_{gs} }{2V_{effn}} \right)$	Nonlinear transconductance for large input signals. (Same for PMOS with V_{effp} .)
$V_{dsmin} = V_{DS} - V_{effn}$	Negative output signal limit, NMOS.
$V_{dsmin} = V_{SD} - V_{effn}$	Negative output signal limit, PMOS.
$V_{dsplus} = V_{DD} - V_{DS} = I_D R_D$	Positive output signal limit, NMOS.
$V_{dsplus} = V_{SS} - V_{SD} = I_D R_D$	Positive output signal limit, PMOS.
$a_v = -\frac{g_m R_D}{1 + g_m R_S}$	Voltage gain of common-source stage with source resistor.
$G_m = \frac{g_m}{1 + g_m R_S}$	Circuit transresistance of common-source stage with source resistor.

5.7. Exercises and Projects

Project Mathcad Files

Exercise05.mcd - Project05.mcd

Laboratory Project 5

[PMOS Common-Source Amplifier](#)

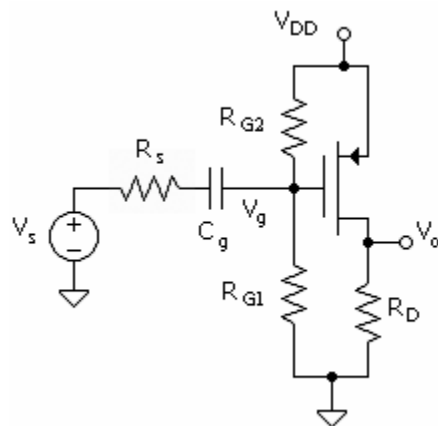
Unit 6. Coupling and Bypass Capacitors and Frequency Response

As noted in [Unit 1](#), due to the external capacitors that are attached to the circuit for configuring the signal circuit, the gain of the amplifiers will tend to fall off at the low-frequency end of the frequency spectrum. The design basis for choosing the capacitors for a given application is considered in the following. The gain-frequency dependence due to the input coupling capacitor, the source-resistor bypass capacitor, and the load coupling capacitor is discussed. A discussion of response roll-off on the high end of the spectrum is deferred to [Unit 11](#).

6.1. Grounded-Source Amplifier: Coupling Capacitor

A version of the circuits of [Fig. 5.2](#), as given in [Fig. 6.1](#), is generalized to include a resistance, R_s , as part of the input signal source. The input signal at the gate (and thus the output) will fall off for decreasing frequencies as the magnitude of reactance of the capacitor increases. The value of the capacitor is selected such that the reactance will be small compared to the value of the sum of the resistors, at the lowest operating frequency.

Figure 6.1. Circuit with coupling capacitor. Capacitor blocks the signal at low frequencies. C_g and $R_G = R_{G1} \parallel R_{G2}$ must be selected for $|X_{C_g}| \ll R_s + R_G$ at the gain measurement frequency.



In this amplifier, all the gain-frequency dependence is between V_g and V_s . The relation between the gate signal, V_g , and source signal, V_s , based on a voltage-divider relation is

Equation 6.1

$$V_g = \frac{R_G}{R_G + R_s + \frac{1}{j2\pi f C_g}} V_s = \frac{R_G}{R_G + R_s} \frac{1}{1 - \frac{jf_g}{f}} V_s$$

with

Equation 6.2

$$f_g = f_{3dB} = \frac{1}{2\pi (R_G + R_s) C_g}$$

where $R_G = R_{G1} \parallel R_{G2}$.

With R_s included, the f_{3dB} expression applies to an actual amplifier. In [Project 6](#), the signal-source voltage is applied directly to the gate. We select a combination of R_G and C_g to obtain a suitably low f_{3dB} . This must be more than a factor of 10 lower than the gain measurement frequency assure that the capacitor is not influencing the measurements.

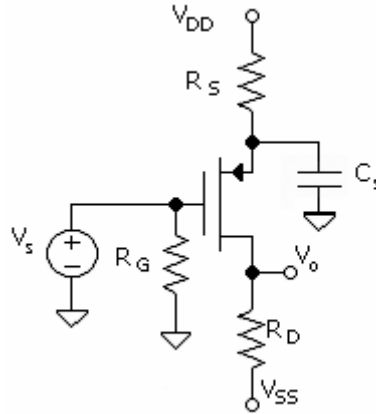
In Projects 5 and 7, no capacitor is used at the input as the signal is superimposed on the dc gate voltage. This is to facilitate the need for both DAQ output channels to provide the bias. However, [Project 6](#), one channel provides the bias, as in [Figs. 5.1\(a\)](#) and [6.1](#), and the other channel is used for the signal with connection to the input through a coupling capacitor.

6.2. Current-Source Bias Amplifier: Bypass Capacitor

The PMOS common-source amplifier circuit with the bypass capacitor is shown in [Fig. 6.2](#). At some low frequency, the effect of the capacitor is absent, and the gain is computed from [\(5.15\)](#), which is

$$a_v = -\frac{g_m R_D}{1 + g_m R_s}$$

Figure 6.2. Dual-power-supply common-source amplifier stage with bypass capacitor. The object is to determine the frequency for which the source is effectively at signal ground.



Over a range of frequencies, the gain evolves from (5.15) to (5.2), $a_v = -g_m R_D$ (neglecting the output resistance of the PMOS). The frequency-dependent transition region is determined by replacing R_S in (5.15) with the impedance of R_S in parallel with the reactance of C_s . This is

Equation 6.3

$$a_v(f) = - \frac{g_m R_D}{1 + g_m \frac{R_S}{1 + j2\pi f R_S C_s}}$$

or

Equation 6.4

$$a_v(f) = - \frac{g_m R_D \left(1 + j \frac{f}{f_z}\right)}{1 + j \frac{f}{f_z} + g_m R_S} = - \frac{g_m R_D}{1 + g_m R_S} \frac{1 + j \frac{f}{f_z}}{1 + j \frac{f}{f_z}}$$

with

Equation 6.5

$$f_z = \frac{1}{2\pi R_S C_S}$$

and

Equation 6.6

$$f_s = \frac{1}{2\pi \frac{R_S}{1 + g_m R_S} C_S} = (1 + g_m R_S) f_z$$

Note that the "RC" factor has an "R," which is R_S in parallel with $1/g_m$, the latter being the output resistance looking into the source of the transistor. The two frequencies f_s and f_z are technically the pole and zero of the response function.

We note that $g_m R_S = 2I_D R_S / V_{eff}$. Since V_{eff} could be as low as $V_{eff} \approx 0.2$ V, then $g_m R_S \gg 1$ in some cases. The corner frequency, f_{3dB} , occurs at

Equation 6.7

$$a_{v3dB} = \frac{g_m R_D}{\sqrt{2}} = \frac{g_m R_D}{1 + g_m R_S} \frac{\sqrt{1 + \left(\frac{f_{3dB}}{f_z}\right)^2}}{\sqrt{1 + \left(\frac{f_{3dB}}{f_s}\right)^2}} = g_m R_D \frac{\sqrt{f_z^2 + f_{3dB}^2}}{\sqrt{f_s^2 + f_{3dB}^2}}$$

which gives

Equation 6.8

$$f_{3dB} = \sqrt{f_s^2 - 2f_z^2} = \sqrt{(1 + g_m R_S)^2 f_z^2 - 2f_z^2} = \sqrt{(1 + g_m R_S)^2 - 2} \cdot f_z \approx f_s$$

such that

Equation 6.9

$$f_{3dB} \approx f_s = \frac{1}{2\pi \frac{R_s}{1 + g_m R_s} C_s} \approx \frac{g_m}{2\pi C_s}$$

where the far-right-hand term applies for $g_m R_s \gg 1$. This is typically only marginally satisfied in MOSFET circuits.

The derivation carried out here was initiated from (5.15), which neglects the output resistance of the transistor. In the case of the MOSFET devices of our projects, the simplification is valid for the NMOS transistor but marginal for the PMOS transistor. This is because $\lambda_p \gg \lambda_n$. The result (6.9) still serves to estimate the required value for C_s , even for the case of the PMOS. Nevertheless, a more detailed derivation is carried out in the next unit. This permits comparisons, in a project, of SPICE and Mathcad solutions with amplifier gains and frequency response.

6.3. Precision Formulation of the Low-Frequency Gain and Characteristic Frequencies

In Unit 8, the gain of a common-source stage with source resistor, which includes the effect of the output resistance, is shown to be [(8.36)]

$$a_{volo} = \frac{-g_m R_D}{1 + g_{ds} R_D + (g_m + g_{ds}) R_s}$$

This is designated here as a_{volo} to emphasize that it is the constant low-frequency asymptotic gain. If the source-branch impedance is substituted for R_s , this becomes

Equation 6.10

$$a_v(f) = \frac{-g_m R_D}{1 + g_{ds} R_D + (g_m + g_{ds}) \frac{R_s}{1 + jf/f_z}}$$

where f_z is (6.5). This can be rearranged in the form of (6.4), which is

Equation 6.11

$$a_v(f) = -a_{v0} \frac{1 + j \frac{f}{f_z}}{1 + j \frac{f}{f_s}}$$

where the new pole frequency is

Equation 6.12

$$f_s = \left[1 + \frac{(g_m + g_{ds}) R_S}{1 + g_{ds} R_D} \right] f_z$$

and a_{v0} is (8.36).

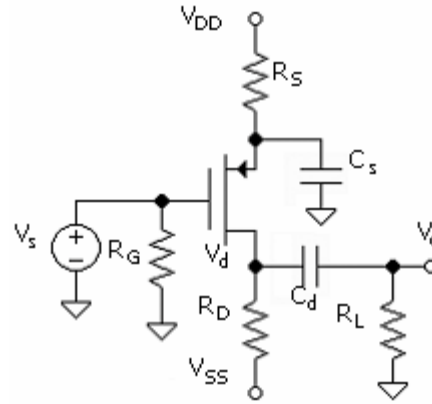
6.4. Load Coupling Capacitor

The complete practical amplifier includes a load, R_L , at the output. This requires an additional coupling capacitor. The amplifier with an attached load is shown in [Fig. 6.3](#). We will assume that the effect of the output resistance, r_{ds} , can be neglected. The transistor appears as a current source of magnitude I_d . The equivalent signal circuit at the output can be represented as shown in [Fig. 6.4\(a\)](#) and (b). In [Figure 6.4\(b\)](#), the current source and drain resistor are replaced with a voltage-source equivalent. From [Fig. 6.4\(b\)](#), the output voltage is

Equation 6.13

$$V_o(f) = \frac{R_L}{R_D + R_L + 1/j\omega C_d} I_d R_D$$

Figure 6.3. Amplifier with capacitively coupled load resistor, R_L . The capacitor must be large enough not to attenuate the output between the drain and load resistor.



The expression with the high-frequency asymptotic value and frequency dependence is

Equation 6.14

$$V_o(f) = \frac{R_D R_L}{R_D + R_L} I_d \frac{1}{1 + 1/j2\pi f C_d (R_D + R_L)}$$

which is

Equation 6.15

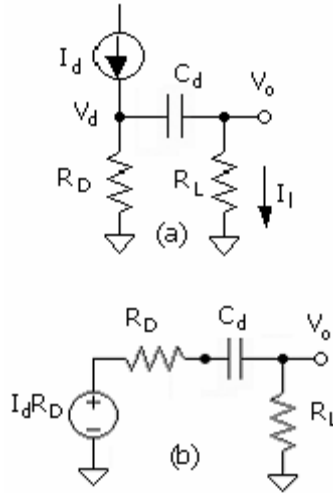
$$V_o(f) = \frac{R_L R_D}{R_D + R_L} I_d \frac{1}{1 + f_d / jf}$$

where

Equation 6.16

$$f_d = \frac{1}{2\pi (R_D + R_L) C_d}$$

Figure 6.4. (a) Equivalent circuit consisting of a current source and the load components. (b) Conversion of current source to voltage source.



Current source I_d is not dependent on frequency (at the low end of the spectrum) when no other capacitors are considered. However, in general, with both C_d and C_s attached, the combined response is

Equation 6.17

$$a_v(f) = -\frac{g_m R_D'}{1 + g_m R_S} \frac{1 + j \frac{f}{f_z}}{\left(1 + j \frac{f}{f_s}\right) \left(1 + j \frac{f}{f_d}\right)}$$

where f_d is (6.16), f_z is (6.5), f_s is (6.6), and R_D' is R_D in parallel with R_L . With $C_d = C_s$, usually $f_d \ll f_s$ such that (6.9) is the approximate f_{3dB} for the circuit with both capacitors connected.

Combining the frequency-dependent terms to obtain (6.17) assumes that current source I_d is independent of components C_d , R_D , and R_L . This is a correct assumption as long as the output resistance of the transistor is neglected. When better precision is required, the derivation starts with the general form of the circuit transconductance, as deducible from (8.36), and replaces both R_D and R_S with the impedance in those respective branches, Z_D and Z_S . This gives

Equation 6.18

$$G_m(f) = \frac{-g_m}{1 + g_{ds} Z_D + (g_m + g_{ds}) Z_S}$$

The gain follows from multiplying the transconductance by the load at the drain, Z_D , and multiplying by the voltage divider, $V_o/V_d = R_L/(R_L + 1/j2\pi f C_d)$. The result is

Equation 6.19

$$a_v(f) = \frac{-g_m R'_D}{1 + g_{ds} Z_D + (g_m + g_{ds}) Z_S} \frac{1}{1 + \frac{f_d}{jf}}$$

6.5. Summary of Equations

$f_g = f_{3dB} = \frac{1}{2\pi(R_G + R_S)C_g}$	MOSFET coupling capacitor corner-frequency equation.
$f_z = \frac{1}{2\pi R_S C_S}$ $f_p = \frac{1}{2\pi \frac{R_S}{1+g_m R_S} C_S} = (1+g_m R_S) f_z$	Zero and pole equations for bypass capacitor response function.
$f_{3dB} = \sqrt{f_s^2 - 2f_z^2} = \sqrt{(1+g_m R_S)^2 - 2} \cdot f_z \approx f_s$	Computation of f_{3dB} frequency with one pole and one zero.
$f_{3dB} \approx f_s = \frac{1}{2\pi \frac{R_S}{1+g_m R_S} C_S} \approx \frac{g_m}{2\pi C_S}$	Approximate f_{3dB} frequency for capacitor C_S .
$f_s = \left[1 + \frac{(g_m + g_{ds}) R_S}{1 + g_{ds} R_D} \right] f_z$	Pole frequency f_s with g_{ds} included in the derivation.
$f_d = \frac{1}{2\pi(R_D + R_L) C_d}$	Load coupling capacitor corner-frequency equation.
Note: Any one of the frequencies, f_g , f_s (approximate), or f_d is the f_{3dB} (corner) frequency if it is the dominant (high) frequency. Otherwise, the corner frequency is some combination.	

6.6. Exercises and Projects

Project Mathcad Exercise06.mcd - Project06.mcd - Exercise07.mcd, Project07_1.mcd
Files - Project07_2.mcd.

Project Mathcad Files	Exercise06.mcd - Project06.mcd - Exercise07.mcd, Project07_1.mcd - Project07_2.mcd.
Laboratory Project 6	PMOS Common-Source Amplifier Stage with Current-Source Biasing
P6.3	PMOS Current-Source Amplifier DC Setup
P6.4	Amplifier Gain
P6.5	Amplifier Frequency Response
Laboratory Project 7	NMOS Common-Source Amplifier Stage with Source-Resistor Bias
P7.2	NMOS Common-Source Amplifier DC Evaluation
P7.3	Amplifier Gain at Optimum Bias for Linear Output
P7.4	Optimum Bias Stability Test
P7.5	Amplifier Frequency Response

Unit 7. MOSFET Source-Follower Buffer Stage

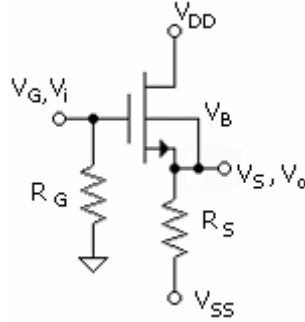
High-gain stages in amplifiers are usually some form of differential stage or common-source stage. These are inherently high-output-resistance stages and are not suitable for low-impedance loads. The solution for a requirement to drive low-impedance loads is to insert a buffer stage between the output of the common-source stage and the load. The buffer will be a source-follower stage. The name follower comes from the fact that to a good approximation, the output voltage follows the input voltage; that is, it has an open-circuit gain of unity and low output resistance.

Although the output voltage/input voltage relation is sometimes loosely referred to as the gain, it would be more accurate to label it the voltage transfer function, as the "gain" is less than 1. The transfer function is explored in the following for various possibilities of degrees of approximation in the linear circuit.

7.1. DC (Bias) Circuit

The circuit diagram shown in [Fig. 7.1](#) uses for the transistor a symbol that includes the body connection. This is to emphasize the fact that the output (source) of the source follower is generally (and usually) at different dc and signal potentials from the body. In [Project 8](#) we measure the gain for both the connection shown (which we can make for this special case of one transistor on the chip being used) and with the body connected to V_{SS} , which is signal ground. The latter demonstrates the extent of the influence that the body effect has on the gain when it is not possible to connect the body and source together as shown in [Fig. 7.1](#).

Figure 7.1. Source-follower (common-drain) stage (NMOS transistor) with the input at the gate and the output at the source. In this example, the source is connected to the body. Usually, in a circuit, the body will be attached to the power supply V_{SS} (NMOS).



The design of the stage will often consist of setting the dc output at $V_O = 0$ V (for a dual power supply), as this will be the final (output) stage. Therefore, R_S will be selected based on the design drain current from $R_S = |V_{SS}|/I_D$. It is then necessary to determine V_{GS} in order to determine the $V_G = V_{GS}$ required (which will be set by the dc circuitry of the preceding circuit).

For the usual practical application, body voltage, $V_B = V_{SS}$. With $V_S = 0$ V, $V_{SB} = |V_{SS}|$. Thus, the threshold voltage, including body effect, will be

Equation 7.1

$$V_{tn} = V_{tn0} + \gamma_n \left(\sqrt{|V_{SS}| + 2\phi} - \sqrt{2\phi} \right)$$

and the gate bias voltage will be obtainable from

Equation 7.2

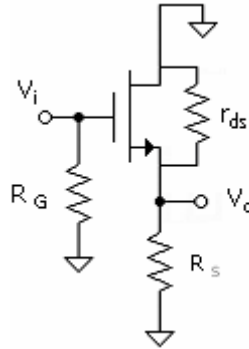
$$V_G = \sqrt{\frac{I_D}{k_n}} + V_{tn}$$

If the source and body are connected, as in [Fig. 7.1](#), V_{tn} is replaced with V_{tn0} .

7.2. Source-Follower Voltage Transfer Relation

A signal circuit is shown in [Fig. 7.2](#), where the dc supplies have been replaced with ground. Also, the internal resistance, $r_{ds} = 1/g_{ds}$, of the transistor has been included in the signal circuit. Note that it is in parallel with R_S since the drain is tied to signal ground. Assume for now that source and body are connected as in [Fig. 7.1](#).

Figure 7.2. Signal circuit of the source follower. Body effect is absent since the source and body are assumed connected.



Regardless of the terminal configuration, for infinite output resistance and with the body and source connected, the relationship between the drain current and gate – source control voltage of the transistor is [\(4.1\)](#) with $g_{ds} = g_{mb} = 0$, which is [[\(2.5\)](#)]

$$I_d = g_m V_{gs}$$

In the source-follower circuit, the output resistance of the transistor can be included in the source-follower load and [\(2.5\)](#) effectively applies.

The transconductance for the circuit configuration with a source resistor was shown in [Unit 4](#) to be [[\(4.8\)](#)]

$$G_m = \frac{g_m}{1 + g_m R'_S}$$

where, for this case, R'_S is the source-follower load, which includes the transistor output resistance and is

Equation 7.3

$$R'_S = R_S || r_{ds} = \frac{R_S}{1 + g_{ds}R_S}$$

Then using

Equation 7.4

$$V_o = I_d R'_S$$

the "gain" (the transfer function is less than unity) is determined to be

Equation 7.5

$$a_v = \frac{V_o}{V_g} = \frac{g_m R'_S}{1 + g_m R'_S}$$

Ideally, this is a unity gain, as would be the case for $g_m R'_S \gg 1$. In bipolar transistors, this condition is closely approached. However, in MOSFET source followers, the ideal case is in general not reached. As we will see below, this deficiency is enhanced when the body effect is included.

Note that a_v is

Equation 7.6

$$a_v = \frac{R'_S}{\frac{1}{g_m} + R'_S}$$

This is the expression for a simple voltage divider. Thus the source follower is the equivalent of a unity-gain circuit with load R'_S and output resistance $1/g_m$. We can conclude that the transistor viewed from the source has an output resistance of $1/g_m$.

Finally, with R'_S separated into its component parts, the source-follower gain becomes

Equation 7.7

$$a_v = \frac{g_m}{g_m + \frac{1}{R_S} + g_{ds}}$$

In general, the g_{ds} parameter can be dropped. The ratio of g_m/g_{ds} is

Equation 7.8

$$\frac{g_m}{g_{ds}} \approx \frac{2}{\lambda_p V_{eff}}$$

7.3. Body Effect and Source-Follower Voltage Transfer Relation

Suppose now, as in [Fig. 7.3](#), that the body terminal is connected to V_{SS} , which is signal ground. Following the definition of the body-effect transconductance, g_{mb} , the component of I_d due to source-body voltage V_{sb} is [from (4.1)]

Equation 7.9

$$I_{db} = g_{mb} V_{sb}$$

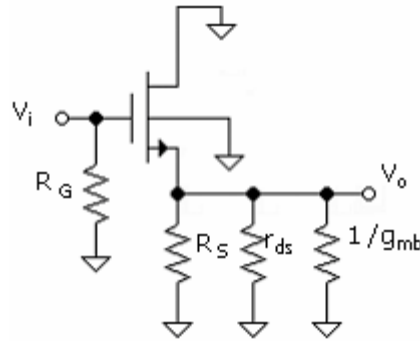
Since the body is at zero potential (signal), this is

Equation 7.10

$$I_{db} = g_{mb} V_s = g_{mb} V_o$$

This component of current can be included in the circuit, as shown in [Fig. 7.3](#), with an additional resistance in parallel with R_S of value $1/g_{mb}$. That is, the body-effect contribution to the drain current is a voltage-dependent current source, where the voltage is the same as that across the current source.

Figure 7.3. Source-follower signal equivalent circuit with body connected to V_{SS} , which is signal ground. The body effect is taken care of for this case with a resistance of magnitude $1/g_{mb}$ placed in parallel with R_S along with $r_{ds} = 1/g_{ds}$.



With the body-effect addition, the source-follower gain expression is modified to the following:

Equation 7.11

$$a_v = \frac{g_m}{g_m + g_{mb} + \frac{1}{R_S} + g_{ds}}$$

The body-effect parameter is related to g_m by [(4.11)]

$$g_{mb} = \eta g_m$$

with

$$\eta = \frac{\gamma_n}{2\sqrt{V_{SB} + 2\phi_F}}$$

With the substitution of (4.11) in (7.11), the gain is

Equation 7.12

$$a_v = \frac{g_m}{g_m (1 + \eta) + \frac{1}{R_S} + g_{ds}}$$

In modern integrated circuits, resistor R_S will be replaced with a current-source output resistance, which is significantly greater than R_S . The source-follower gain is, in this case, approximately (with no external load)

Equation 7.13

$$a_v = \frac{1}{1 + \eta}$$

Parameter η will typically be in the range $0.1 < \eta < 0.2$.

Note that the body effect degrades the "gain" significantly below the ideal unity. In the project on the source-follower circuit, we measure the voltage transfer function of the source follower for a range of dc drain current with body and source connected and with the body connected to

V_{SS} . Τη ρεσυλτινγ χυρπεσ ωιλλ βε χομπαρεδ το δεμονστρατε τηε βοδψ εφφεχτ ο ν τηε τρανσφερ φυνχτιον. Α χυρπε φιτ οφ τηε ΣΠΙΧΕ εθυατιον το μεασυρεδ δατα ωιλλ προπιδε φορ αν αλτερνατιβε δετερμινατιον οφ παραμετερ γ_n . This value should be close to the value obtained from dc measurements.

7.4. Summary of Equations

$V_{tn} = V_{tn0} + \gamma_n \left(\sqrt{ V_{SS} + 2\phi} - \sqrt{2\phi} \right)$	Threshold voltage $V_S = 0$ zero volts and negative power supply V_{SS} .
$a_v = \frac{g_m}{g_m + \frac{1}{R_S} + g_{ds}}$	Source-follower voltage gain with $V_{sb} = 0$.
$a_v = \frac{g_m}{g_m (1 + \eta) + \frac{1}{R_S} + g_{ds}}$	Source-follower voltage gain with $V_{sb} = I_d R_S$.
$g_{mb} = \eta g_m$	Body-effect transconductance.
$\eta = \frac{\gamma_n}{2\sqrt{V_{SB} + 2\phi_F}}$	Source-follower body-effect degradation factor.

7.5. Exercises and Projects

Project Mathcad Files Exercise08.mcd - Project08.mcd

Laboratory Project 8 [NMOS Source-Follower Stage](#)

P8.2 [Source-Follower DC Evaluation](#)

P8.3 [Source-Follower Voltage Transfer Relation](#)

Unit 8. MOSFET Differential Amplifier Stage

The MOSFET differential amplifier stage is a relatively high-gain stage with grounded dc inputs, which requires no bypass capacitor. The stage also has the important advantage of having two inputs, one inverting and one noninverting. This is essential in operational amplifiers where one or the other (or both) is used, depending on the application. The two inputs also provide for operating the stage in the differential mode, where the input is not referenced to ground. The differential amplifier stage discussed here is an all-resistor circuit with source and drain resistors. In a modern integrated circuit, these are replaced with transistor current sources. However, the principle of operation is the same, and the resistor version can be implemented in our laboratory project.

Various segments of the differential amplifier comprise amplifier stages from all three terminal configurations: the common source, the common gate, and the source follower (common drain). In this unit, a detailed theoretical analysis of all three stages is undertaken (based on Level 1 SPICE). The linear models used include, progressively, the output conductance, g_{ds} , and the body-effect transconductance, g_{mb} .

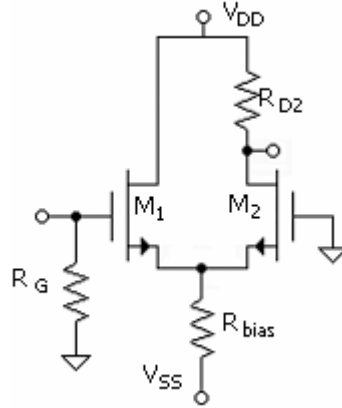
8.1. DC (Bias) Circuit

The equation for the dc circuit ([Fig. 8.1](#)) is the same as for the current-source common-source stage except that the bias resistor has current $2I_D$ as the current from both transistors is flowing through this resistor. For now, the stage will be assumed to be perfectly balanced such that $I_{D1} = I_{D2} = I_D$. The bias equation is

Equation 8.1

$$I_D = \frac{V_{SS} - V_{GS}}{2R_{bias}} = \frac{V_{SS} - \sqrt{\frac{I_D}{k_n}} - V_{tn}}{2R_{bias}}$$

Figure 8.1. Dc circuit for differential-amplifier stage with resistance bias. Source and body are normally at different potentials. In the project with this circuit, we attach the two sources to the transistor body, which is allowed in a simple circuit.



Note that the threshold voltage is that for $V_{SB} \neq 0$ V. The solution can be obtained with the inclusion of the threshold voltage equation, also a function of I_D ,

Equation 8.2

$$V_{tn} = V_{tn0} + \gamma_n \left(\sqrt{2I_D R_{bias} + 2\phi_F} - \sqrt{2\phi_F} \right)$$

In the project on the differential amplifier, we can connect the sources and bodies of the two transistors. This is because only NMOS transistors are used in the circuit, and both transistors have the source connected to the same node. In this case, $V_{tn} = V_{tn0}$.

8.2. DC Imbalances

In a real transistor, parameters k_n , V_{tn0} , and λ_n can be slightly different such that, in general, $I_{D1} \neq I_{D2}$. The ratio of the currents is given by

Equation 8.3

$$\frac{I_{D2}}{I_{D1}} = \frac{k_{n2} (V_{GS} - V_{tn02})^2 (1 + \lambda_{n2} V_{DS2})}{k_{n1} (V_{GS} - V_{tn01})^2 (1 + \lambda_{n1} V_{DS1})}$$

where, in the circuit with the gate of M_1 grounded, $V_{GS1} = V_{GS2} = V_{GS}$. For the specific case of the project on the amplifier, we can assume that $V_{DS1} \approx V_{DS2}$ such that

Equation 8.4

$$\frac{I_{D2}}{I_{D1}} = \frac{k_{n2}}{k_{n1}} \frac{(V_{GS} - V_{tno2})^2}{(V_{GS} - V_{tno1})^2}$$

To simplify the relation, it is advantageous to express parameters in terms of increments. For example, threshold voltage will be expressed as a mean value

Equation 8.5

$$V_{tno} = \frac{V_{tno1} + V_{tno2}}{2}$$

and a difference between the threshold voltage of the two transistors

Equation 8.6

$$V_{t\epsilon} = V_{tno2} - V_{tno1}$$

such that

Equation 8.7

$$V_{tno2} = V_{tno} + \frac{V_{t\epsilon}}{2}$$

and

Equation 8.8

$$V_{tno1} = V_{tno} - \frac{V_{t\epsilon}}{2}$$

Using these definitions along with $V_{eff} = V_{GS} - V_{tno}$, we have

Equation 8.9

$$\frac{I_{D2}}{I_{D1}} = \frac{k_{n2} \left(V_{eff} - \frac{V_{tno}}{2} \right)^2}{k_{n1} \left(V_{eff} + \frac{V_{tno}}{2} \right)^2} = \frac{k_{n2} \left(1 - \frac{V_{t\epsilon}}{2V_{eff}} \right)^2}{k_{n1} \left(1 + \frac{V_{t\epsilon}}{2V_{eff}} \right)^2}$$

or, approximately,

Equation 8.10

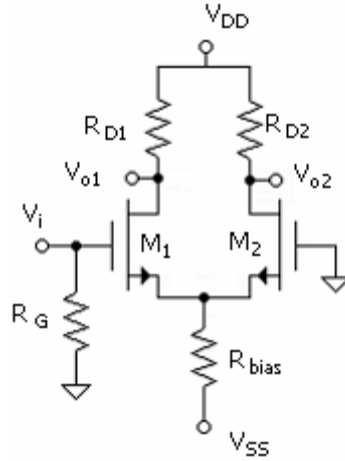
$$\frac{I_{D2}}{I_{D1}} \approx \frac{k_{n2}}{k_{n1}} \left(1 - \frac{2V_{t\epsilon}}{V_{eff}} \right)$$

In the differential amplifier project (PMOS), we measure the drain currents along with V_{SG} over a range of drain current values. These are used to obtain, in the Mathcad project file, k_{n1} , k_{n2} , and the threshold voltages and thus, $V_{t\epsilon}$. This will provide a comparison between the measured drain-current ratios and (8.10) as a function of V_{eff} and drain current.

8.3. Signal Voltage Gain of the Ideal Differential Amplifier Stage

This initial assessment of the gain of the differential amplifier neglects the effects of the bias resistor, R_{bias} , and of the output resistance of the transistors as well as the body effect. These effects are considered below. To generalize the discussion to include the possibility of inverting and noninverting modes of operation, we base the discussion on the circuit (Fig. 8.2) with a drain resistor for both transistors. The inverting and noninverting modes take, respectively, the outputs at $V_{d1} = V_{o1}$ (a_{v1}) and $V_{d2} = V_{o2}$ (a_{v2}). Also, this is the configuration that is studied in the project on the differential amplifier. Note that the input could arbitrarily be moved to the gate of M_2 , in which case the output taken at V_{d1} becomes the noninverting mode, and so forth.

Figure 8.2. Circuit for our project on the differential amplifier. The circuit has resistors in both drain branches for comparing the gain from the noninverting and inverting outputs and for sensing the bias drain currents.



Assume for the present that the amplifier is perfectly symmetrical such that, for example, $I_{D1} = I_{D2}$ and $g_{m1} = g_{m2}$. A signal voltage V_i is applied to the gate of M_1 as shown in the diagram. The voltage divides between the two transistors (from gate 1 to gate 2) to give

Equation 8.11

$$V_i = V_{gs1} + V_{sg2} = V_{gs1} - V_{gs2}$$

Since $V_{gs1} = |V_{sg2}|$ (due to the symmetry of this case), then

Equation 8.12

$$V_{gs1} = \frac{V_i}{2} = -V_{gs2}$$

The negative V_{gs2} results from the fact that the total v_{GS2} is reduced from bias V_{GS2} upon the application of a positive signal voltage, V_i , at the gate of M_1 (i.e., $v_{GS2} = V_{GS2} - V_i/2$). It follows that total i_{D2} decreases, which leads to a positive signal voltage at the drain, V_{d2} . Thus, the amplification is noninverting.

As always, the relation between the signal drain current and the signal drain-source voltage is (4.1), which is (with $g_{ds} = g_{mb} = 0$)

$$I_d = g_m V_{gs}$$

Eliminating V_{gs} in (4.1), applied to M_2 , with (8.12), the drain current for M_2 is related to the input voltage, V_i , by

Equation 8.13

$$I_{d2} = -g_{m2} \frac{V_i}{2}$$

Combining (8.13) with $V_{o2} = -I_{d2}R_{D2}$, the output voltage is related to the input voltage by

Equation 8.14

$$V_{d2} = V_{o2} = \frac{1}{2}g_{m2}R_{D2}V_i$$

such that the gain is

Equation 8.15

$$a_{v2} = \frac{V_{d2}}{V_i} = \frac{1}{2}g_{m2}R_{D2}$$

Note that there is no minus in the relation; that is, this is the noninverting amplifier mode. Also note that the gain for this amplifier is one-half of that for the common-source amplifier. This is a reasonable price to pay for not having a bypass capacitor. The amplification can be viewed as a source-follower stage in cascade with a common-base stage, neither of which is an inverting stage.

The inverting mode of the amplifier is for the output taken at the drain of M_1 . For this case, $V_{o1} = -I_{d1}R_{D1}$, and from (8.12) and (4.1) applied to M_1 , $I_{d1} = g_{m1}V_g/2$ and the voltage relationship is

Equation 8.16

$$V_{d1} = V_{o1} = -\frac{1}{2}g_{m1}R_{D1}V_i$$

This leads to the inverting gain, which is

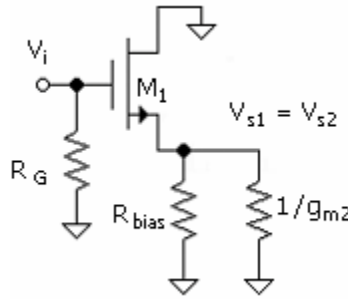
Equation 8.17

$$a_{v1} = \frac{V_{d1}}{V_{g1}} = -\frac{1}{2} g_{m1} R_{D1}$$

8.4. Effect of the Bias Resistor on Voltage Gain

The results above ignore the effect of resistance, R_{bias} . When it is included, it is no longer valid to assume that $V_{gs1} = V_{sg2}$, even with $I_{D1} = I_{D2}$. The bias-resistor effect is examined here based on a signal equivalent circuit of [Fig. 8.3](#). The resistance at the source of M_1 consists of the bias resistor and the linear-model resistance $1/g_{m2}$ (which neglects $1/g_{ds2}$), looking into the source of M_2 .

Figure 8.3. Source circuit as viewed from M_1 . Source resistance includes actual resistor R_{bias} in parallel with input resistance at source of M_2 , which is $1/g_{m2}$ for the conditions of this unit.



For the noninverting mode, the circuit transconductance, $G_{m2} = I_{d2}/V_i$, is

Equation 8.18

$$G_{m2} = \frac{\frac{R_{bias}}{1 + g_{m2}R_{bias}}}{\frac{R_{bias}}{1 + g_{m2}R_{bias}} + \frac{1}{g_{m1}}} g_{m2}$$

Parameter g_{m2} is the transconductance I_{d2}/V_{s2} (i.e., from the source of M_2 to the output V_{o2}). Note that the fraction is just a voltage divider, V_{s2}/V_i , since the output resistance at the source of M_1 is $1/g_{m1}$, as noted in connection with (7.6). With $g_{m1} = g_{m2}$, the multiplying fraction assumes the value of $1/2$ for very large R_{bias} .

The noninverting gain, which includes the effect of the bias resistor, is

Equation 8.19

$$a_{v2} = G_{m2}R_{D2} = \frac{\frac{R_{bias}}{1 + g_{m2}R_{bias}}}{\frac{R_{bias}}{1 + g_{m2}R_{bias}} + \frac{1}{g_{m1}}} g_{m2}R_{D2} = \frac{g_{m1}R_{bias}}{1 + 2g_{m1}R_{bias}} g_{m1}R_{D2}$$

where the right-hand side uses $g_{m1} = g_{m2}$. The gain result is technically that of the cascade of a source-follower stage (M_1) and a common-gate stage, $g_{m2}R_{D2}$ (M_2). The multiplying fraction is $< 1/2$.

The gain for the inverting side can be obtained by using (5.15), which is for a common-source amplifier with source resistor. This gives for the inverting-mode gain

Equation 8.20

$$a_{v1} = \frac{-g_{m1}R_{D1}}{1 + g_{m1} \frac{R_{bias}}{1 + g_{m2}R_{bias}}} = -\frac{1 + g_{m1}R_{bias}}{1 + 2g_{m1}R_{bias}} g_{m1}R_{D1}$$

where the right-hand side again uses $g_{m1} = g_{m2}$. The multiplying fraction is $> 1/2$. Note that for both inverting and noninverting gains, the effect of R_{bias} can be neglected for most purposes, as the multiplying fractions in (8.19) and (8.20) are both close to $1/2$.

The choice of the input at gate 1 is arbitrary and all of the results obtained here apply equally to the input taken at gate 2. The inverting output is always at the side of the input, and the noninverting output is always at the opposite side.

8.5. Differential Voltage Gain

Suppose that an input voltage, $V_{g12} = V_{g1} - V_{g2}$, is applied between the inputs. Due to symmetry, the voltage magnitudes at the inputs with respect to ground are $V_{g1} = V_{g12}/2$ and $V_{g2} = -V_{g12}/2$, respectively. The noninverting output, V_{d2} , for this case is, by superposition,

Equation 8.21

$$V_{d2} = \frac{g_{m1}R_{bias}}{1 + 2g_{m1}R_{bias}} g_{m1}R_{D2} \frac{V_{g12}}{2} + \frac{1 + g_{m1}R_{bias}}{1 + 2g_{m1}R_{bias}} g_{m1}R_{D2} \frac{V_{g12}}{2} = \frac{1}{2} g_{m1}R_{D2} V_{g12}$$

The gains for the contributions from V_{g1} and V_{g2} are (8.19) and (8.20), respectively. This is the case of a pure differential input with resulting gain

Equation 8.22

$$a_{vd2} = \frac{1}{2} g_m R_{D2}$$

A similar approach applied to obtain the gain for the output taken as V_{d1} produces

Equation 8.23

$$a_{vd1} = -\frac{1}{2} g_m R_{D1}$$

Assume, for a numerical example, that $I_D = 100 \mu A$, $R_{D1} = 50 K\Omega$, and $g_m = 200 \mu A/V$ ($V_{effn} = 0.5 V$). In this case the gain is $a_{vd1} = -10$. This would be consistent with $V_{DD} = |V_{SS}| = 10 V$.

The gain for the case of the differential output, $a_{vd12} = (V_{d1} - V_{d2})/(V_{g1} - V_{g2})$, can be obtained from $(V_{d1} - V_{d2}) = -(1/2)g_{m1}R_{D1}V_{g12} - (1/2)g_{m2}R_{D2}V_{g12}$ and is

Equation 8.24

$$a_{vd12} = -g_m R_D$$

where $g_{m1} = g_{m2}$ and $R_{D1} = R_{D2} = R_D$. Note that (8.22) and (8.23) are the same as (8.15) and (8.17). However, (8.15) and (8.17) are the limiting forms of (8.19) and (8.20) for $R_{bias} \rightarrow \infty$, whereas (8.22) and (8.24) apply for a finite R_{bias} .

8.6. Common-Mode Voltage Gain

The common-mode gain is defined for the same voltage applied simultaneously to both inputs. The output must be the same at either output terminal (again assuming that $g_{m1} = g_{m2}$ and $R_{D1} = R_{D2}$). For example, for the output V_{d2} , the gain can be determined by a superposition of gains, inverting (input, V_{g2}) and noninverting (input, V_{g1}), with R_{D2} in both equations.

Using (8.19) (noninverting) and (8.20) (inverting), the gain for finite R_{bias} is, accordingly,

Equation 8.25

$$a_{vcm} = \left(\frac{g_m R_{bias}}{1 + 2g_m R_{bias}} - \frac{1 + g_m R_{bias}}{1 + 2g_m R_{bias}} \right) g_m R_{D2}$$

which is

Equation 8.26

$$a_{vcm} = \frac{-g_m R_{D2}}{1 + 2g_m R_{bias}}$$

The result is that of a common-source stage with source resistance $2R_{bias}$. This is intuitively correct as taken from the half-circuit viewpoint, where the circuit is completely symmetrical. The input from either side looks at a common-source stage except that the opposite side is contributing an equal amount of source current, thus giving an effective source resistance equal to twice the actual value. A valid approximate form for well-designed circuits (in terms of common-mode gain) is

Equation 8.27

$$a_{vcm} \approx \frac{-R_{D2}}{2R_{bias}}$$

The same result applies to the case of the output taken at the opposite drain, with the substitution of R_{D1} for R_{D2} . Using the circuit values following (8.23) plus $R_{bias} = 85 \text{ k}\Omega$, $a_{vcm} \approx -0.3$. Note that this result in combination with the gain from (8.23) would indicate that this is not a particularly good design. The goal is for $a_{vd} \gg a_{vcm}$.

8.7. Voltage Gains Including Transistor Output Resistance

In [Project 9](#) we measure the gain of a "balanced" stage with drain resistors for both transistors. The amplifier is the PMOS version of the NMOS amplifier of [Fig. 8.2](#). We obtain an exact Level 1 SPICE solution for the gains for both outputs. The example is used to explore the use of a simulator for obtaining precision results to compare with simple hand calculations.

For larger λ_n values (NMOS), the output resistance can influence the gain and complicate the gain expressions considerably. Here we consider the effect due to g_{ds1} and g_{ds2} while

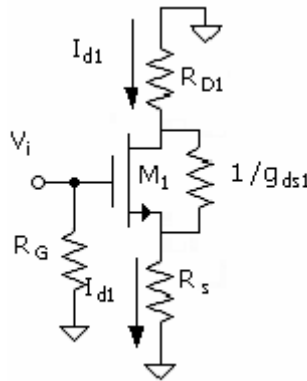
retaining the effect of R_{bias} . In the following, the gain of the inverting input, a_{v1} , is obtained again as a common-source stage with source resistance. The effect of g_{ds2} on the effective source resistance is included (input at the source of M_2). The effects due to g_{ds1} are also taken into consideration.

The gain of the noninverting case, a_{v2} , is obtained by considering the cascade of the source follower stage (M_1) and the common-gate stage (M_2), as, in effect, was done in the development of (8.19). The source-follower gain takes into account effects from g_{ds1} and g_{ds2} , and the gain of the common-gate stage depends on g_{ds2} .

8.7.1. Gain of the Common-Source Stage with Transistor Output Conductance and Source Resistor

The circuit transconductance for a common-source stage with source resistor, with the inclusion of g_{ds} , was developed in Unit 4 [(4.18)]. This will be reviewed and reinforced here in the form of a slightly different approach to the result. The signal circuit for this case is again given in Fig. 8.4. Using the variables of Fig. 8.4, the circuit transconductance is $G_{m1} = I_{d1}/V_i$. The object is thus to obtain a relation between these two variables.

Figure 8.4. Circuit for obtaining the gain for the inverting output with the transistor output resistance included. R_s includes all resistance contributions at the source.



The fraction of the current produced by the intrinsic transistor, $g_{m1}V_{gs1}$, which flows into R_s is

Equation 8.28

$$I_{R_s} = I_{d1} = \frac{1/g_{ds}}{1/g_{ds} + R_D + R_s} g_{m1} V_{gs1} = \frac{g_m}{1 + g_{ds}(R_D + R_s)} V_{gs1}$$

This is the portion of current source $g_{m1}V_{gs1}$, shared between $1/g_{ds1}$ and $R_D + R_s$, which flows through $R_D + R_s$, that is, I_{d1} . Note that $R_D + R_s$ and $1/g_{ds1}$ are in parallel and shunt the transistor current source.

A relation for V_{gs1} in terms of I_{d1} follows, which is

Equation 8.29

$$V_{gs1} = \frac{1 + g_{ds1}(R_D + R_s)}{g_{m1}} I_{d1}$$

The input Voltage is the sum of V_{gs1} and the drop across R_s . That is,

Equation 8.30

$$V_i = V_{gs1} + I_{d1}R_s$$

Using (8.29) in (8.30) gives

Equation 8.31

$$V_i = \left[\frac{1 + g_{ds1}(R_D + R_s)}{g_{m1}} + R_s \right] I_{d1}$$

or

Equation 8.32

$$I_{d1} = \frac{g_{m1}V_i}{(R_{D1} + R_s)g_{ds1} + (1 + g_{m1}R_s)}$$

The circuit transconductance follows as

Equation 8.33

$$G_m = \frac{g_{m1}}{(R_{D1} + R_s)g_{ds1} + (1 + g_{m1}R_s)}$$

The gain for this case is then

Equation 8.34

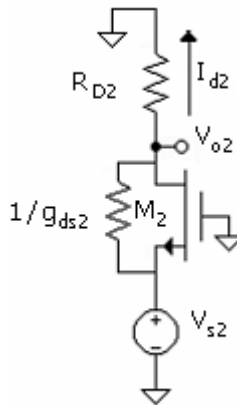
$$a_{v1} = \frac{-g_{m1}R_{D1}}{(R_{D1} + R_s)g_{ds1} + (1 + g_{m1}R_s)}$$

A discussion of R_s as affected by g_{ds2} follows.

8.7.2. Common-Gate Amplifier Stage

The circuit diagram of [Fig. 8.5](#) is for the M_2 portion of the differential amplifier. The input is applied at the source, V_{s2} , and the output is taken at the drain, $V_{d2} = V_{o2}$, while the gate is grounded. This is a common-gate configuration. Here we analyze the input resistance, for evaluating the effect on R_s , and gain of the common-gate stage.

Figure 8.5. Circuit that includes the output resistance of M_2 . The circuit is for obtaining input resistance at the source of M_2 and the gain of the common-gate stage of M_2



Without g_{ds2} , the input resistance at the source is just $1/g_{m2}$. With g_{ds2} in place, there is a positive feedback from the drain output to the source input. This causes the input resistance to increase. With g_{ds2} , the current into the source terminal is

Equation 8.35

$$I_{d2} = g_{m2}V_{s2} - (I_{d2}R_{D2} - V_{s2})g_{ds2}$$

It is noted that the g_{ds2} term reduces the input current, which has the effect of increasing the input resistance. From (8.35), the resulting input resistance at the source, $R_{is2} = V_{s2}/I_{d2}$, is

Equation 8.36

$$R_{is2} = \frac{1 + g_{ds2}R_{D2}}{g_{m2} + g_{ds2}}$$

The input resistance goes to $1/g_{m2}$, as noted above, for $g_{ds2} = 0$. The expression for the equivalent R_s is now $R_s = R_{bias} \parallel R_{is2}$, where R_{is2} is (8.36). In modern integrated circuits, R_{D2} may be replaced with a high-resistance transistor current source, and the input resistance in this case can be much greater than $1/g_{m2}$.

The gain of the common-gate stage is obtained as follows: The output voltage is

Equation 8.37

$$V_{d2} = I_{d2}R_{D2} = \left(g_{m2}V_{s2} - I_{d2}g_{ds2} \right) R_{D2} = \left[g_{m2}V_{s2} - (V_{d2} - V_{s2})g_{ds2} \right] R_{D2}$$

which gives

Equation 8.38

$$a_{vcg} = \frac{(g_{m2} + g_{ds2})R_{D2}}{1 + g_{ds2}R_{D2}}$$

Note that for $g_{ds2} = 0$, the gain has the same magnitude as the simple case for the common-source stage.

8.7.3. Voltage Gain for the Noninverting Output

The noninverting amplifier gain is based on a cascade of a source-follower stage (M_1) and a common-gate stage (M_2). The source-follower transconductance, I_{d1}/V_i , is (8.33). Using this with $V_{s1} = I_{d1}R_s$ gives

Equation 8.39

$$a_{vsf} = \frac{g_{m1}R_s}{(R_{D1} + R_s)g_{ds1} + (1 + g_{m1}R_s)}$$

Note that the magnitude is about 1/2. Overall gain is the product of (8.38) and (8.39), which is

Equation 8.40

$$a_{v2} = a_{vsf}a_{vcg} = \frac{g_{m1}R_s}{(R_{D1} + R_s)g_{ds1} + (1 + g_{m1}R_s)} \frac{(g_{m2} + g_{ds2})R_{D2}}{1 + g_{ds2}R_{D2}}$$

The equations from this unit are summarized below in [Unit 8.11](#).

Recall from the discussion of MOSFET model parameters that g_{ds} is given by (4.13), which is

$$g_{ds} = \frac{I_D \lambda_n}{1 + \lambda_n V_{DS}}$$

Thus, the gain expression depends on parameter λ , and especially if λ is somewhat large. In [Project 9](#) we measure the gains from the two drains and use the results to find the value of λ that makes the theory fit the measurements. In this way, we are getting a signal-derived experimental number to compare with that obtained in the parameter-determination project. This will be done using the PMOS configuration since the value of λ_p is large and the effect is significant. All of the gain expressions, which are based on NMOS transistors, apply exactly to the PMOS stage with substitution of subscripts; change n to p (parameters) and reverse the order for dc voltage variables.

For hand calculations, approximate forms must reasonably be used. This applies to approximate forms for device parameters and gain. The basic gain equations are, again, assuming that $g_{ds} = 0$ and $R_{bias} = \infty$, simply, as given by (8.15) and (8.17),

$$a_{v2} = -a_{v1} = \frac{1}{2}g_m R_D$$

with $R_{D1} = R_{D2}$ and $g_{m1} = g_{m2}$. In the differential amplifier project, we will compare these with the more precision forms.

8.8. Body Effect and Voltage Gain

In [Project 9](#) we are able to connect the sources of the transistors to the chip body. Certainly, in general this cannot be done such that there is a body effect associated with the differential-amplifier-stage transistors. The necessary alterations to the gain equations are determined in the following. At the end of this unit, we will have the complete, precision-gain calculation equations of Level 1 SPICE. It will be informative to consider numerical results that are based on various degrees of approximations, and this is done below.

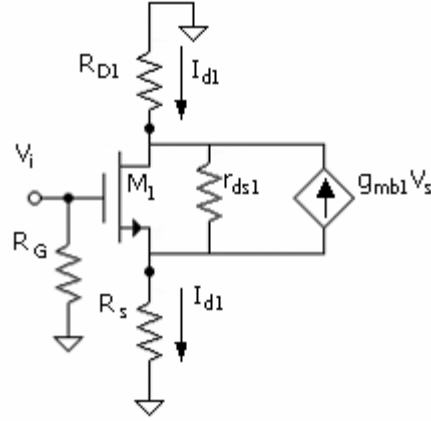
8.8.1. Common-Source Stage and Body Effect

With the body effect present, a component of current, $g_{mb1}V_s$ ([Fig. 8.6](#)), is subtracted from $g_{m1}V_{gs1}$ such that ([8.28](#)) for this case is modified to become

Equation 8.41

$$I_{d1} = \frac{1/g_{ds1}}{1/g_{ds1} + R_{D1} + R_S} (g_{m1}V_{gs1} - g_{mb1}V_s)$$

Figure 8.6. Circuit for obtaining the inverting gain of the differential stage with body effect included. Body effect is accounted for by the current source added to the transistor signal (linear) equivalent circuit.



Additionally using $V_s = I_{d1}R_s$ and $V_{gs1} = V_i - I_{d1}R_s$ (8.30) in (8.41), a relation between I_{d1} and V_i is obtained, which is

Equation 8.42

$$I_{d1} = \frac{1}{1 + g_{ds1}(R_{D1} + R_s)} \left[g_{m1}(V_i - I_{d1}R_s) - g_{mb1}I_{d1}R_s \right]$$

Solving for I_{d1} , the circuit transconductance is, for the body-effect case,

Equation 8.43

$$G_{m1} = \frac{I_{d1}}{V_i} = \frac{g_{m1}}{[1 + g_{ds1}(R_{D1} + R_s)] + g_{m1}R_s + g_{mb1}R_s}$$

It follows that the gain for the inverting mode, with the addition of body effect, is

Equation 8.44

$$a_{v1} = - \frac{g_{m1}R_{D1}}{[1 + g_{ds1}(R_{D1} + R_s)] + (1 + \eta_n)g_{m1}R_s}$$

The effect of η_n in the denominator tends to make the gain smaller. However, the body effect, as shown below, will decrease R_s such that the two effects tend to cancel one another.

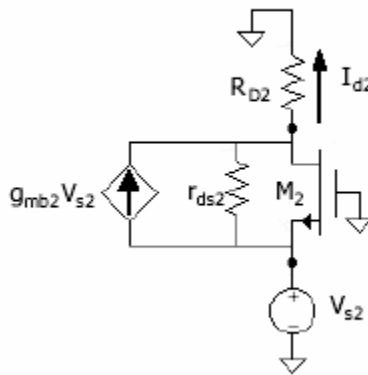
8.8.2. Common-Gate Stage and Body Effect

The voltage applied to the common-gate stage is $V_{s2} = V_{sg2}$ (Fig. 8.7). Recall that in the g model for the transistor, as discussed for the common-source mode, current sources $g_{mb}V_{sb}$ and g_mV_{gs} are in parallel but in opposite directions. Thus, the current sources g_mV_{sg} and $g_{mb}V_{sb}$ are, for the common-gate mode, in the same direction since $V_{sb} = V_{sg} = -V_{gs}$; the common-gate has an effect transconductance of $(1 + \eta)g_m$. It follows that the input resistance of the common-gate stage is as obtained before the body effect was included [(8.38)], except for the addition of η_n , as in the following:

Equation 8.45

$$R_{is2} = \frac{1 + g_{ds2}R_{D2}}{g_{m2}(1 + \eta_n) + g_{ds2}}$$

Figure 8.7. Circuit that includes the voltage-dependent current source due to body effect associated with M_2 . Body effect affects the input resistance into the source of M_2 and the gain of the common-gate stage of M_2 .



Similarly, the common-gate gain, (8.38), is readily modified with the addition of the multiplying factor $(1 + \eta_n)$. This is

Equation 8.46

$$a_{v_{cg}} = \frac{[g_{m2}(1 + \eta_n) + g_{ds2}]R_{D2}}{1 + g_{ds2}R_{D2}}$$

Note that in the absence of g_{ds2} , the gain for the common-source stage reduces to $a_{v_{cd}} = g_{m2}(1 + \eta_n)R_{D2}$, where, with body effect, the effective transconductance is, again, $(1 + \eta_n)g_{m2}$.

8.8.3. Source-Follower Stage with Body Effect

The transconductance relation obtained for the common-source stage given by (8.43) also applies to the source-follower stage. Combining this with $V_{s1} = I_{d1}R_s$ leads to the source-follower gain associated with M_1 , which is

Equation 8.47

$$a_{v_{sf}} = \frac{g_{m1}R_s}{(R_{D1} + R_s)g_{ds1} + [1 + g_{m1}(1 + \eta_n)R_s]}$$

The overall gain is again

$$a_{v2} = a_{vsf} a_{v_{cg}}$$

Note that the body effect for the source-follower stage increases the denominator of (8.49) while it increases the numerator in the common-gate result, (8.48). These tend to cancel, as in the case of the inverting gain.

8.9. Amplifier Gain with Differential and Common-Mode Inputs

For inputs at either or both gates, there exists a common-mode and differential-mode voltage. These are, for applied voltages V_{g1} and V_{g2} , common mode

Equation 8.48

$$V_{cm} = \frac{V_{g1} + V_{g2}}{2}$$

and differential mode

Equation 8.49

$$V_{dm} = V_{g1} - V_{g2}$$

Based on these definitions, the output, for example, V_{d1} , for a given set of inputs is

Equation 8.50

$$V_{d1} = a_{vd1} V_{dm} + a_{vcm} V_{cm}$$

where a_{vd1} and a_{vcm} are (8.23) and (8.26) (with R_{D1}), respectively. For the output V_{d2} , a_{vd2} is substituted for a_{vd1} . V_{dm} is a pure differential input and is not with respect to ground; the effect from the bias resistor is accounted for in the common-mode gain.

For the case of a single-ended input, for example, $V_{g1} = V_i$ and $V_{g2} = 0$, the output is

Equation 8.51

$$V_o = \left(a_{vd1} + \frac{a_{vcm}}{2} \right) V_i$$

This is identical to (8.20). The common-mode contribution can be significant, for example, in a resistance feedback amplifier (Unit 11). In this case, V_{dm} in (8.50) can be very small compared to V_{cm} .

8.10. Comparison of Numerical Gain Results

Gain calculations were made using $k_n = 1000 \mu\text{A}/\text{V}^2$, $\lambda_n = 0.05 \text{ V}^{-1}$, $R_D = 150 \text{ k}\Omega$, $R_{bias} = 100 \text{ k}\Omega$, $\eta_n = 0.15$, $I_D = 50 \mu\text{A}$, and $V_{DS} = 5 \text{ V}$. The values are given in Table 8.1. For the comparison, g_m was calculated with the precision form [(4.5)] ($\lambda_n \neq 0$) for all cases. Possibly a more valid consistency (hand calculation versus precision calculation) would be achieved with the use of the approximate form for g_m up to $\lambda_n \neq 0$. This would reduce some of the difference in the results. For example, for the first case, $a_{v1} = -a_{v2} = -33.5$, with the approximate form for g_m ($\lambda_n \neq 0$).

TABLE 8.1		
Gain Magnitudes	a_{v1}	a_{v2}
$R_{bias} = \infty$, $\lambda_n = 0$, $\eta_n = 0$	37.5	37.5
plus $R_{bias} = 100 \text{ k}\Omega$	37.9	37.1

TABLE 8.1

Gain Magnitudes	a_{v1}	a_{v2}
plus $\lambda_n = 0.05V^{-1}$	29.21	28.48
plus $\eta_n = 0.15$	29.17	28.53

It is notable that the various factors do not have a major effect on the results, even though the value of λ_n is relatively large. This is especially true for the differential amplifier with resistive load, as considered here. The conclusion can be made that for initial hand-calculation purposes, the simplest form is satisfactory. Precision results can be obtained with a simulator.

8.11. Summary of Equations

$I_D = \frac{V_{SS} - V_{GS}}{2R_{bias}} = \frac{V_{SS} - \sqrt{I_D/k_n} - V_{tn}}{2R_{bias}}$	Differential-stage bias equation.
$V_{tn} = V_{tn0} + \gamma_n \left(\sqrt{2I_D R_{bias}} + 2\phi_F - \sqrt{2\phi_F} \right)$	Threshold voltage for with $V_B = V_{SS}$ and thus $V_{SB} = 2I_D R_{bias}$.
$\frac{I_{D2}}{I_{D1}} \approx \frac{k_{n2}}{k_{n1}} \left(1 - \frac{2V_{t\epsilon}}{V_{eff}} \right)$	Approximate relation for drain-current imbalance due to $k_{n1} \neq k_{n2}$ and $V_{tn01} \neq V_{tn02}$.
$a_{v1} = -\frac{1}{2} g_m R_{D1}$ $a_{v2} = \frac{1}{2} g_m R_{D2}$	Ideal voltage gain, $R_{bias} \rightarrow \infty$ and $g_{ds} = 0$, $g_{m1} = g_{m2} = g_m$.
$a_{v2} = \frac{g_m R_{bias}}{1 + 2g_m R_{bias}} g_m R_{D2}$	Gain $V_{d2}/V_{g1} = V_{o2}/V_i$ for noninverting output with finite R_{bias} .
$a_{v1} = -\frac{1 + g_m R_{bias}}{1 + 2g_m R_{bias}} g_m R_{D1}$	Gain $V_{d1}/V_{g1} = V_{o1}/V_i$ for inverting output with finite R_{bias} .
$a_{v1} = \frac{-g_{m1} R_{D1}}{(R_{D1} + R_s) g_{ds1} + (1 + g_{m1} R_s)}$ $R_s = R_{is2} \parallel R_{bias}$	Gain $V_{d1}/V_{g1} = V_{o1}/V_i$ for inverting output for source resistance, R_s , with $g_{ds} \neq 0$.
$R_{is2} = \frac{1 + g_{ds2} R_{D2}}{g_{m2} + g_{ds2}}$	Input at the source of M_2 with g_{ds} 0.

$I_D = \frac{V_{SS} - V_{GS}}{2R_{bias}} = \frac{V_{SS} - \sqrt{I_D/k_n} - V_{tn}}{2R_{bias}}$	Differential-stage bias equation.
$a_{vcg} = \frac{(g_{m2} + g_{ds2})R_{D2}}{1 + g_{ds2}R_{D2}}$	Gain $V_{d2}/V_{s2} = V_{o2}/V_{s2}$ for common-gate stage of M_2 for $g_{ds} \neq 0$.
$a_{vsf} = \frac{g_{m1}R_s}{(R_{D1} + R_s)g_{ds1} + (1 + g_{m1}R_s)}$	Gain $V_{s1}/V_{g1} = V_{s1}/V_i$ of source-follower with input at gate of M_1 and output at source of M_1 .
$a_{v2} = \frac{g_{m1}R_s}{(R_{D1} + R_s)g_{ds1} + (1 + g_{m1}R_s)} \frac{(g_{m2} + g_{ds2})R_{D2}}{1 + g_{ds2}R_{D2}}$	Gain $V_{d2}/V_{g1} = V_{o2}/V_i$ for noninverting output and $g_{ds} \neq 0$.
$a_{vd12} = -g_{m1}R_{D1}$	Gain for differential input, differential output with $R_{D1} = R_{D2}$ and $g_{m1} = g_{m2}$.
$a_{vcm} = \frac{-g_{m1}R_{D2}}{1 + 2g_{m1}R_{bias}}$	Common-mode gain.
$a_{v1} = -\frac{g_{m1}R_{D1}}{1 + g_{ds1}(R_{D1} + R_s) + (1 + \eta)g_{m1}R_s}$ $R_s = R_{is2} \parallel R_{bias}$	Inverting-input gain $V_{d1}/V_{g1} = V_{o1}/V_i$, including body effect.
$R_{is2} = \frac{1 + g_{ds2}R_{D2}}{g_{m2}(1 + \eta) + g_{ds2}}$	Input at the source of M_2 , including body effect and $g_{ds} \neq 0$.
$a_{vcg} = \frac{[g_{m2}(1 + \eta) + g_{ds2}]R_{D2}}{1 + g_{ds2}R_{D2}}$	Common-gate gain $V_{d2}/V_{s2} = V_{o2}/V_{s2}$ of M_2 , including body effect.
$a_{vsf} = \frac{g_{m1}R_s}{(R_{D1} + R_s)g_{ds1} + [1 + g_{m1}(1 + \eta)R_s]}$	Source-follower gain $V_{s1}/V_{g1} = V_{s1}/V_i$ of M_1 , including body effect.
$a_{v2} = a_{vsf}a_{vcg}$	Gain $V_{d2}/V_{g1} = V_{o2}/V_i$, including body effect, $g_{ds} \neq 0$.

8.12. Exercises and Projects

Project Mathcad Files Exercise09.mcd - Project09.mcd

Laboratory Project 9 [MOSFET Differential Amplifier Stage](#)

P9.2 [DC Evaluation of the Single-Power-Supply Differential Amplifier](#)

P9.3 [Determination of the PMOS Parameters](#)

P9.4 [Amplifier Gain Measurement](#)

P9.5 [Transistor Parameters and DC Imbalance](#)

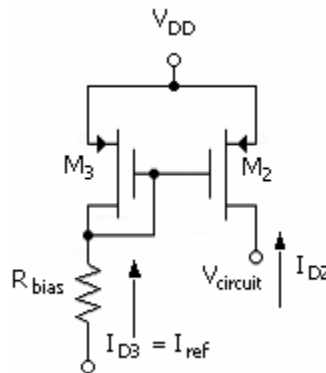
Unit 9. MOSFET Current Sources

With the evolution of integrated circuits, it was necessary to reduce the number of resistors in the circuit, and the solution was the transistor current source. An added bonus was that the circuit was generally greatly improved as well. Here, the basic principle is introduced, followed by a discussion of the standard method of increasing the current output resistance with source degeneration. The application of a current-source configuration for balancing a differential amplifier stage is discussed.

9.1. Basic Current Source

An example of a current source based on PMOS transistors is shown here in [Fig. 9.1](#). It consists of the reference circuit, consisting of transistor M_3 and bias resistor R_{bias} . The reference-circuit transistor is diode connected. The induced $V_{SG3} = V_{SD3}$ is applied to the current-source transistor M_2 . Thus, the drain current, or current-source current of M_2 is the mirror of the reference current. In an integrated circuit, any number of current sources can be referenced to the reference current or voltage. Current ratios are implemented with the selection of the relative gate widths of the transistors.

Figure 9.1. PMOS current source. The current set up by the reference circuit of M_3 and R_{bias} is mirrored as the current of M_2 .



The design of the reference circuit is based on a dc solution to the reference current, I_{D3} . The solution is obtained from the loop equation

Equation 9.1

$$I_{D3} = \frac{V_{DD} + |V_{SS}| - V_{SG3}}{R_{bias}}$$

and transistor equation

Equation 9.2

$$V_{SG3} = \sqrt{\frac{I_{D3}}{k_{p3} (1 + \lambda_p V_{SG3})}} + V_{tp0}$$

The current in M_2 is

Equation 9.3

$$I_{D2} = k_{p2} (V_{SG3} - V_{tp0})^2 (1 + \lambda_p V_{SD2})$$

where

$$k_{p2} = \frac{K P_p}{2} \frac{W_2}{L}$$

and

$$k_{p3} = \frac{K P_p}{2} \frac{W_3}{L}$$

W_2 and W_3 are the transistor gate widths of M_2 and M_3 , respectively, and L is the channel length. In general, the ratio of currents is

Equation 9.4

$$\frac{I_{D2}}{I_{D3}} = \frac{W_2}{W_3} \frac{1 + \lambda_p V_{SD2}}{1 + \lambda_p V_{SG3}} \approx \frac{W_2}{W_3}$$

where the approximation can often be used for simplicity with long-channel devices. This avoids the necessity of knowing the source – drain voltages. The signal output resistance at the drain of M_2 is

Equation 9.5

$$r_{ds2} = \frac{1 + \lambda_p V_{SD2}}{\lambda_p I_{D2}} \approx \frac{1}{\lambda_p I_{D2}}$$

In the project of the current source, we will scan V_{SD2} and compare the source current with the reference current.

9.2. Current Source with Source Degeneration

It is normally desirable to have the output resistance of a current source as large as possible. It can be improved over the circuit of [Fig. 9.1](#) by adding resistance in the source branch, as shown in [Fig. 9.2](#), to establish source degeneration (negative feedback). In the discussion of the signal circuit, it is necessary to represent the diode-connected transistor with its linear equivalent, which is just a resistance of magnitude, $1/g_m$. This can be seen by inspection of the circuit of [Fig. 9.3](#). The voltage, V_d , is applied directly to the gate such that $V_{gs} = V_d$ and the drain current is $I_d = g_m V_d$. Thus the resistance of the diode is just $V_d/I_d = 1/g_m$. The result is the same for the PMOS and NMOS as is always true for signal linear models.

Figure 9.2. Current-source circuit with source resistors to improve output resistance at drain of current-source transistor, M_2 .

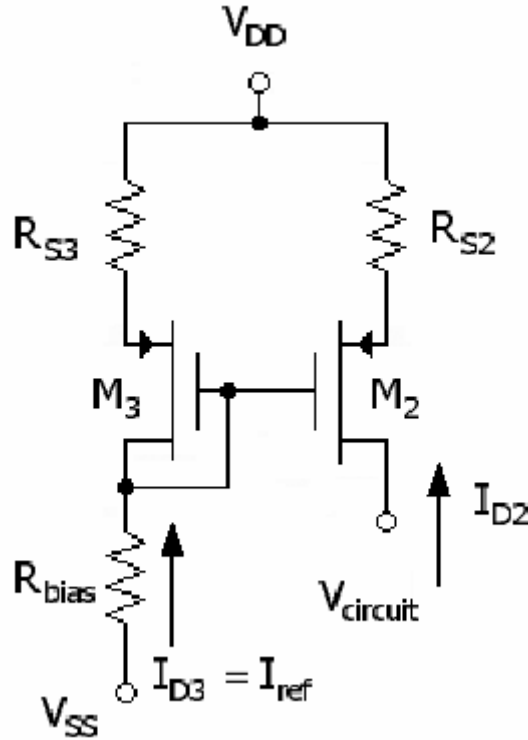
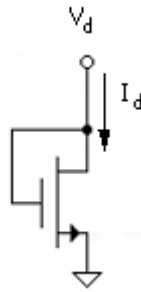
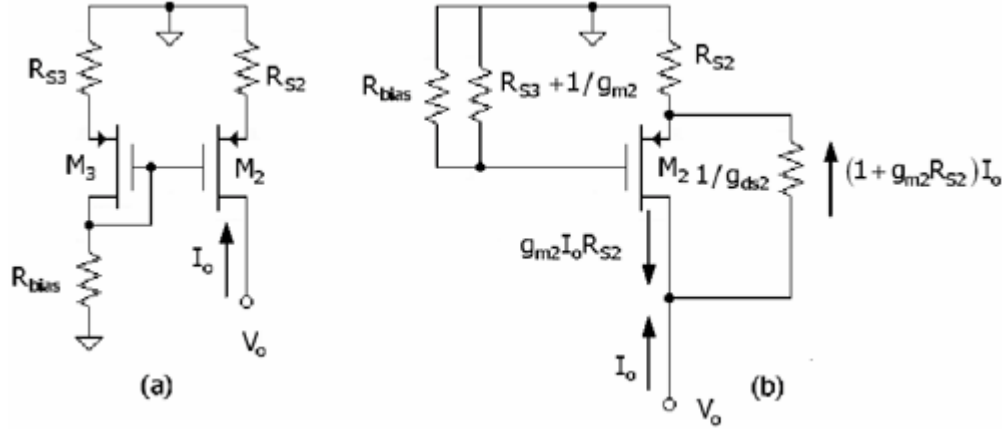


Figure 9.3. Signal circuit for diode-connected NMOS.



The new output resistance at the drain of M_2 for the current-source circuit with source resistors (Fig. 9.2) can be derived based on the signal circuits shown in Fig. 9.4. The output resistance is defined as $R_o = V_o/I_o$, where I_o is the drain current flowing in conjunction with the application of the test voltage V_o . Figure 9.4(b) replaces M_3 with the signal model equivalent and M_2 with the ideal, intrinsic transistor model (current source) along with output resistance, $1/g_{ds2}$. The transistor symbol represents the ideal transistor with output current $g_{m2}V_{sg2}$, positive in the direction shown.

Figure 9.4. (a) Signal circuit of Fig. 9.2. (b) Reference transistor circuit replaced with the linear model. The effective resistance at the gate of the current source transistor plays no role other than to return the gate to ground since $I_{g2} = 0$.



The current I_o flowing through the transistor and up through the resistor, R_{S2} , develops a voltage across the resistor, $V_{RS2} = I_oR_{S2}$. Since $I_{g2} = 0$, an input circuit loop equation is simply $V_{gs2} = -I_dR_{S2}$. The transistor linear-model current source, $g_{m2}V_{gs2}$, thus is down, as shown in the signal-circuit diagram of Fig. 9.4(b). The sum of currents through $1/g_{ds}$ is

Equation 9.6

$$I_{gds} = I_o(1 + g_m R_{S2})$$

The voltage V_o is thus

Equation 9.7

$$V_o = \frac{I_o(1 + g_m R_{S2})}{g_{ds}} + I_o R_S$$

and the output resistance is

Equation 9.8

$$R_O = \frac{V_O}{I_O} = \frac{1 + g_{m2} R_{S2}}{g_{ds2}} + R_{S2}$$

This magnitude can be assessed with the use of (4.5), which is $g_m = 2I_D / V_{eff}$. Eliminating g_m in (9.8) results in

Equation 9.9

$$R_O = \left(1 + \frac{2I_{D2} R_{S2}}{V_{eff2}} \right) \frac{1}{g_{ds2}} + R_{S2}$$

If the magnitude of the voltage across R_{S2} is several volts, then R_O is much greater than $1/g_{ds2}$, since V_{eff2} is typically a fraction of a volt. In integrated circuits, R_S may be replaced with an additional current source, in which case the expression becomes

Equation 9.10

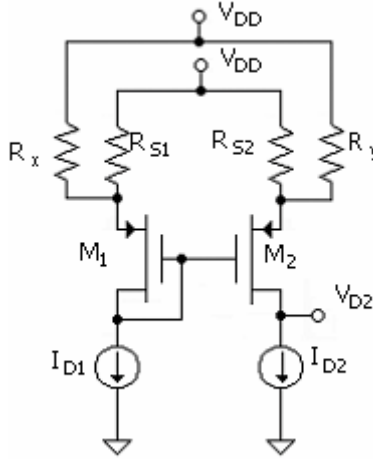
$$R_O \approx \frac{2}{V_{eff2} \lambda_p} \frac{1}{g_{ds2}}$$

where the output resistance of the added current source is approximately $1/I_{D2} \lambda_p$ [(4.13)].

9.3. Differential Amplifier Balancing Circuit

The principle of the balancing of the differential-amplifier stage (e.g., in an opamp) is often based on a circuit similar to that in Fig. 9.2, as shown in Fig. 9.5. Imbalance could be due, for example, to $V_{tpo1} \neq V_{tpo2}$ and $k_{p1} \neq k_{p2}$. Balancing is implemented by adjusting $R_1 \neq R_2$, where $R_1 = R_{S1} // R_x$ and $R_2 = R_{S2} // R_y$.

Figure 9.5. Balancing circuit. Voltage V_{D2} is controlled by selecting the relative values of R_x and R_y .



The relationship among the currents, parameters, and resistors is obtained by writing the loop equation around the source resistors and the gate – source terminals, which is

Equation 9.11

$$I_{D1}R_1 + \sqrt{\frac{I_{D1}}{k_{p1}(1 + \lambda_p V_{GS1})}} + V_{tpo1} = I_{D2}R_2 + \sqrt{\frac{I_{D2}}{k_{p2}(1 + \lambda_p V_{GS2})}} + V_{tpo2}$$

A solution for V_{SD2} can be obtained from (9.11) for a given set of parameters, resistors, and bias variables; on the other hand, R_x and R_y are adjusted to obtain a certain V_{SD2} .

For the special case of $I_{D1} = I_{D2} = I_D$, the difference between the resistors in (9.11) is

Equation 9.12

$$\Delta R = R_1 - R_2 = \frac{V_{tpo2} - V_{tpo1} + \sqrt{\frac{I_D}{k_{p2}(1 + \lambda_p V_{GS2})}} - \sqrt{\frac{I_D}{k_{p1}(1 + \lambda_p V_{GS1})}}}{I_D}$$

For a numerical example, suppose the goal is to set $V_{SD2} = V_{SD1}$ ($V_{D2} = V_{D1}$). By design, in (9.12),

Equation 9.13

$$V_{SG1} = V_{SD1} = \sqrt{\frac{I_D}{k_{p1}(1 + \lambda_p V_{SG1})}} + V_{t_{no1}} = V_{SD2}$$

Any slight possible difference in λ_p is neglected.

In a practical opamp with this balancing configuration, the source nodes are connected to external pins. These pins are connected through external resistors R_x and R_y to V_{DD} as shown in [Fig. 9.5](#). For adequate adjustment sensitivity, R_x and R_y are greater than $R_{S1} \approx R_{S2}$ by at least a factor of 10. In practice, the external resistances are implemented with a potentiometer.

Assume that the transistor parameter values are $V_{tpo1} = 1.49$ V, $V_{tpo2} = 1.51$ V, $k_{p1} = 345$ $\mu\text{A}/\text{V}^2$, $k_{p2} = 335$ $\mu\text{A}/\text{V}^2$, and $\lambda_p = 0.02$ V^{-1} . Also assume that the bias current $I_D = 100$ μA . We select $R_{S1} = R_{S2} = R_S = 3$ $\text{k}\Omega$ (Voltage $I_D R_S = 0.3$ V) and we use $R_x + R_y = R_{\text{pot}} = 25$ $\text{k}\Omega$; the two resistors are segments of a 25-k Ω potentiometer. The segments of the potentiometer are then determined from a solution to

Equation 9.14

$$\frac{R_x}{R_x + R_S} - \frac{R_{\text{pot}} - R_x}{R_{\text{pot}} - R_x + R_S} = \frac{\delta R}{R_S}$$

where δR is obtained from [\(9.12\)](#) with $V_{SD2} = V_{SG1}$ as obtained from [\(9.13\)](#). The resistor values are $R_x = 16$ $\text{k}\Omega$ and $R_y = 9$ $\text{k}\Omega$ to give $R_1 = 2.53$ $\text{k}\Omega$ and $R_2 = 2.25$ $\text{k}\Omega$.

An evaluation of V_{SD2} without the balancing circuit can be made by setting $\delta R = 0$ in [\(9.12\)](#) and for solving V_{SD2} to obtain

Equation 9.15

$$V_{SD2} = \left[\frac{I_D}{k_{p2}(V_{SG1} - V_{t_{po2}})^2} - 1 \right] \frac{1}{\lambda_p}$$

V_{SG1} is again obtained from [\(9.13\)](#).

With the numbers from the example above, $V_{SD2} = 7.87$ V. We note that with $\lambda_p = 0.01$ V^{-1} , the solution is $V_{SD2} = 13.2$ V and the circuit might well have exceeded the power-supply limits. If $V_{tpo1} = 1.45$ V, $V_{tpo2} = 1.55$ V, and $\lambda_p = 0.02$ V^{-1} that is, for an extreme case of the difference of threshold voltage, the balancing circuit will still function but now $R_x = 21.8$ k Ω and $R_y = 3.24$ k Ω with a small $R_2 = 1.56$ k Ω .

9.4. Summary of Equations

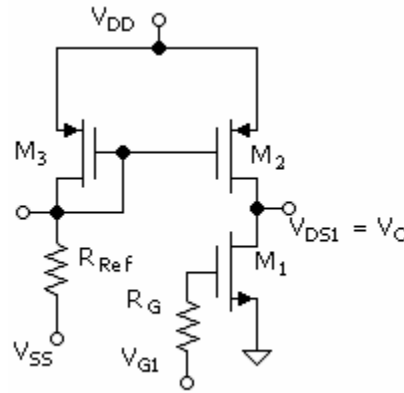
$\frac{I_{D2}}{I_{D3}} = \frac{\frac{W_2}{W_3} \frac{1+\lambda_p V_{SD2}}{1+\lambda_p V_{SG3}}}{\frac{W_2}{W_3}} \approx \frac{W_2}{W_3}$	Reference current, I_{D3} , and current-source current, I_{D2} , relation.
$R_O = \frac{V_O}{I_O} = \frac{1+g_{m2}R_{S2}}{g_{ds2}} + R_{S2}$	Output resistance of current-source with source degeneration.
$R_O = \left(1 + \frac{2I_{D2}R_{S2}}{V_{eff2}}\right) \frac{1}{g_{ds2}} + R_{S2}$	Output resistance of current-source with source degeneration and $g_m = 2I_D/V_{eff}$.
$\delta R = \frac{V_{tpo2} - V_{tpo1} + \sqrt{\frac{I_D}{k_{p2}(1+\lambda_p V_{SD2})}} - \sqrt{\frac{I_D}{k_{p1}(1+\lambda_p V_{SG1})}}}{I_D}$	Relation for setting V_{D2} in balancing circuit with $R_1 = R_{S1}/R_x$ and $R_2 = R_{S2} R_y$. $\delta R = R_1 - R_2$
$V_{SD1} = \sqrt{\frac{I_D}{k_{p1}(1+\lambda_p V_{SD1})}} + V_{tno1} = V_{SD2}$	Relation for $V_{D2} = V_{D1}$ for $I_{D1} = I_{D2} = I_D$.
$V_{SD2} = \left(\frac{I_D}{k_{p2}(V_{SG1} - V_{tpo2})^2} - 1 \right) \frac{1}{\lambda_p}$	V_{SD2} for unbalanced circuit with $\delta R = 0$ and $I_{D1} = I_{D2} = I_D$.

Unit 10. Common-Source Amplifier with Current-Source Load

The amplifier in this part is the same, in principle, as the basic common-source amplifiers of [Figs. 2.4](#) and [5.1](#). However, now the actual resistor R_D is replaced with a current-source load as shown in the circuit of [Fig. 10.1](#). As noted in [Unit 9](#), a benefit of replacing bias resistors with current sources is a reduced requirement for resistors in the circuit; the reference voltage associated with M_3 can be used elsewhere in a typical circuit. Additionally, as will be shown, the gain of the common-source stage is substantially improved over that with actual bias resistor R_D . The specific circuit of [Fig. 10.1](#) is that for the project amplifier. The bias and gain of the amplifier are evaluated in the following.

Figure 10.1. Common-source amplifier with driver transistor M_1 and current-source load from drain of M_2 . In the amplifier project, an output channel sets

V_{G1} for a given current and another output channel will search for the V_{SS} to set up bias $V_O = V_{DD}/2$. For the signal measurements, a signal voltage is superimposed on the dc V_{G1} .



10.1. DC (Bias) Circuit

The bias circuit is critical in terms of getting the dc output voltage at a value near the project design value of $V_O = V_{DD}/2$ (for maximum output magnitude and linearity). The relation, from the dc circuit analysis, for obtaining this condition is based on $I_{D1} = I_{D2}$. This is, in terms of the transistor characteristic equations, (3.8),

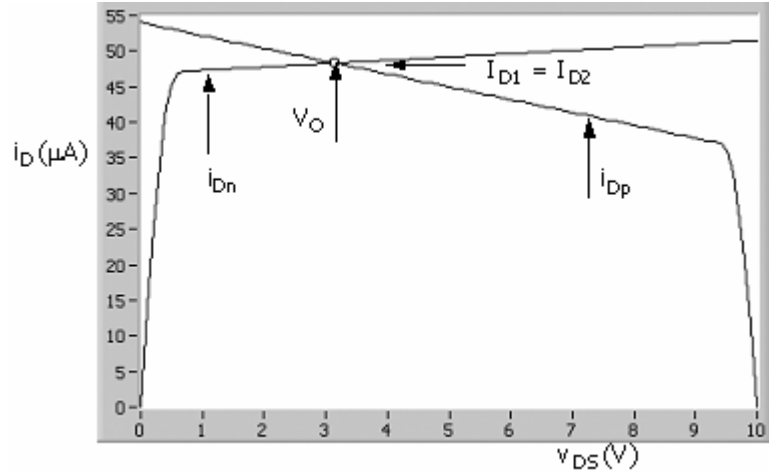
Equation 10.1

$$k_{p2} (V_{SG3} - V_{tp0})^2 (1 + \lambda_p V_{DD}/2) = k_{n1} (V_{GS1} - V_{tn0})^2 (1 + \lambda_n V_{DD}/2)$$

Note that the source-gate voltage of M_2 is referenced to M_3 . A certain combination of V_{GS1} and V_{SS} will satisfy $V_O = V_{DD}/2$. In the project on the amplifier, a LabVIEW VI sends out a $V_{G1} = V_{GS1}$ to set up a given $I_{D1} = I_{D2}$ and then adjusts V_{SS} to obtain the desired bias output voltage.

[Figure 10.2](#) shows a SPICE plot of the current for the PMOS and NMOS transistors as a function of V_{DS1} . The source – drain voltage for the PMOS is $V_{SD2} = V_{DD} - V_{DS1}$. Both transistors have a specific gate – source voltage. The solution for the output voltage for this case is about 3 V. A slight decrease in V_{GS1} or a slight increase in V_{SG2} is required to bring V_{DS1} to 5 V, the design result in this example. The increase in V_{SG2} would be implemented by making V_{SS} more negative to increase the reference current. The steeper slope in the active region of the PMOS device is consistent with $\lambda_p > \lambda_n$, as in the project amplifier.

Figure 10.2. Plot of driver and load transistor output characteristics on common voltage scale. Since $I_{D1} = I_{Dn} = I_{D2} = I_{Dp}$, $V_{DS1} = V_O$ is where currents intersect. The current from the current source load needs, in this example, to be slightly increased to set $V_{DS1} = V_O$, with $V_{DD} = 5V$.



10.2. Signal Voltage Gain

The expression for the gain of the basic common-source amplifier, (5.4), which includes the output resistance of the driver transistor, is

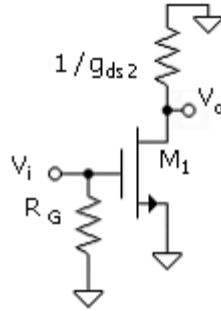
$$a_v = -g_m \frac{R_D}{1 + g_{sd}R_D}$$

In the signal circuit, as shown in Fig. 10.3, R_D is replaced by $1/g_{ds2}$. The new equation for this circuit is

Equation 10.2

$$a_v = -\frac{g_{m1}}{g_{ds1} + g_{ds2}}$$

Figure 10.3. Linear circuit of the common-source amplifier. The load is now the output resistance of the current source load, $1/g_{ds2}$.



or with $g_m = 2I_D/V_{eff}$, (4.5), and $g_{ds} \approx I_D \lambda$, (4.13),

Equation 10.3

$$a_v \approx -\frac{g_{m1}}{(\lambda_n + \lambda_p)I_{D1}} \approx -\frac{2}{(\lambda_n + \lambda_p)V_{eff1}}$$

The result shows that the gain increases for decreasing V_{eff1} . This is limited by the fact that the MOSFET ceases to behave in the normal manner at some lower limit on V_{eff1} .

Using $g_{m1} \approx 2\sqrt{k_{n1}I_{D1}}$ an alternative form is obtained, which reveals the I_{D1} dependence of the gain. This is

Equation 10.4

$$a_v \approx -\frac{2}{\lambda_n + \lambda_p} \sqrt{\frac{k_{n1}}{I_{D1}}}$$

Increasing gain is achieved with decreasing the level of bias current. The lower limit is, as noted above, associated with a lower limit on V_{eff1} . Also, there is an approximate inverse trade-off between frequency response limitations, on the high end of the spectrum, and gain.

The gain, though, for the amplifier with current source load, will generally be several times as large as that with the resistive load. An amplifier with even higher gain can be obtained by replacing the current source load with one with source degeneration as discussed in [Unit 9.2](#), and as discussed extensively for the case of the BJT in [Unit C](#).

10.3. Summary of Equations

$k_{p2} (V_{SGS} - V_{tp0})^2 (1 + \lambda_p V_{DD} / 2) = k_{n1} (V_{GS1} - V_{tn0})^2 (1 + \lambda_n V_{DD} / 2)$	Output drain-current balance equation.
$a_v = - \frac{g_{m1}}{g_{ds1} + g_{ds2}}$	Voltage-gain equation.
$a_v \approx - \frac{g_{m1}}{(\lambda_n + \lambda_p) I_{D1}} \approx - \frac{2}{(\lambda_n + \lambda_p) V_{effn1}}$	Voltage-gain equation in terms of λ 's and V_{effn1} .
$a_v \approx - \frac{2}{\lambda_n + \lambda_p} \sqrt{\frac{k_{n1}}{I_{D1}}}$	Voltage-gain equation in terms of λ 's and I_D .

10.4. Exercises and Projects

Project Mathcad Exercise10.mcd - Project10.mcd
Files

Laboratory Project 10 [Current Mirror and Common-Source Amplifier with Current-Source Load](#)

P10.2 [Evaluation of the Current-Source Circuit](#)

P10.3 [Evaluation of the Mirror-Current Circuit](#)

P10.4 [Evaluation of the Bias Setup](#)

P10.5 [Measurement of the Amplifier Gain versus Drain Current](#)

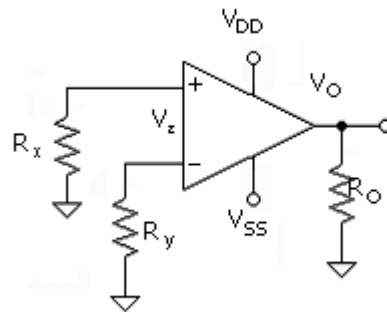
Unit 11. Operational Amplifiers with Resistor Negative Feedback

In the following units we consider the operational amplifier in the basic resistive feedback amplifier configurations. We discuss gain, dc offset, and frequency response. In projects, the opamp is implemented in the noninverting amplifier mode to evaluate voltage gain, opamp offset, bias stabilization, and amplifier and opamp frequency response.

11.1. Operational Amplifiers with Resistance Feedback

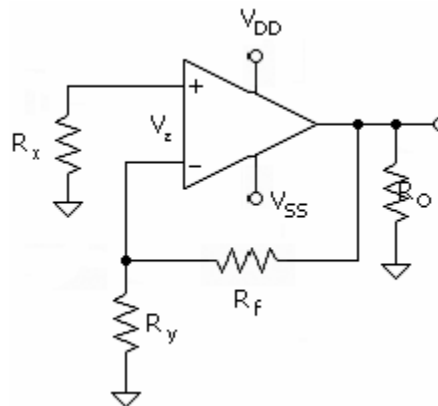
In this unit, the gain characteristics of the operational amplifiers with resistive feedback are discussed. These are dc amplifiers that are configured for specific gain and input and output resistance characteristics. The operational amplifier without feedback is in the open-loop mode. The dc (bias) configuration is shown in [Fig. 11.1](#). Due to imbalances in the amplifier circuit, which are a result of variations in the parameters of the transistors and circuit components from the values used in the design, the output will probably be latched at either the plus or minus power supply.

Figure 11.1. Open-loop amplifier. Inputs, output, and power-supply pins are connected. Circuit will be bias unstable. Dc V_O is likely to be latched at near V_{DD} or V_{SS} .



The circuit can be set into a stable, active mode with the output approximately at zero volts by providing resistive negative feedback as shown in the circuit diagram of [Fig. 11.2](#).

Figure 11.2. Opamp resistor network, including feedback resistor for bias stabilization. Signal output voltage is $V_o = a_{vo} V_{\epsilon}$.



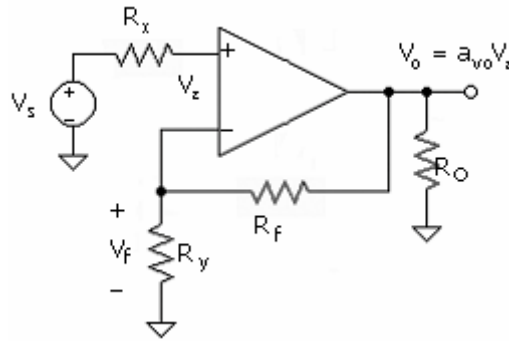
The resistor connected between the output, V_O , and the inverting (minus) input effectively applies the output voltage to the opamp input and it is of such a polarity as to drive the output toward zero volts, where it tends to stay. That is, attaching the resistor completes the negative feedback loop from V_{ϵ} to V_{Ry} . The quantitative aspects of stabilization with the feedback resistor are discussed in [Unit 11.6](#).

This circuit becomes a dc amplifier by installing a signal at either input. These two possibilities are discussed in the following units.

11.1.1. Voltage Gain of the Noninverting Resistor Feedback Amplifier

In the circuit diagram of [Fig. 11.3](#), an input signal voltage V_s is attached to resistor R_x . By definition, the output terminal voltage is positive for a plus input. Hence, this is the noninverting amplifier. Here we obtain the relationship between the amplifier gain, $a_{vo} = V_o/V_\epsilon$, and the open-loop gain (opamp gain), $a_{vo} = V_o/V_\epsilon$, and the circuit resistors.

Figure 11.3. Non-inverting or voltage amplifier (series-shunt). The input resistance is essentially infinite.



With a signal applied to the plus input terminal, the responding output voltage is fed back to the resistor R_y . The voltage across R_y , V_f , and V_o (signal) are related by

Equation 11.1

$$V_f = \frac{R_y}{R_y + R_f} V_o$$

This is just the voltage-divider relation, which applies in this case, as negligible current flows into the input terminals of the opamp. Variable V_f is used in lieu of V_{R_y} to distinguish it from the dc value of the voltage across R_y . This use is also consistent with the fact that V_f is technically a signal feedback voltage.

The input signal voltage V_s adds up to

Equation 11.2

$$V_s = V_\epsilon + V_f = \frac{V_o}{a_{vo}} + \frac{R_y}{R_y + R_f} V_o$$

where (11.1) is used to eliminate V_f . (The voltage drop across R_x is essentially zero.) This leads directly to the relation for amplifier gain, which is

Equation 11.3

$$A_v = \frac{V_o}{V_s} = \frac{1}{\frac{1}{a_{vo}} + \frac{1}{1 + \frac{R_f}{R_y}}} = \frac{A_{vNI}}{1 + \frac{A_{vNI}}{a_{vo}}}$$

with (ideal noninverting gain).

Equation 11.4

$$A_{vNI} = 1 + \frac{R_f}{R_y}$$

A_{vNI} is the limiting form of the noninverting amplifier gain for $a_{vo} \rightarrow \infty$. This is consistent with the fact that in the limit, $V_e \rightarrow 0$ and $V_s = V_{Ry}$. Thus, the output and input voltages are simply related by the voltage-divider relation. The result, (11.3), indicates that for $a_{vo} \gg A_{vNI}$, the voltage gain can simply be expressed in terms of the resistors and therefore is very predictable. If we make, for example, $R_f = 10R_y$, $A_{vNI} = 11$, and (11.3) gives $A_v = 10.998$, with $a_{vo} = 40,000$. The value for the opamp gain is typical for our project opamp. Note that due to the high resistance at the opamp input terminals, R_x has no influence on the gain.

The noninverting amplifier has a very high input resistance and a low output resistance, as discussed in Unit 11.4. The amplifier technically falls into the category of a series – shunt feedback configuration or a voltage amplifier.

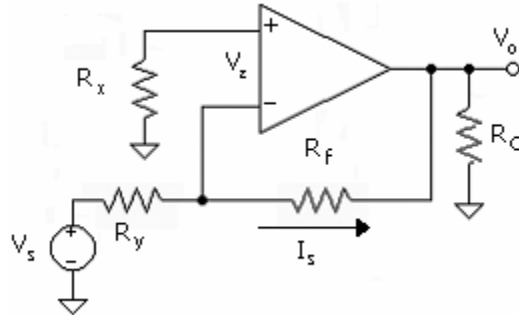
11.1.2. Voltage Gain of the Inverting Resistor Feedback Amplifier

To obtain the inverting amplifier, the signal is applied to the negative or inverting terminal as shown in Fig. 11.4. A positive signal results in a negative output voltage. The gain expression can be obtained with the loop equation from output to input:

Equation 11.5

$$V_s - V_o = I_s R_y + I_s R_f$$

Figure 11.4. Inverting or transresistance amplifier (shunt – shunt). The input resistance is equal to the signal-source resistance, R_y .



and the loop equation at the input (voltage drop across R_x is zero)

Equation 11.6

$$V_s = I_s R_y + V_z = I_s R_y + \frac{V_o}{a_{vo}}$$

Eliminating I_s between (11.5) and (11.6) gives

Equation 11.7

$$V_s = \frac{V_o}{a_{vo}} + \frac{V_s - V_o}{R_y + R_f} R_y$$

This expression can be manipulated to give the gain as

Equation 11.8

$$\frac{V_o}{V_s} = \frac{R_f}{R_y} \frac{\frac{1}{R_y + R_f}}{\frac{1}{a_{vo} R_y} - \frac{1}{R_y + R_f}}$$

which is

Equation 11.9

$$A_v = A_{vI} \frac{1}{1 - \frac{1 - A_{vI}}{a_{vo}}}$$

where

Equation 11.10

$$A_{vI} = -\frac{R_f}{R_y}$$

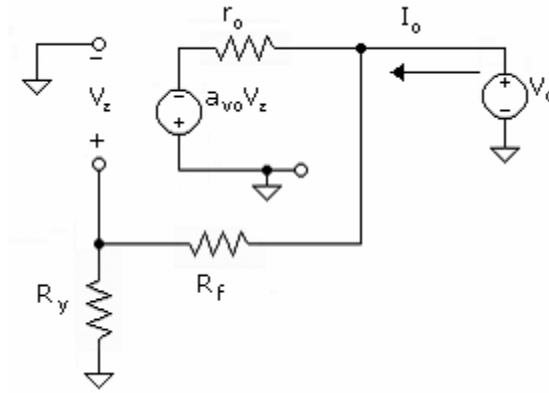
A_{vI} is the gain of the ideal inverting amplifier.

The ideal gain, as in the case of the noninverting amplifier, depends only on resistor values. The approximate form is based on the approximation $V_E \rightarrow 0$, in which case the negative input terminal is at virtual ground. Thus, $|v_o|$ and V_s are proportional to R_f and R_y , respectively. The inverting-amplifier gain result includes the fact that the current into the negative terminal is zero. The circuit is a shunt – shunt feedback configuration or a transresistance amplifier.

11.2. Output Resistance of the Resistor Feedback Amplifier

The negative feedback of the amplifiers will alter the output resistance of the circuit. In the case of series – shunt feedback (noninverting amplifier) and the shunt – shunt (inverting amplifier), the output resistance will be reduced from that of the open-loop amplifier. This can be explained with the use of the circuit of [Fig. 11.5](#), which shows a linear-model circuit for the amplifiers of [Figs. 11.3](#) and [11.4](#), with the inputs set to zero.

Figure 11.5. Linear equivalent circuit for deriving opamp output resistance. Test voltage, V_o , is applied at the opamp output. The inputs are grounded for deriving the output resistance.



For the present purposes, a voltage-dependent voltage source equivalent circuit for the opamp is chosen. The parameter r_o is the output resistance of the open-loop opamp. For example, the output resistance would be about $1/g_m$ in a CMOS opamp with a source-follower stage-output stage.

A test voltage, V_o , is applied at the output terminal with the input grounded. This results in an input to the opamp of magnitude $V_{in} = V_o/A_{vNI}$. The voltage-dependent voltage source of the opamp is thus $(a_{vo}/A_{vNI})V_o$. The total current, I_o , flowing from the test voltage is then

Equation 11.11

$$I_o = \left(\frac{1}{R_f + R_y} + \frac{1 + a_{vo}/A_{vNI}}{r_o} \right) V_o = \left(\frac{r_o}{R_f + R_y} + 1 + a_{vo}/A_{vNI} \right) \frac{V_o}{r_o}$$

Therefore, the output resistance is

Equation 11.12

$$R_o = \frac{V_o}{I_o} = \frac{r_o}{\frac{r_o}{R_y + R_f} + 1 + \frac{a_{vo}}{A_{vNI}}} \approx \frac{r_o}{1 + T}$$

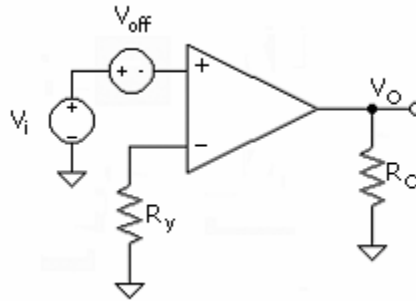
where $T = a_{vo}/A_{vNI}$ (i.e., the loop gain of the feedback amplifier). The approximate form comes from the expectation that $r_o \ll R_y + R_f$; it neglects the current through the feedback resistors. $R_o \approx r_o/T$ is a valid approximation in most cases and $R_o \ll r_o$. For example, for $A_{vNI} \approx 10$ and $a_{vo} \approx 40,000$, $T = 4000$, and $R_o \approx r_o/4000$.

11.3. Operational Amplifier Offset

Ideally, an opamp is balanced such that the dc output is zero (with dual power supply) with zero input voltage (as in [Fig. 11.1](#)). In practice, this is not realized, due to the variation of parameters between similar transistors and components. As mentioned above, the opamp configuration of [Fig. 11.1](#) will probably be locked at near the positive or negative rail voltage. Thus, a given opamp requires an input voltage V_{in} to set the output to zero. The magnitude of this voltage is defined as offset voltage V_{off} .

An equivalent circuit that includes the offset voltage is given in [Fig. 11.6](#). The imperfections of the amplifier are reflected into the offset voltage and the opamp is ideal. If V_i is made equal to V_{off} , the output V_O will be zero. Suppose that $V_i = 0$. In this case we have the equivalent of V_{off} applied to the input and the output is driven toward the plus or minus power-supply voltages (depending on the polarity of V_{off}). V_{off} is usually large enough to cause the amplifier to be driven out of the active mode. For example, a typical V_{off} is $V_{off} = 1$ mV. Thus, if $a_{vo} = 40,000$, the output will attempt to become -40 V, which is probably greater than the supply voltage value; the output is set at the negative limit. In our project opamp, this will be roughly $V_{SS} + 50$ mV if driven negative or $V_{DD} - 1$ V if driven toward the positive supply.

Figure 11.6. Ideal opamp with offset voltage. With dc input voltage $V_i = V_{off}$, $V_O = 0$. With $V_i = 0$ (grounded input), $V_O \approx V_{DD}$ or V_{SS} with $|a_{vo}V_{off}| > V_{DD}$ or $|V_{SS}|$ for V_{off} negative or positive, respectively.



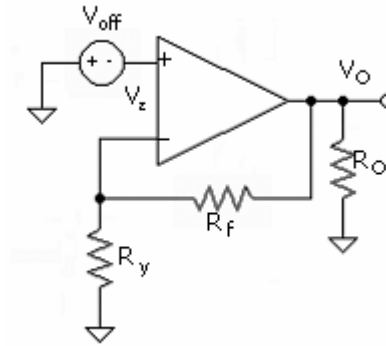
11.4. DC Stabilization with the Feedback Resistor

The dc output voltage can be set close to zero with negative feedback for normal operation as a linear amplifier as illustrated in [Fig. 11.7](#). A portion of the output voltage is applied back to the input as the voltage across R_y . This is

Equation 11.13

$$V_{R_y} = \frac{R_y}{R_y + R_F} V_O$$

Figure 11.7. Opamp dc stabilized with feedback resistor network. The output voltage is applied back to the input to then drive the output toward a smaller value. The voltage across R_y will be about equal to V_{off} and V_O is forced to near zero.



(Dc V_O is used, as this is a no-signal or bias state under consideration.) This voltage will cancel most of V_{off} , such as to drive V_O close to zero if R_f is small enough compared to R_y . The value of V_O for a given R_f can be obtained as follows: A loop equation at the input side ([Fig. 11.7](#)) gives

Equation 11.14

$$\frac{R_y}{R_y + R_f} V_O + V_z + V_{off} = 0$$

The opamp circuit is a dc amplifier. Therefore, the input – output relation holds for dc as well as for signals. Thus, $V_{\epsilon} = V_O/a_{vo}$ such that

Equation 11.15

$$\frac{R_y}{R_y + R_f} V_O + \frac{V_O}{a_{vo}} + V_{off} = 0$$

and

Equation 11.16

$$V_O = \frac{-A_{vNI}}{1 + \frac{A_{vNI}}{a_{vo}}} V_{off} \approx -A_{vNI} V_{off}$$

where the approximation usually applies and is equivalent to $V_E = 0$.

For example, with $V_{off} = 1$ mV and $A_{vNI} = 100$, $V_O \approx -0.1$ V. This would be a very satisfactory dc state for using the circuit as an amplifier. Also, the approximation of the right-hand side of (11.16) is quite valid. Note that without the feedback network, $V_O = -a_{vo} V_{off} = -40,000 \cdot 1$ mV = -40 V. Assuming that the power-supply voltage is, for example, ± 15 V, the output is locked at $V_O \approx -15$ V and the linear gain relation does not apply. In our project with the opamp, we determine V_{off} from a measurement of V_O and the application of (11.16).

11.5. Frequency Response of the Operational Amplifier and Resistor Feedback Amplifier

The MOSFET has a number of capacitances, including those associated with the gate structure and source and drain pn junctions as well as other parasitic capacitors of the transistor and the circuit. Therefore, the open-loop opamp will tend to be unstable due to the poles and zeros associated with the capacitors. Opamps are usually stabilized with the addition of a capacitor. The added pole is well below the lowest naturally occurring pole. This pole will dominate the frequency response of the opamp. In a lab project, the pole frequency will be measured.

We will obtain the frequency response of the feedback amplifier based on the single-pole response. The form of the response of the open-loop opamp is

Equation 11.17

$$a_v(f) = \frac{a_{vo}}{1 + j \frac{f}{f_{3dB \text{ opamp}}}}$$

The response function has value a_{vo} at low frequencies and has magnitude $a_{vo}/\sqrt{2}$ at $f = f_{3dB \text{ opamp}}$, where $f_{3dB \text{ opamp}}$ is the characteristic frequency of the opamp. This is indirectly, $f_{3dB \text{ opamp}} = \text{GBP}/a_{vo}$, where GBP is the gain – bandwidth product of the opamp as given in the datasheet. Datasheet value a_{vo} is also given (A_{vd} , large-signal voltage gain).

The noninverting feedback amplifier has a gain as given by (11.3). That expression is used here along with (11.17) in place of a_{vo} . The frequency response of the feedback amplifier is thus

Equation 11.18

$$A_v(f) = \frac{A_{vNI}}{1 + \frac{A_{vNI}}{a_v(f)}}$$

which is

Equation 11.19

$$A_v(f) = A_{vNI} \frac{1}{1 + \frac{A_{vNI}}{a_{vo}} \left[1 + j \frac{f}{f_{3dB \text{ op amp}}} \right]} = \frac{A_{vNI}}{1 + \frac{A_{vNI}}{a_{vo}}} \frac{1}{1 + j \frac{f}{f_{3dB \text{ op amp}} \left(\frac{a_{vo}}{A_{vNI}} + 1 \right)}}$$

This can be simplified to

Equation 11.20

$$A_v(f) = \frac{A_{vNI}}{1 + \frac{A_{vNI}}{a_{vo}}} \frac{1}{1 + j \frac{f}{f_{BW}}} = A_{vo} \frac{1}{1 + j \frac{f}{f_{BW}}}$$

where A_{vo} is the amplifier gain at low frequencies and where

Equation 11.21

$$f_{BW} = f_{3dB \text{ op amp}} \left(1 + \frac{a_{vo}}{A_{vNI}} \right) = f_{3dB \text{ op amp}} \left(1 + a_{vo} \frac{R_y}{R_y + R_f} \right)$$

Frequency parameter f_{BW} is the bandwidth of the amplifier for a specific A_{vNI} . Parameter f_{BW} , by definition of bandwidth, is also f_{3dB} of the amplifier. Adding feedback by reducing R_f always increases bandwidth at the expense of gain. Note that for R_f very small, f_{BW} can be relatively high and the frequency response is complicated by the fact that other poles of the opamp come into play.

Given in [Table 11.1](#) is a segment from the manufacturer's datasheet for the opamp (TS271C) used in our project. As mentioned above, the parameter for large-signal voltage

gain is designated as A_{vd} and is identical to a_{vo} . The GBP indicated is 700 kHz. Thus for a low-frequency amplifier gain of, for example, $A_v = 5000$, the bandwidth is $f_{BW} = 700 \text{ kHz}/5K = 140 \text{ Hz}$, which can be readily measured using LabVIEW and the DAQ in the computer.

TABLE 11.1					
Symbol	Parameter	TS271C/AC/BC			Unit
		Min.	Typ.	Max	
A_{vd}	Large Signal Voltage Gain				
	$V_O = 1 \text{ V to } 6 \text{ V}, R_L = 100 \text{ k}\Omega, V_{ic} = 5 \text{ V}$	30	50		V/mV
	$T_{min.} < T_{amb} < T_{max.}$	20			
GBP	Gain Bandwidth Product ($A_v = 40\text{dB}, R_L = 100 \text{ K}\Omega, C_L = 100\text{pF}, f_{in} = 100 \text{ KHz}$)		0.7		MHz

Note that for the limiting case of $R_f \rightarrow \infty$, the bandwidth of the open-loop amplifier is only $f_{3dB} = 700 \text{ kHz}/50K = 14 \text{ Hz}$, where the typical (Typ.) number, $A_{vd} = 50 \text{ k}$ has been used.

11.6. Summary of Equations

$A_v = \frac{A_{vNI}}{1 + \frac{A_{vNI}}{a_{vo}}}$	Noninverting feedback amplifier voltage gain.
$A_{vNI} = 1 + \frac{R_f}{R_y}$	Ideal noninverting amplifier voltage gain.
$A_v = A_{vI} \frac{1}{\left(1 - \frac{1 - A_{vI}}{a_{vo}}\right)}$	Inverting feedback amplifier voltage gain.
$A_{vI} = -\frac{R_f}{R_y}$	Ideal inverting amplifier voltage gain.
$R_o = \frac{r_o}{\frac{r_o}{R_y + R_f} + 1 + \frac{a_{vo}}{A_{vNI}}} \approx \frac{r_o}{1 + T}$	Output resistance of the resistance-feedback opamp amplifier.
$T = a_{vo}/A_{vNI}$	Loop gain of resistance-feedback opamp amplifier.

$A_v = \frac{A_{vNI}}{1 + \frac{A_{vNI}}{a_{vo}}}$	Noninverting feedback amplifier voltage gain.
$V_O = \frac{-A_{vNI}}{1 + \frac{A_{vNI}}{a_{vo}}} V_{off} \approx -A_{vNI} V_{off}$	Output voltage of resistance-feedback opamp with offset voltage, V_{off} .
$A_v(f) = \frac{A_{vNI}}{1 + \frac{A_{vNI}}{a_{vo}}} \frac{1}{1 + j \frac{f}{f_{BW}}} = A_{vo} \frac{1}{1 + j \frac{f}{f_{BW}}}$	Frequency-response function of resistance-feedback opamp amplifier.
$f_{BW} = f_{3dB} \left(1 + \frac{a_{vo}}{A_{vNI}} \right) = f_{3dB} \left(1 + a_{vo} \frac{R_y}{R_y + R_f} \right)$	Bandwidth (corner frequency) of opamp amplifier with opamp open-loop corner frequency, f_{3dB} .

11.7. Exercises and Projects

Project Mathcad Files Exercise11.mcd - Project11.mcd

Laboratory Project 11 [Operational Amplifier with Resistor Feedback](#)

P11.2 [Bias Circuit Setup](#)

P11.3 [Opamp Offset Voltage](#)

P11.4 [Evaluation of the Bias Balancing Circuit](#)

P11.5 [Evaluation of the Gain and Signal Limits with Swept Input](#)

P11.6 [Evaluation of the Gain with Sine-Wave and Square-Wave Signals](#)

P11.7 [Determination of the Opamp Frequency Response](#)

Unit 12. Operational Amplifier Applications with Capacitors

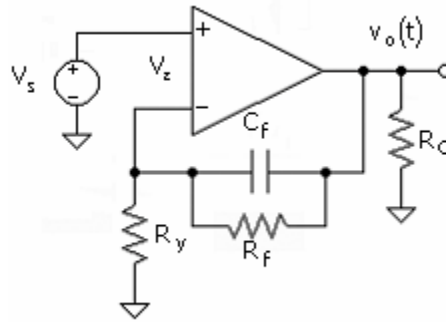
In this unit we consider the most common applications of operational amplifiers where the circuit uses a capacitor as the key component. Included are the opamp integrator and a basic opamp square-wave oscillator. Both forms are investigated in projects.

12.1. Operational Amplifier Integrator

The noninverting or inverting resistance-feedback amplifier can be converted into an integrator with the addition of a feedback capacitor, C_f , as shown in [Fig. 12.1](#).

Specifically, the feedback resistor is replaced with C_f , except that the resistor R_f must remain for dc stabilization purposes.

Figure 12.1. Opamp integrator. The resistor R_f in shunt with C_f is for opamp stabilization purposes.



The operation of the integrator can be explained as follows. For the initial discussion, assume that R_f is infinite. Upon application of a constant voltage V_s , a current is induced in R_y , which is

Equation 12.1

$$i_{R_f} = i_{C_f} = \frac{V_s - V_{\epsilon}}{R_y} \approx \frac{V_s}{R_y}$$

The approximation relies on the fact that $V_{\epsilon} \approx 0$, which can be expected. The resulting output voltage is

Equation 12.2

$$v_o(t) = V_s + \frac{1}{C_f} \int I_{C_f} dt = V_s + \frac{1}{R_y C_f} \int V_s dt$$

To a good approximation, $v_o(t)$ is proportional to the integral of the input. Performing the integration, the result is

Equation 12.3

$$v_o(t) = V_s + \frac{V_s}{R_y C_f} t$$

If V_s is the magnitude of a square-wave, the output will be a triangular wave. In the integrator project we explore the integrator by using it to generate a triangular wave. The project design calls for a peak of the triangular wave equal to a few volts, that is, a significant portion of the power-supply voltage. The time of integration for the plus or minus segment of the square-wave input is $T/2$, where T is the period and $T = 1/f$, that is, the inverse of the square-wave frequency. The peak of the triangular wave then is based on

Equation 12.4

$$2V_{\text{opeak}} \approx \frac{V_s}{R_y C_f} \frac{T}{2}$$

The $2 V_{\text{opeak}}$ is due to the fact that the integration is from minus V_{opeak} to plus V_{opeak} . The voltage drop across R_y has been neglected in the output.

Suppose we choose C_f as the design parameter where V_s , T , V_{opeak} , and R_y (from [Unit 11](#)) are given. The capacitor is then selected according to

Equation 12.5

$$C_f = \frac{V_s}{2 \cdot 2V_{\text{opeak}}} \frac{T}{R_y}$$

For example, for $V_s = 1 \text{ mV}$, $V_{\text{opeak}} = 5 \text{ V}$, $R_y = 220 \text{ } \Omega$ and $T = 1 \text{ mS}$ ($f = 1 \text{ kHz}$), $C_f = 23 \text{ nF}$.

In the integrator project, a sine-wave source is used. The integrated output for this case is

Equation 12.6

$$v_o(t) = V_s \sin\left(2\pi \frac{t}{T}\right) + \frac{T}{2\pi R_y C_f} V_s \cos\left(2\pi \frac{t}{T}\right)$$

The sine term can be neglected and the resulting formulation can be used to predict the peak of the cosine for a given C_f and peak V_s .

A more general solution for the output voltage for the noninverting integrator includes the current through the resistor R_f . If we neglect the voltage across R_y , (12.2) becomes, for this case (constant input or square-wave input)

Equation 12.7

$$v_o(t) = \frac{1}{C_f} \int \left(\frac{V_s}{R_y} - \frac{v_o(t)}{R_f} \right) dt$$

The solution is

Equation 12.8

$$v_o(t) = \frac{V_s}{R_y C_f} \left(1 - e^{-t/R_f C_f} \right)$$

Replacing the exponential with the first three terms of an infinite series gives

Equation 12.9

$$v_o(t) \approx \frac{V_s}{R_y C_f} t \left(1 - \frac{t}{2R_f C_f} \right)$$

This demonstrates that for the circuit to function properly as an integrator, the choice of R_f should satisfy $R_f C_f \gg T/2$. In the above example, $C_f = 23$ nF. With this capacitor and $R_f = 1$ M Ω , for example, $R_f C_f = 23$ ms, compared with $T/2 = 0.5$ ms. The magnitude of R_f must also satisfy the requirement of sustaining the operational amplifier in the active mode. The solution for the periodic square-wave input, giving an approximate triangular-wave output, unlike (12.8), includes the fact that the transient does not start at zero volts. This case is considered in Exercise12.mcd, Part 2. The output voltage function equivalent to (12.2) for the inverting-mode integrator is

Equation 12.10

$$v_o(t) = \frac{-1}{R_y C_f} \int V_s dt$$

This would apply to the circuit of [Fig. 12.1](#) with the plus input grounded and the source, V_s , attached at the grounded end of R_y .

12.2. Operational Amplifier Oscillator

The opamp can serve as a square-wave oscillator by adding positive feedback with R_1 and R_3 and replacing R_y with a capacitor C_o , as shown in [Fig. 12.2](#). The frequency of the operation can be computed on the basis of the following derivation. With positive feedback, the circuit is unstable and the output voltage is either V_{op} or $-V_{om}$, that is, the positive or negative limit. The limits are close to V_{DD} or V_{SS} . These voltage values are applied to the input through the positive feedback network such that the plus input terminal is either at

Equation 12.11

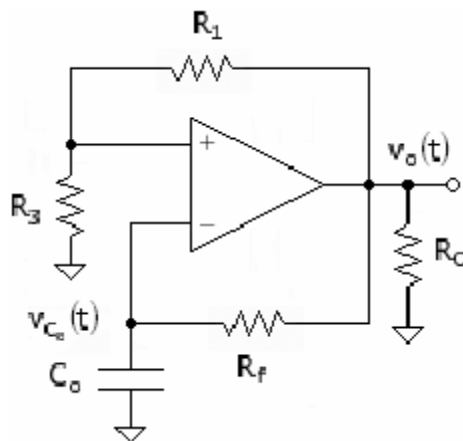
$$V_{+p} = \frac{R_3}{R_3 + R_1} V_{op}$$

or

Equation 12.12

$$V_{+m} = -\frac{R_3}{R_3 + R_1} V_{om}$$

Figure 12.2. Opamp oscillator. The frequency is determined by the value of resistors and the capacitor.



where V_{om} is positive.

However, the negative-feedback circuit consisting of R_f and C_o will cause the capacitor to charge up toward the voltage at the output. For example, suppose that for an interval of time, $v_o(t) = V_{op} \approx V_{DD}$. Also assume that $R_3 = R_1$ such that the plus input is at $V_{op}/2$. When the capacitor charges up to and just beyond $V_{op}/2$, the opamp input V_E changes sign and the output will switch over to $v_o(t) = -V_{om}$. At this point, the capacitor begins charging toward the opposite polarity, and so forth.

For a given output, the time-dependent capacitor voltage is a simple RC transient. For example, for $v_o(t) = V_{op}$,

Equation 12.13

$$v_{C_o}(t) = K e^{\frac{-t}{R_f C_o}} + V_{op}$$

with a steady-state value of V_{op} . As an oscillator, the capacitor voltage will never reach the steady-state value; the transient terminates at the value given by (12.11).

The output is switching back and forth from V_{op} to $-V_{om}$. Therefore, the initial value of the capacitor voltage in (12.13) is

Equation 12.14

$$v_{C_o}(0) = \frac{-R_3}{R_3 + R_1} V_{om}$$

Using this to obtain a complete solution for (12.13) results in

Equation 12.15

$$v_{C_o+}(t) = \left(-V_{op} - \frac{R_3}{R_3 + R_1} V_{om} \right) e^{\frac{-t}{R_f C_o}} + V_{op}$$

We define $t = T_{plus}$ as the positive time segment of the square-wave period. As noted, this corresponds to the time where the output switches to $-V_{om}$; that is,

Equation 12.16

$$v_{C_o}(T_{plus}) = \frac{R_3}{R_3 + R_1} V_{op}$$

Using (12.16) in the left-hand side of (12.15) gives an equation for T_{plus} as follows:

Equation 12.17

$$\frac{R_3}{R_3 + R_1} V_{op} = \left(-V_{op} - \frac{R_3}{R_3 + R_1} V_{om} \right) e^{\frac{-T_{plus}}{R_f C_o}} + V_{op}$$

Then, solving for T_{plus} results in

Equation 12.18

$$T_{plus} = R_f C_o \ln \left(\frac{V_{op} + \frac{R_3}{R_3 + R_1} V_{om}}{\frac{R_1}{R_3 + R_1} V_{op}} \right)$$

Following the reasoning that led to (12.15), the falling capacitor-voltage transient is

Equation 12.19

$$v_{C_o}(t) = \left(V_{om} + \frac{R_3}{R_3 + R_1} V_{op} \right) e^{\frac{-t}{R_f C_o}} - V_{om}$$

The total time interval (negative segment of the square-wave period) of the falling transient, $t = T_{minus}$, corresponds to the capacitor voltage, decreasing to

Equation 12.20

$$v_{C_o}(T_{minus}) = -\frac{R_3}{R_3 + R_1} V_{om}$$

Using (12.20) in (12.19) gives the equation for T_{minus} , which is

Equation 12.21

$$\frac{-R_3}{R_3 + R_1} V_{om} = \left(V_{om} + \frac{R_3}{R_3 + R_1} V_{op} \right) e^{\frac{-T_{\text{minus}}}{R_f C_o}} - V_{om}$$

This leads to

Equation 12.22

$$T_{\text{minus}} = R_f C_o \ln \left(\frac{V_{om} + \frac{R_3}{R_3 + R_1} V_{op}}{\frac{R_1}{R_3 + R_1} V_{om}} \right)$$

A special case is for $R_1 = R_3$, where (12.18) and (12.22) become

Equation 12.23

$$T_{\text{plus}} = R_f C_o \ln \left(\frac{2V_{op} + V_{om}}{V_{op}} \right)$$

and

Equation 12.24

$$T_{\text{minus}} = R_f C_o \ln \left(\frac{2V_{om} + V_{op}}{V_{om}} \right)$$

The oscillator period is $T = T_{\text{plus}} + T_{\text{minus}}$ or, for this special case,

Equation 12.25

$$T = R_f C_o \left[\ln \left(\frac{2V_{op} + V_{om}}{V_{op}} \right) + \ln \left(\frac{2V_{om} + V_{op}}{V_{om}} \right) \right]$$

A good estimate comes from the approximation $V_{op} \approx V_{om}$. The result is

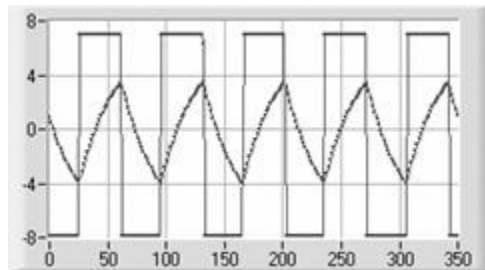
Equation 12.26

$$T = 2R_f C_o \ln(3) \approx 2.2R_f C_o$$

Note that although T_{minus} and T_{plus} can be significantly different [from (12.18) and (12.22)], the sum is well represented by (12.26). Note also that the waveform is strictly a square-wave only if $V_{op} = V_{om}$.

LabVIEW uses (12.25) to calculate the actual value of the capacitor used in the project on the oscillator. In the project, period T is a measured value. Shown in Fig. 12.3 is a project result of the measurement of the output voltage and capacitor voltage for the oscillator. Power-supply voltages were set at plus and minus eight volts and $R_1 = R_3$.

Figure 12.3. Oscillator $v_o(t)$ and $v_{co}(t)$ versus t (ms). Note that the switching takes place where the capacitor voltage reaches about one-half of the square-wave peak magnitude. This is for $R_1 = R_3$.



12.3. Summary of Equations

$2V_{\text{opeak}} \approx \frac{V_s}{R_y C_f} \frac{T}{2}$	Opamp-integrator equation for determining triangular-wave peak amplitude for applied square-wave with peak V_s and period T .
$v_o(t) = V_s \sin\left(2\pi \frac{t}{T}\right) + \frac{T}{2\pi R_y C_f} V_s \cos\left(2\pi \frac{t}{T}\right)$	Opamp-integrator equation for determining output waveform for sine-wave input with peak V_s and period T .

$2V_{\text{opeak}} \approx \frac{V_s}{R_y C_f} \frac{T}{2}$	Opamp-integrator equation for determining triangular-wave peak amplitude for applied square-wave with peak V_s and period T .
$T = R_f C_o \left[\ln \left(\frac{2V_{op} + V_{om}}{V_{op}} \right) + \ln \left(\frac{2V_{om} + V_{op}}{V_{om}} \right) \right]$	Equation for the opamp-oscillator period, for the case of $R_1 = R_3$.
$T = 2R_f C_o \ln(3) \approx 2.2R_f C_o$	Equation for opamp-oscillator period, for the case of $R_1 = R_3$ and equal positive and negative peak magnitudes.

12.4. Exercises and Projects

Project Mathcad Files

Exercise12.mcd - Project12.mcd

Laboratory Project 12

[Operational Amplifier Integrator and Oscillator](#)

P12.2

[Opamp Integrator](#)

P12.3

[Opamp Oscillator](#)

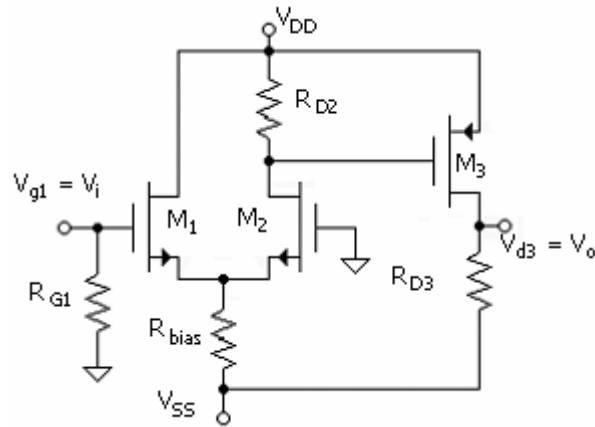
Unit 13. Cascaded Amplifier Stages

The cascade electronic circuit configuration is that formed by connecting the output of one stage to the input of the one following. In analog amplifiers, such as the operational amplifier, high gain is achieved through cascading a number of stages. Mixing NMOS and PMOS circuits provides the major advantage of direct coupling between stages. The amplifier investigated is the cascade of a differential stage and a common-source stage. Circuit bias stability is explored as an example of problems in general with direct stage coupling.

13.1. Combining NMOS and PMOS Circuits in Cascade

The circuit of [Fig. 13.1](#) illustrates the concept of combining NMOS circuits with PMOS circuits for enabling direct coupling of the stages. The added feature of a dual power supply is included, which provides for having the input and outputs at ground potential. One of the important advantages of this is that connecting input sources and output loads does not affect the bias.

Figure 13.1. Cascade of a differential stage and a common-source stage. No coupling capacitors or bypass capacitors are required. This is therefore a dc amplifier.



The input can be set equal to zero volts, as it is the gate of a differential amplifier stage in a dual-power-supply configuration. The output can be set equal to zero volts by selecting the drop across the bias resistor R_{D3} to be equal to $|V_{SS}|$. This flexibility is provided by the PMOS as the output stage; it is approximately a current source with termination at the negative power supply. With $V_O = 0$, the available output signal range is nearly equal to the magnitude of the power-supply voltage; the drain voltage of M_3 can swing anywhere from near V_{DD} down to V_{SS} . Note that this would not be possible if the common-source stage transistor were based on an NMOS.

The output from M_2 of the differential-amplifier stage is used arbitrarily. Since the common-source stage is an inverting stage, the gates of M_1 and M_2 are the inverting (–) and noninverting (+) inputs, respectively. A dc stabilization resistor would be connected between the output (drain of M_3) and the input at the gate of M_1 . This type of feedback would, of course, represent an additional load on the output stage.

13.2. Amplifier Gain of Differential Amplifier and Common-Source Stage in Cascade

In the example of [Fig. 13.1](#), a differential stage and a common-source stage are connected in a cascade configuration; the input of the common-source stage is the output of the differential stage. The overall voltage gain is defined as $a_v = v_o/v_i = v_{d3}/v_{g1}$. But this is also

Equation 13.1

$$a_v = \frac{v_{d2}}{v_{g1}} \frac{v_{d3}}{v_{g3}} = a_{vd} a_{vcs}$$

Therefore, the gain can be calculated by using the separate expressions considered previously for the differential amplifier stage [(8.15)] and the common-source stage [(5.5)]. This leads to

Equation 13.2

$$a_v = \left(\frac{1}{2} g_{m1} R_{D2} \right) \left(-g_{m3} \frac{R_{D3}}{1 + \lambda_p I_{D3} R_{D3}} \right)$$

or using, from (4.5), $g_m = 2I_D/V_{eff}$,

Equation 13.3

$$a_v = - \frac{I_{D2} R_{D2}}{V_{effn1}} \frac{2I_{D3}}{V_{effp3}} \frac{R_{D3}}{1 + \lambda_p I_{D3} R_{D3}} = - \frac{V_{R_{D2}}}{V_{effn1}} \frac{2V_{R_{D3}}}{V_{effp3}} \frac{1}{1 + \lambda_p V_{R_{D3}}}$$

Throughout this unit, the assumption is made that $g_{m1} = g_{m2}$ and $v_{effn1} = V_{effn2}$.

The gain expression neglects the transistor output resistance in the differential amplifier gain for simplicity without a great loss of accuracy, due to the relatively small value of R_{D2} . It also neglects the effect on the gain of the differential stage of R_{bias} . [R_{bias} and the output resistance are included in gain result (8.40), for comparison.] For a calculation of a numerical value of a representative gain, assume that $V_{effn1} = V_{effp3} = 0.3$ V, $V_{tno} = V_{tpo} = 1.0$ V (giving $V_{RD2} = 1.3$ V), $V_{RD3} = 5$ V (for $V_{ss} = -5$ V) and $\lambda_p = 0.05$ V⁻¹. The voltage-gain magnitude is $|a_v| = 4.33 \cdot 26.7 = 116$. We compare this below with an amplifier that has better bias stability but which loses gain in a trade-off.

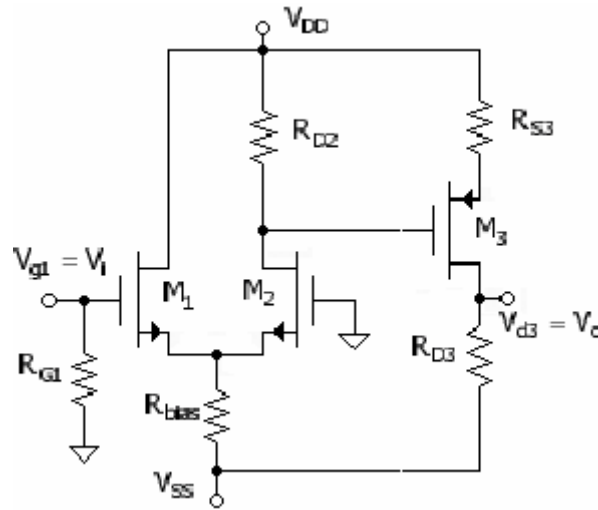
13.3. Stabilization of Signal Gain and Bias Current with a Source Resistor

The amplifier of Fig. 13.1 is not a good design in terms of bias stability. Bias current I_{D3} (of M_3) in the circuit of Fig. 13.1 is very sensitive to the voltage across R_{D2} , which, in a relative sense, is only marginally predictable, given the normal variation in device parameters and circuit components. Also, as in the bias-stability discussion of Unit 5.5 on the design of the NMOS common-source amplifier, for a constant V_{GS3} , bias current I_{D3} is sensitive to changes in transistor parameters. (Note that the differential-amplifier-stage bias current is relatively stable, as it is a current-source bias current.)

13.3.1. Gain and Gain Stabilization

As noted, it was shown in [Unit 5.5](#) that a degree of bias stability is provided with a source resistor as in the circuit of [Fig. 13.2](#) (in this case, R_{S3}). This is an additional example of providing some stabilization with the addition of ac and dc negative feedback in a circuit. The addition of the resistor results in an increase in the gain of the differential amplifier stage but a decrease in the gain of the common-source stage. The net result is a reduction of gain as a trade-off between the gain and bias stability.

Figure 13.2. Amplifier circuit with the addition of a source resistor, R_{S3} , which serves to improve the gain and bias stability. This again demonstrates a general principle, often used in electronic circuits, wherein a linear component is installed to dominate the voltage of voltages in series.



The gain expression for the circuit with R_{S3} is

Equation 13.4

$$a_v = -\frac{1}{2} g_{m2} R_{D2} \frac{g_{m3} R_{D3}}{1 + g_{m3} R_{S3}}$$

where (5.15) is now used for the gain of the common-source stage. It is useful again to express the result in terms of V_{effn} and V_{effp} and the voltages across resistors to make a quantitative assessment of the gain. Using $g_m = 2I_D/V_{eff}$ [(4.5)], the gain expression is

Equation 13.5

$$a_v = - \frac{V_{RD2}}{V_{effn1}} \frac{\frac{2V_{RD3}}{V_{effp3}}}{1 + \frac{2V_{RS3}}{V_{effp3}}}$$

For example, we set $V_{RS3} = 1$ V, with a new $V_{RD2} = 2.3$ V. The gain magnitude is now $|a_v| = 7.67 \cdot 4.35 = 33.3$, compared with 116 for the circuit of [Fig. 13.1](#). Note that the contribution from the differential stage increases from 4.3 to 7.7.

The output-resistance effect on the gain of the common-source stage is neglected [as in [\(5.15\)](#)]. This is justified as the output resistance at the drain of M_3 is increased significantly with R_{S3} . [Precision gain equation [\(8.34\)](#) gives 4.21 compared with 4.35 using [\(5.15\)](#)]. This is discussed further below in conjunction with the common-source amplifier with source resistor from the viewpoint of a feedback circuit.

If resistor V_{RS3} is sufficiently large, $g_{m3}R_{S3} \gg 1$. In this limit the gain is

Equation 13.6

$$a_v \approx - \frac{V_{RD2}}{V_{effn1}} \frac{R_{D3}}{R_{S3}} = - \sqrt{k_{n2} I_{D2} R_{D2}} \frac{R_{D3}}{R_{S3}}$$

The magnitude of gain given by [\(13.6\)](#) is about 38, a fair approximation to the value of 33 from [\(13.5\)](#). It is apparent from the form of [\(13.6\)](#) that a significant degree of gain stabilization has been achieved, as the gain depends primarily only on the bias currents $I_{D1} = I_{D2}$, which is quite stable in the differential amplifier circuit.

13.3.2. Transistor Parameter Variation and I_{D3} Bias Stability

The benefit to I_{D3} bias stability associated with transistor parameter variation can be assessed by a consideration of [\(5.18\)](#) applied to this case, which is

$$I_{D3} = \frac{I_{D2} R_{D2} - \sqrt{\frac{I_{D3}}{k_{p3}}} - V_{tp03}}{R_{S3}}$$

For a quantitative example, suppose that $I_{D3} = 100$ μ A. To be consistent with the numbers used in [Unit 13.2](#), $k_{p3} = 1110$ μ A/V² with $V_{effp3} = 0.3$ V.

Assume that $V_{RD2} = I_{D2}R_{D2}$ is constant. Now make k_{p3} 5% larger and V_{tp03} 50 mV smaller. For the $V_{RD2} = 2.3$ V, we have a new $I_{D3} = 105$ μ A for $V_{D3} = 0.25$ V, which is an acceptable bias value. For the circuit of [Fig. 13.1](#), and the same parameter changes, the new bias current is $I_{D3} = 143$ μ A for $V_{D3} = 2.15$ V, significantly away from the design zero volts.

13.3.3. Bias Stability of I_{D3} Due to Changes in I_{D2}

The actual differential-stage bias current will vary from the basic design due to the variation in circuit components, transistor parameters, and power-supply voltage. Suppose that in lieu of the value of $V_{RD2} = 2.3$ V as in the example above, V_{RD2} is 5% higher or 2.42 V. Assume that the transistor parameters of the common-source stage are at the original design values.

An estimate of the new output voltage (different from 0V) can be made using the linear relation between the input and output of the common-source stage, that is, the common-source contribution of [\(13.4\)](#). The "signal" is the change of voltage across R_{D2} , which is -0.12 V. (The gate voltage v_{G3} decreases.) The common-source gain from [\(13.4\)](#) for the original parameters associated with M_3 is -4.35 . The output bias voltage is now $(-4.35)(-0.12\text{V}) = 0.5$ V (instead of 0 V). Therefore, the circuit is reasonably bias stable.

The gain of the unstabilized circuit of [Fig. 13.1](#) is -27 , which leads to an output voltage, for the same change of voltage across R_{D2} , of 3.2 V, using the linear approximation. Note that the linear approximation is marginally valid only for such a large value of the common-source input "signal" voltage, as in this case of no source resistor. (A dc calculation gives a bias output voltage of 4.8 V.)

13.4. Common-Source Stage as a Series – Series Feedback Circuit

The circuit of [Fig. 13.2](#) with the source resistor is technically a feedback circuit. It falls into the category of series – series feedback, or the feedback network samples the output current and feeds back a voltage in series (and the same polarity, negative feedback) with the input to the common-source stage, that is, V_{gs3} . The series – series feedback amplifier is referred to as a transconductance amplifier. The noninverting and inverting opamp configurations are series – shunt (voltage amplifier) and shunt – shunt (transresistance amplifier), respectively. An additional alternative is shunt – series (current amplifier). An example of this type of feedback amplifier is discussed in [Unit 13.5](#).

In Unit 9.9, an expression for the output resistance of the current source (drain current) with source degeneration was obtained [[\(9.9\)](#)]. This is

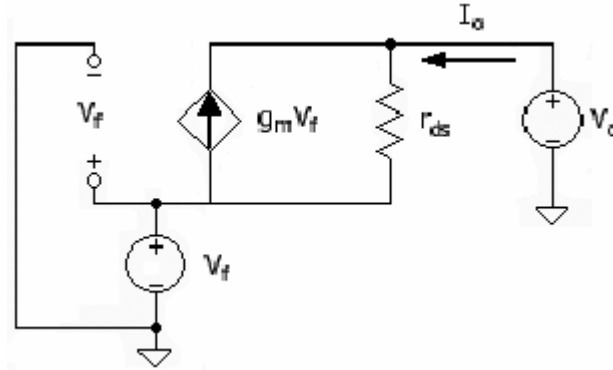
$$R_o = (1 + g_m R_s) \frac{1}{g_{ds}} + R_s$$

The perspective of the common-source amplifier in a series – series feedback mode is shown in [Fig. 13.3](#). The circuit configuration is for determining the output resistance. A voltage V_o is applied at the output terminals with the input grounded. In this series – series circuit, a feedback voltage, V_f , which is proportional to the current I_o , is induced as indicated in the circuit diagram. With the input grounded, voltage V_f is applied to the input terminals of the transistor. The loop equation at the output is then

Equation 13.7

$$V_o = (I_o + g_m V_f) r_{ds} + V_f$$

Figure 13.3. Linear circuit for deriving the output resistance for the series – series feedback circuit. The feedback voltage is proportional to the output current.



By comparison of the feedback circuit of [Fig. 13.3](#) with the circuit of [Fig. 13.2](#), $V_f = I_o R_S$. Thus,

Equation 13.8

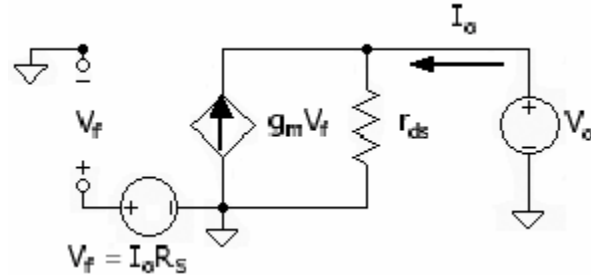
$$V_o = [(1 + g_m R_S) r_{ds} + R_S] I_o \approx (1 + g_m R_S) r_{ds} I_o$$

This produces (9.9) from $R_o = V_o / I_o$ and with $r_{ds} = 1 / g_{ds}$. Intuitively, the feedback is such as to cause a much larger current through r_{ds} than I_o , and therefore a much larger V_o is required for a given I_o .

Dropping the R_S as in the right-hand side of (13.8) renders the result equivalent to the ideal series – series feedback amplifier, as shown in [Fig. 13.4](#). In the ideal series – series amplifier, the sense function has zero resistance; that is, the output voltage is directly

across r_{ds} . With $g_m/g_{ds} \gg 1$, the ideal circuit provides a very good approximation to the actual circuit. For example, with $V_{effn} = 0.5$ V and $\lambda_n = 1/50$ V, the ratio is 100.

Figure 13.4. An ideal series – series feedback amplifier. V_o is applied directly across the output resistance and V_f is applied to the grounded input.



For both feedback amplifier types with series as the second term (series – series and shunt – series), the output resistance is increased by a factor $(1 + T)$, where T is the loop gain, as in the discussion of the opamp feedback amplifiers in [Unit 11](#). The loop gain for the common-source circuit of [Fig. 13.2](#) is, by definition, $T = V_f/V_{gs3}$. By inspection of [\(9.9\)](#) we conclude that $T = g_{m3}R_{S3}$. In terms of T , the output resistance is then

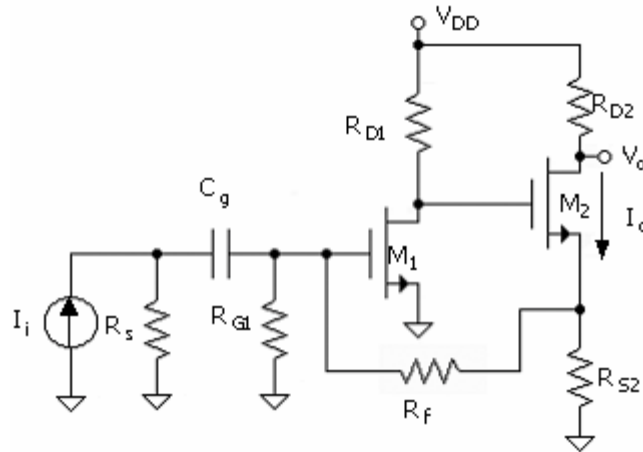
Equation 13.9

$$R_o = (1+T)r_{ds3} + R_{S3}$$

13.5. Shunt – Series Cascade Amplifier

The cascading of two common-source amplifier stages will now be explored. This will expand the discussion on feedback amplifiers to include the shunt – series (current) amplifier. As shown in [Fig. 13.5](#), the current I_o is sensed, with the feedback network consisting of R_{S2} and R_f , and a fraction of I_o is summed at the gate input node along with the input source current. Gate resistor R_{G1} provides for an addition bias design variable but otherwise is simply combined with the source resistance, R_s , in the amplifier performance analysis or design.

Figure 13.5. Shunt-series feedback amplifier. This is a current amplifier with feedback amplifier gain, $A_i = I_o/I_i$.



13.5.1. DC Design

For the design, assume that a choice has been made for I_{D1} and I_{D2} and $V_{G2} = V_{D1}$. The latter must be such that $V_{G2} > V_{GS1} + V_{GS2}$. Then, for R_{D1} , use

Equation 13.10

$$R_{D1} = \frac{V_{DD} - V_{G2}}{I_{D1}}$$

As will be indicated below, the ideal current gain is

Equation 13.11

$$A_{II} = 1 + \frac{R_f}{R_{S2}}$$

Use this, based on the feedback-amplifier design current gain goal, to obtain a relation between R_f and R_{S2} . This gives

Equation 13.12

$$R_f = (A_{II} - 1)R_{S2}$$

Now determine the value of R_{S2} which satisfies the design value of I_{D2} . This is obtained from

Equation 13.13

$$I_{D2} = \frac{V_{S2}}{R_{S2}} + \frac{V_{S2} - V_{GS1}}{(A_{II} - 1)R_{S2}}$$

where [13.12](#) has been used for R_f and where $V_{S2} = V_{G1} - V_{GS2}$ and

Equation 13.14

$$V_{GS1} = \sqrt{\frac{I_{D1}}{k_{p1}}} + V_{tno}$$

and

Equation 13.15

$$V_{GS2} = \sqrt{\frac{I_{D2}}{k_{p2}}} + V_{tno}$$

Finally, select R_{G1} to satisfy the V_{GS1} requirement from

Equation 13.16

$$V_{GS1} = \frac{R_{G1}}{R_{G1} + R_f} V_{S2}$$

13.5.2. DC Stability

The feedback network provides significant bias stabilization and the effect can be assessed by the same formulation as for the opamp. In [Unit 11.4](#), it was shown that the output voltage for a given offset voltage is [[\(11.16\)](#)]

$$\Delta V_O = \frac{-A_{vNI}}{1 + \frac{A_{vNI}}{a_{vo}}} V_{off} \approx -A_{vNI} V_{off}$$

As applied to our shunt – series circuit (by equivalency), $A_{vNI} = 1 + R_f/R_{G1}$, and a_{vo} is the voltage-gain magnitude (minus terminal equivalent) from the gate, V_{g1} , to the source V_{s2} , that is, the gain of the case of a common-source stage (M_1) and a source-follower stage (M_2). Also, for this case, $\Delta V_o \equiv \Delta V_{s2}$. Note that for this case, ΔV_o is a bias-voltage change, whereas for the opamp, it is the output voltage (different from zero volts).

Suppose that the dc (bias) is at a set state. Then the bias is altered such as by a change of a transistor parameter (either by temperature change or substituting one transistor for a new one). For example, a change in V_{tno} of M_1 is equivalent to installing an offset voltage, and [(11.16)] can be applied directly to this case. Make $R_{G1} = R_f$ for $A_{vNI} = 2$. Also assume a typical $a_{vo} = 40$ (common-source stage). For a change of $\delta V_{tno} \equiv -V_{off} = -0.1$ V, $\Delta V_O \approx 2 \cdot 0.1$ V = 0.2 V. Note that based on $a_{vo} = 40$, with no feedback stabilization, the change in output would be approximately $a_{vo}\delta V_{tno} = 4$ V.

13.5.3. Signal Current Gain

We first compute the open-loop transconductance and then the open-loop current gain. This is done, for a good approximation, by disconnecting the right side of R_f from the source of M_2 and connecting it to ground. (This disables the shunt – series feedback and otherwise alters the circuit only slightly.) The result is

Equation 13.17

$$a_i = R_i G_m$$

where $R_i = R_s || R_{G1} || R_f$. (This assumes that $R_f \gg R_{S2}$ which is consistent with $A_{i1} \gg 1$.) Transconductance G_m is for the complete cascade circuit (open loop), that is,

Equation 13.18

$$G_m = \frac{I_o}{V_i} = \frac{I_{d2}}{V_{gs1}} = g_{m1} R_{D1} \frac{g_{m2}}{1 + g_{m2} R_{S2}}$$

Note that the series – series feedback effect is retained in the open-loop computation and that R_{S2} has been used as an approximation for $R_{S2} \parallel R_f$. This once again assumes that $R_f \gg R_{S2}$. The loop gain is

Equation 13.19

$$T = a_i \frac{R_{S2}}{R_{S2} + R_f}$$

Following the discussion in [Unit 11](#), which led to (11.3), the current gain with feedback is then

Equation 13.20

$$A_i = \frac{a_i}{1 + T} = \frac{A_{iI}}{1 + \frac{1}{T}}$$

where (ideal current gain of the feedback amplifier)

Equation 13.21

$$A_{iI} = 1 + \frac{R_f}{R_{S2}}$$

As a design example, suppose the design goal is $A_{iI} = 10$. Assume that the transistors have $V_{th0} = 1$ V, $k_{n1} = 1000 \mu\text{A}/\text{V}^2$, and $k_{n2} = 2000 \mu\text{A}/\text{V}^2$. We select $I_{D1} = 100 \mu\text{A}$, $I_{D2} = 200 \mu\text{A}$, $V_{DD} = 10$ V, and $V_{G2} = 3.5$ V. From (13.12) and (13.13) we obtain $R_{S2} = 11.4 \text{ k}\Omega$ and $R_f = 103 \text{ k}\Omega$. Based on (13.16), the value of R_{G1} is $R_{G1} = 156 \text{ k}\Omega$. To satisfy the drain voltage requirements, $R_{D1} = 65 \text{ k}\Omega$ and $R_{D2} = 25 \text{ k}\Omega$. The latter is based on $V_{D2} = V_{DD}/2$. Also, we assume that $R_s = 100 \text{ k}\Omega$.

The open-loop transconductance is $G_m = 3372 \mu\text{A}/\text{V}$ and $R_i = 38 \text{ k}\Omega$ for $a_i = 129$. Loop gain $T = 12.9$ such that $A_i = 9.28$. We note that since $A_i \approx A_{iI} = 10$, the current gain is very predictable, despite bias and parameter variations.

13.6. Summary of Equations

$a_v = \left(\frac{1}{2} g_{m1} R_{D2} \right) \left(-g_{m3} \frac{R_{D3}}{1 + \lambda_p I_{D3} R_{D3}} \right)$	Gain of cascade of differential stage and common-source stage, including output resistance of M_3 .
$a_v = -\frac{1}{2} g_{m2} R_{D2} \frac{g_{m3} R_{D3}}{1 + g_{m3} R_{S3}}$	Gain of cascade circuit for case of common-source stage with source resistor.
$a_v \approx -\sqrt{k_{p2} I_{D2}} R_{D2} \frac{R_{D3}}{R_{S3}},$ $a_v \approx -\frac{V_{R_{D2}}}{V_{effm2}} \frac{R_{D3}}{R_{S3}}$	Approximate form for gain which assumes that $g_{m3} R_{S3} \gg 1$.
$I_{D3} = \frac{I_{D2} R_{D2} - \sqrt{\frac{I_{D3}}{k_{p3}}} - V_{tp0}}{R_{S3}}$	Bias equation for transistor M_3 .
$R_{D1} = \frac{V_{DD} - V_{G2}}{I_{D1}}$ $\frac{R_{G1}}{R_{G1} + R_f} V_{S2} = V_{GS1}$ $I_{D2} = \frac{V_{S2}}{R_{S2}} + \frac{V_{S2} - V_{GS1}}{(A_{iI} - 1) R_{S2}}$	Shunt – series amplifier dc design equations.
$\Delta V_O = \frac{-A_{vNI}}{1 + \frac{A_{vNI}}{a_{vo}}} V_{off} \approx -A_{vNI} V_{off}$	Shunt – series amplifier dc stability relation.
$a_i = R_i G_m$ $G_m = g_{m1} R_{D1} \frac{g_{m2}}{1 + g_{m2} R_{S3}}$	Shunt – series amplifier open-loop current gain relations.
$T \approx a_i \frac{R_{S3}}{R_{S3} + R_f}$ $A_{iI} = \frac{a_i}{1+T} = \frac{A_{iI}}{1 + \frac{1}{T}} \quad A_{iI} = 1 + \frac{R_f}{R_{S2}}$	Feedback amplifier current-gain relations.

Following the discussion of transistor current sources in [Unit 9](#), the ratio of the reference current and the current-source current is

Equation 14.1

$$\frac{I_{D11}}{I_{D10}} = \frac{W_{11}}{W_{10}} \frac{1 + \lambda_n V_{DS11}}{1 + \lambda_n V_{DS10}} \approx \frac{W_{11}}{W_{10}}$$

The ratio of the gate widths will thus be selected to satisfy the design of the differential-stage bias. The current ratio will deviate from the simple ratio due to the λ_n terms, but this is not critical, as the design will not call for an exact value for $I_{D1} \approx I_{D2}$.

14.2. Current-Source Output Resistance and Common-Mode Gain

As discussed in [Unit 8](#), the common-mode gain is the gain for the same signal being applied to both inputs. In [Unit 8.6](#) the common-mode voltage gain expression, (8.26), was obtained for the bias resistor, R_{bias} . This is

$$a_{vcm} = \frac{-g_m R_{D2}}{1 + 2g_m R_{bias}}$$

To apply the common-mode gain equation to the circuit of [Fig. 14.1](#), resistor R_{bias} is replaced by r_{ds11} . The result for the common-mode gain is

Equation 14.2

$$a_{vcm} = -\frac{g_{m1} R_{D2}}{1 + g_{m1} 2r_{ds11}} \approx -\frac{R_{D2}}{2r_{ds11}}$$

The approximate form is consistent with neglecting the change of the gate – source voltage compared to the change of voltage at the drain of the current-source transistor, M_{11} . Intuitively, (14.2) indicates that for $r_{ds11} \rightarrow \infty$, transistor M_{11} , is a pure current source, and $i_{D1} = i_{D2} = I_{D1} = I_{D2}$ regardless of the magnitude of common voltage applied to both gates. In this case, signals $I_{d1} = I_{d2} = 0$ for any (realistic) common-mode input signal.

Common-mode inputs can be a form of noise, and therefore the ideal opamp would reject these signals entirely. An important consideration is the extent of rejection based on the common-mode gain relative to the differential amplifier gain. This is quantitatively assessed with the common-mode rejection ratio, which is the ratio of the differential amplifier gain with differential output divided by the common-mode gain, (14.2). The gain for this case was obtained in [Unit 8.5](#) as (8.24), which is $a_{vd12} = -g_m R_D$.

Using (8.24) and (14.2), the common-mode rejection ratio is

Equation 14.3

$$\text{CMRR} = \frac{a_{vd12}}{a_{vcm}} = 1 + g_{m1}2r_{ds11} \approx 2g_{m1}r_{ds11}$$

With $g_{m1} = 2I_{D1}/V_{eff1}$ [(4.5)], and for the output resistance of M_{11} , $r_{ds11} \approx 1/\lambda_n 2I_{D1}$ [(4.15)], the common-mode rejection ratio is

Equation 14.4

$$\text{CMRR} = \frac{4}{V_{eff1}\lambda_n}$$

By comparison, for resistor bias, the equivalent result is

Equation 14.5

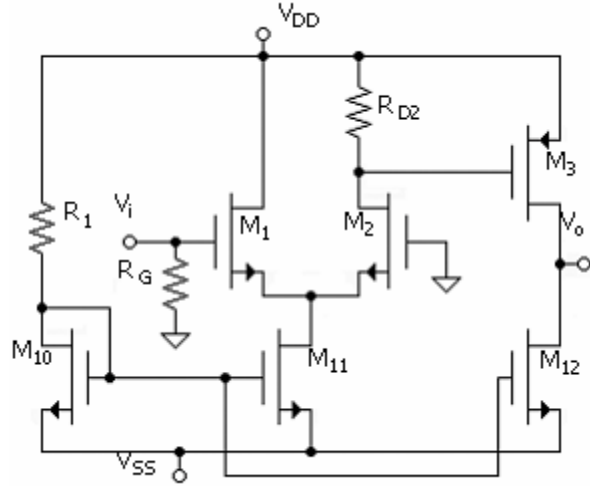
$$\text{CMRR} = \frac{4V_{R_{bias}}}{V_{eff1}}$$

where R_{bias} is the bias resistor of, for example, the circuits of Figs. 13.1 and 13.2.

14.3. Current-Source Load for the Common-Source Stage

We now add to the differential stage the common-source stage to obtain a two-stage amplifier as in Unit 13 (Fig. 14.2). Transistor M_{12} , which replaces bias resistor R_{D3} , provides a current-source load as in the circuit of Fig. 10.1. Note that current I_{D12} mirrors I_{D10} and no additional resistors are required with the addition of the common-source stage. That is, M_{11} and M_{12} both use the reference voltage provided by the diode-connected M_{10} .

Figure 14.2. Cascade of the differential amplifier stage and common-source stage. M_{12} provides a current-source load for the common-source stage. This circuit remains inadequate in terms of dc bias stability of I_{D3} . This is improved in the modification that follows.



The bias design for the common-source stage consists of picking W_{12} to obtain a specified I_{D12} relative to I_{D10} and making $I_{D3} = I_{D12}$. Parameter W_{12} is determined from

Equation 14.6

$$\frac{I_{D12}}{I_{D10}} = \frac{W_{12}}{W_{10}} \frac{1 + \lambda_n V_{DS12}}{1 + \lambda_n V_{DS10}} \approx \frac{W_{12}}{W_{10}}$$

The approximation is sufficient, as the current magnitude, again, is not critical.

The circuit is adjusted to make $I_{D3} = I_{D12}$ by equating the relations for the two currents. This is

Equation 14.7

$$k_{p3} V_{eff3}^2 (1 + \lambda_p V_{DD}) = k_{n12} V_{eff12}^2 (1 + \lambda_n |V_{SS}|)$$

A much simpler equation replaces this rather complicated one when R_{D2} is finally replaced by a transistor as well. V_{eff12} is known from $V_{eff12} = V_{eff10}$. Thus, the current balance can be obtained by a selection of the various remaining parameters.

The load on the common-source stage is now $r_{ds12} = 1/g_{ds12}$. The gain is, including the output resistance of M_3 ,

Equation 14.8

$$a_{vcs} = -g_{m3}(r_{ds12} || r_{ds3}) = -\frac{g_{m3}}{g_{ds12} + g_{ds3}}$$

The overall amplifier gain, $a_v = V_o/V_i$, is now

Equation 14.9

$$a_v = -\frac{1}{2} g_{m1} R_{D2} \frac{g_{m3}}{g_{ds3} + g_{ds12}}$$

This can be evaluated with

Equation 14.10

$$a_v = -\frac{I_{D2} R_{D2}}{V_{eff1}} \frac{2}{V_{eff3} (\lambda_n + \lambda_p)}$$

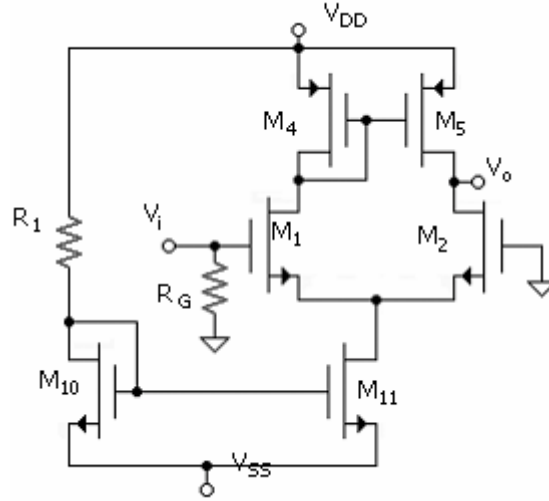
Using the numbers from the previous calculation for the amplifier of [Fig. 13.1](#) with R_{D3} , and adding $\lambda_n = \lambda_p$, the gain magnitude for the circuit of [Fig. 14.2](#) is 289. This compares with the value of 116 for the cascade amplifier of [Fig. 13.1](#). In the next unit, the differential stage will be modified to include a current source load with a considerable additional improvement in gain.

14.4. Current-Source Load for the Differential Stage

The load resistor (or bias resistor) of the differential stage will now be replaced with a current-source load. In addition to eliminating the need again for a resistor, it provides for much better dc stability and a significant gain improvement, including the elimination of the factor of 1/2 associated with the differential stage gain as in [\(8.15\)](#) and [\(8.17\)](#).

As shown in the new circuit in [Fig. 14.3](#), the resistor R_{D2} is replaced by transistors M_4 and M_5 . Transistor M_5 provides a current-source load for transistor M_2 . The diode-connected transistor, M_4 , provides the dc reference voltage for M_5 . The reference current is $I_{D1} = I_{D4}$.

Figure 14.3. Differential amplifier stage with current-source load.



The dc output voltage (of the differential stage) is set automatically at $V_{DD} - V_{SG4}$ due to the relation $V_{SD4} = V_{SG4} = V_{SG5} = V_{SD5}$. This is an idealization that assumes that the parameters of transistors M_4 and M_5 and M_1 and M_2 are identical and that $I_{D1} = I_{D2}$.

Recall that in the differential stage with drain resistor, R_{D2} , the noninverting output signal voltage was induced across R_{D2} due to the signal current I_{d2} . In the new circuit, the output current is the composite of signal currents I_{d2} and I_{d5} . That is, signal current I_{d1} is mirrored at the output as I_{d5} . This is a result of the coupling from M_1 through M_4 to the gate of M_5 . The circuit from the gate of M_1 to the drain of M_5 can be regarded as a cascade of two common-source stages (M_1 and M_5) with a shunt resistor between them of magnitude $1/g_{m4}$ (the signal resistance of the diode-connected transistor).

The gain for this circuit can be explained starting from the transconductance for the two currents I_{d2} and I_{d5} . Assume for a moment that the output voltage is held constant at the dc bias value. This is equivalent to a signal short circuit at the output. As in the case of the resistor-bias differential amplifier stage, we have

Equation 14.11

$$I_{d2} = -\frac{1}{2} g_{m1} V_{g1}$$

where the effect of the output resistance of M_{11} is neglected. But also, as in the resistor-load case,

Equation 14.12

$$I_{d1} = \frac{1}{2} g_{m1} V_{g1}$$

But $I_{d1} = -I_{d4}$ and I_{d4} is mirrored as I_{d5} , which leads to $I_{d5} = -I_{d1}$. This assumes that M_4 and M_5 are identical. Thus,

Equation 14.13

$$I_{d5} = -\frac{1}{2} g_{m1} V_{g1}$$

All signal drain currents are defined as positive into the respective drains.

The output currents summed at the output node, V_o , are (defined positive into the node)

Equation 14.14

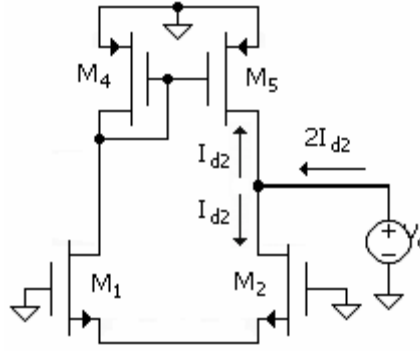
$$I_o = I_{d2} + I_{d5} = -g_{m1} V_{g1}$$

The result is valid for $v_o = V_o$ or $V_o = 0$, as stipulated above. One could imagine attaching a dc supply exactly equal to V_o at the output node. In this case, the output current could readily flow into the node according to (14.14).

For the opposite extreme of having an open circuit at the output, as in Fig. 14.3, the currents I_{d2} and I_{d5} are dependent on $v_{D2} = v_{D5} = v_o$. The deviation of I_o from (14.14) is, as usual, taken care of by including the effect of the output resistance at the output node.

The output resistance is somewhat complicated by feedback effects, which are inherent in this circuit. For assessing the circuit output resistance, a signal circuit is shown in Fig. 14.4 with a test voltage, V_o , applied at the output with both inputs grounded. Resulting currents I_{d2} and I_{d5} will be considered separately and will be superimposed.

Figure 14.4. Signal (linear) circuit for determining the output resistance at the drains of M_2 and M_5 . Currents shown are associated with the assessment of the output resistance associated with M_2 only. The bias transistor (M_{11}) output resistance is neglected.



The currents indicated in [Fig. 14.4](#) are for the case of I_{d2} of M_2 . Looking back into the drain of M_2 , the output resistance is affected by emitter degeneration because of the resistance $1/g_{m1}$ between the emitter of M_2 and ground. That is, the resistance looking back into the emitter of M_1 is $1/g_{m1}$. Thus, the resistance looking only back into M_2 is $2r_{ds2}$. However, I_{d2} feeds through and around the loop composed of M_2 , M_1 , M_4 , and M_5 such that the total current flowing into the node (exclusive of the separate I_{d5} contribution) is $2I_{d2}$. The test voltage is $V_o = I_{d2}2r_{ds2}$, due to current I_{d2} flowing down into the resistance $2r_{ds2}$ of the drain of M_2 . The output resistance associated with the drain of M_2 , R_{oM2} , is based on a total current of $2I_{d2}$ such that it is

Equation 14.15

$$R_{oM2} = \frac{V_o}{2I_{d2}} = \frac{2I_{d2}r_{ds2}}{2I_{d2}} = r_{ds2}$$

The drain resistance $2r_{ds2}$ combined with a test current of $2I_{d2}$ results in an effective drain resistance r_{ds2} despite the emitter degeneration.

Separately, by superposition, for applied V_o , a current I_{d5} of value $I_{d5} = V_o/r_{ds5}$ flows into the drain of M_5 such that the output resistance associated with M_5 is $R_{oM5} = r_{ds5} = 1/g_{ds5}$. This is based on the fact that the output resistance is that of a common-source stage with grounded source. The two contributions are in parallel such that finally, the output resistance is

Equation 14.16

$$R_{o25} = \frac{1}{g_{ds2} + g_{ds5}}$$

Combining this with the effective amplifier transconductance obtained above, (14.14), the gain is

Equation 14.17

$$a_{v2} = \frac{V_{d2}}{V_{g1}} = -\frac{I_o R_{o25}}{V_{g1}} = \frac{g_{m1}}{g_{ds2} + g_{ds5}}$$

or

Equation 14.18

$$a_{v2} = \frac{2}{V_{eff1}(\lambda_n + \lambda_p)}$$

An external load, R_L , appears in parallel with R_o . Hence with an external load, the gain is

Equation 14.19

$$a_{v2} = \frac{g_{m1}}{g_{ds2} + g_{ds5} + \frac{1}{R_L}} = \frac{g_{m1} R_L}{1 + (g_{ds2} + g_{ds5}) R_L}$$

In the discussion leading to (14.14), it was noted that the output current given by (14.14) is for a short-circuit condition at the output, and the effect of finite $V_{d2} = V_{d5}$ is taken care of through the concept of the output resistance of the circuit. A clarification of this statement is provided by the analytical formulation, which includes a load, R_L , given by

Equation 14.20

$$\frac{\partial i_O}{\partial v_{g1}} V_{g1} = \frac{\partial i_{D2}}{\partial v_{D2}} V_{d2} + \frac{\partial i_{D5}}{\partial v_{D2}} V_{d2} + \frac{V_{d2}}{R_L}$$

where $v_{D5} = v_{D2}$, as they are the same node. The linear form is

Equation 14.21

$$g_{m1} V_{g1} = g_{ds2} V_{d2} + g_{ds5} V_{d2} + \frac{V_{d2}}{R_L}$$

and this leads to (14.19), where, again, $a_{v2} = V_{d2}/V_{g1}$. A voltage source attached to the output is equivalent to $R_L \rightarrow 0$, in which case $V_{d2} \rightarrow 0$ and $v_{D2} \rightarrow V_{D2} = V_O$, that is, the bias value. The finite output current magnitude into the short circuit is consistent with $V_{d2}/R_L|_{v_{ds}, R_L \rightarrow 0} = g_{m1} V_{g1}$.

14.4.1. Common-Mode Gain of the Differential Stage with Current-Source Load

As may be deduced from the common-mode gain equation, (14.2), in Unit 14.2, the common-mode transconductance for the differential stage with current-source biasing is $G_m = g_{m1}/(1 + g_{m1}2r_{DS11})$. The incremental load at the drain of M_1 for the case of the circuit shown in Fig. 14.3, however, is the resistance of the diode-connected transistor M_4 of Fig. 14.3, which is $1/g_{m4}$. Due to symmetry, this is also the effective load at the drain of M_2 . The common-mode gain is thus

Equation 14.22

$$a_{vcm} = -\frac{g_{m1}}{1 + g_{m1}2r_{ds11}} \frac{1}{g_{m4}}$$

and the common-mode rejection ratio is, from (4.17) (for single-ended output) and (14.22),

Equation 14.23

$$CMRR = \left(1 + 2\frac{g_{m1}}{g_{ds11}}\right) \cdot \frac{g_{ds2} + g_{ds5}}{g_{m4}} \approx \frac{2g_{m1}g_{m4}}{(g_{ds2} + g_{ds5})g_{ds11}}$$

This is greater than (14.3) by the factor $g_{m4}/(g_{ds2} + g_{ds5}) \approx [V_{eff4}(\lambda_n + \lambda_p)]^{-1}$.

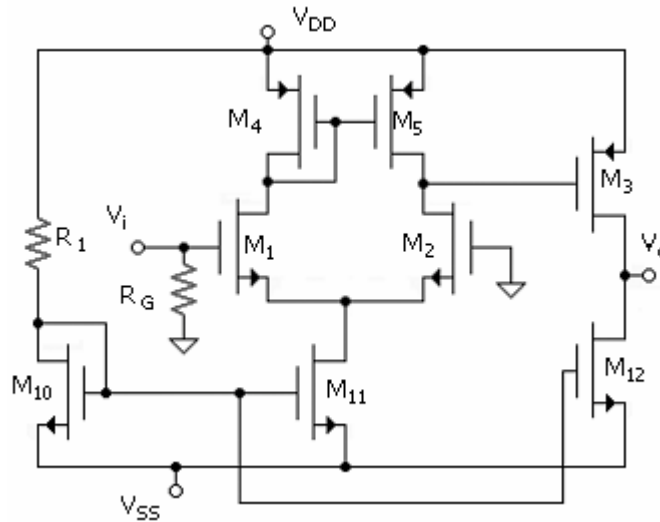
14.5. Two-Stage Amplifier with Current-Source Biasing

The cascade amplifier that includes all features discussed up to this point is given in Fig. 14.5. Note that the only (internal) resistor is that of the reference-current circuit. Bias currents for this circuit can be set up using gate-width proportions. We note that $I_{D3} \propto I_{D5}$ since M_3 and M_5 have a common bias voltage, V_{SG4} . Thus

Equation 14.24

$$\frac{I_{D3}}{I_{D5}} \approx \frac{W_3}{W_5}$$

Figure 14.5. Two-stage amplifier with current-source biasing. R_1 is the only resistor in the internal circuit. R_G is externally connected.



Also, M_{11} and M_{12} are referred to the same reference voltage such that

Equation 14.25

$$\frac{I_{D12}}{I_{D11}} = \frac{W_{12}}{W_{11}}$$

Using $I_{D3} = I_{D12}$ and $I_{D11} = 2I_{D5}$, we obtain

Equation 14.26

$$\frac{I_{D3}}{I_{D5}} = 2 \frac{W_{12}}{W_{11}}$$

Combining (14.24) and (14.26) leads to

Equation 14.27

$$W_3 = 2W_5 \frac{W_{12}}{W_{11}}$$

Assume that W_{11} has been picked to satisfy the design I_{D11} . W_{12} is then selected to give the design I_{D12} using $I_{D12}/I_{D10} = W_{12}/W_{10}$. W_5 is selected on the basis of signal considerations. This leaves the computation of W_3 [from (14.27)].

A precision calculation includes the lambda effects and is from

Equation 14.28

$$W_3 = 2W_5 \frac{1 + \lambda_p V_{SD5}}{1 + \lambda_p V_{SD3}} \frac{W_{12} (1 + \lambda_n V_{DS12})}{W_{11} (1 + \lambda_n V_{DS11})}$$

The simple form should normally suffice. The dc output voltage V_O is very sensitive to some of the parameters and the approximate calculation from (14.27) will not result in $V_O = 0$. However, uncertainties in transistor parameters preclude the justification of using (14.28) in practice.

The overall signal gain is now obtainable using (14.17) for the differential stage and (14.8) for the common-source stage with a current-source load. The result is

Equation 14.29

$$a_v = - \frac{g_{m1}}{g_{ds2} + g_{ds5}} \frac{g_{m3}}{g_{ds12} + g_{ds3}}$$

A useful form for making a quantitative gain assessment is

Equation 14.30

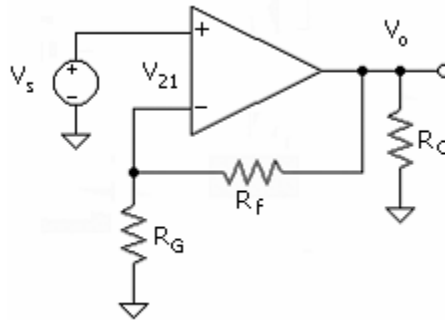
$$a_v = - \frac{2}{V_{eff1} (\lambda_n + \lambda_p)} \frac{2}{V_{eff3} (\lambda_n + \lambda_p)}$$

Based on the same parameters as used previously for gain calculations ($\lambda_n = \lambda_p = 1/20 \text{ V}$ and $V_{\text{effn}} = V_{\text{effp}} = 0.3 \text{ V}$), the gain magnitude is 4440. The value compares with 116 for the all-resistor circuit for Fig. 13.1 and 289 for the circuit of Fig. 14.3. In this special case, both stages contribute the same value.

14.6. Output Buffer Stage

It is evident from the gain result, (14.29), that the high gain depends on having a high resistance at the output; a relatively small load resistor can reduce the gain substantially. For example, suppose that the amplifier is to be used as a resistance feedback amplifier as shown in Fig. 14.6. The input is at the gate of M_2 , and the feedback resistor is connected back to the gate of M_1 . The load at the output is $R_f + R_2 \parallel R_O$.

Figure 14.6. Opamp with feedback resistor R_f . The feedback circuit adds load $R_f + R_2$ in parallel with external load, R_O , at the output.



For a specific example, assume that $R_O \gg R_f \gg R_2$ such that the load is approximately R_f . With the additional load, the gain (14.30) becomes

Equation 14.31

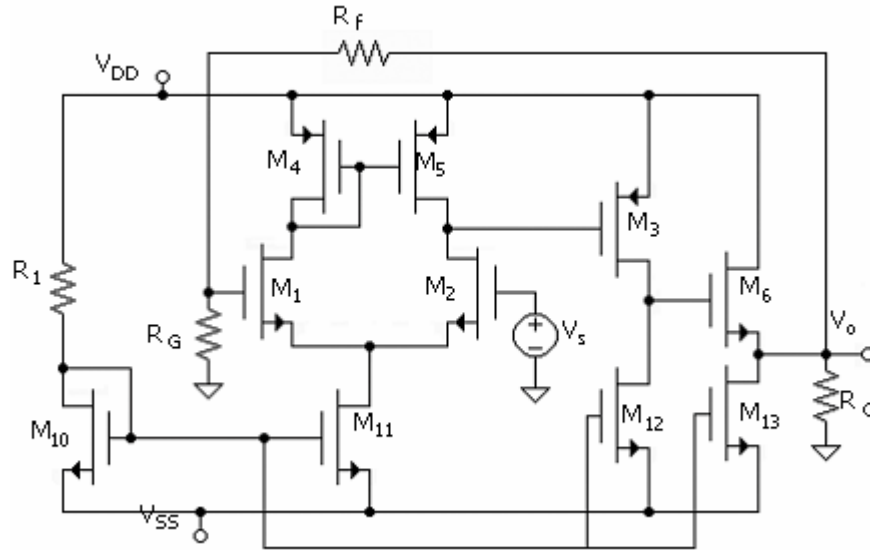
$$a_v = - \frac{2}{V_{\text{eff1}} (\lambda_n + \lambda_p)} \frac{2}{V_{\text{eff3}} (\lambda_n + \lambda_p)} \frac{1}{1 + I_{D3} R_f}$$

which is the result of R_f being in parallel with the output resistance of the common-source stage. Suppose that $R_f = 10 \text{ k}\Omega$ and $I_{D3} = 100 \text{ }\mu\text{A}$. In this case the gain would drop to about 400 (from about 4400), which would be unacceptable, as proper operation of opamps assumes a very high gain.

This loading problem is substantially eliminated with the addition of a buffer stage to isolate the load from the output node of the common-source stage, as shown in the circuit

of [Fig. 14.7](#). The source-follower buffer stage is made up of M_6 and M_{13} . The current-source bias transistor M_{13} also uses the reference voltage provided by M_{10} . The load on the common-source stage of M_3 is now infinite. The feedback network is included in the circuit for comparing with the diagrammatic version of the circuit of [Fig. 14.6](#).

Figure 14.7. Three-stage amplifier with the addition of the source-follower (buffer) stage.



The gain expression of the source follower requires that the body effect be taken into consideration, as the body of M_6 will be at signal ground. [The body of M_6 will be attached to the negative supply voltage (not shown).] From [Unit 7](#) it was shown that the source-follower stage gain, with resistor bias [[\(7.12\)](#)] for this case is

$$a_{vsf} = \frac{g_m}{g_m(1 + \eta) + \frac{1}{R_S} + g_{ds}}$$

For the present case of current-source bias for the source-follower stage, the gain expression becomes, with inclusion now of the feedback network and the external load resistance of [Fig. 14.7](#),

Equation 14.32

$$a_{vsf} = \frac{g_{m6}}{g_{m6}(1 + \eta) + g_{ds13} + g_{ds6} + \frac{1}{(R_f + R_G) || R_O}}$$

To obtain a quantitative sense of the body effect on the gain of the source follower, suppose that $\eta = 0.15$. Thus, in the limit for $g_{m6} \rightarrow \infty$, $a_{vsf} = 0.87$. This is the best possible result, given the body effect.

For a more general assessment, we can substitute the relations $g_m = 2I_D/V_{eff}$, [(4.5)], and $g_{ds} = I_D\lambda_n$ [(14.15)] into (14.32) to obtain

Equation 14.33

$$a_{vsf} = \frac{1}{(1 + \eta) + \frac{V_{eff6}\lambda_n}{2} + \frac{V_{eff13}\lambda_n}{2} + \frac{V_{eff6}}{[(R_f + R_G) \parallel R_O]2I_{D6}}}$$

We note that the terms associated with g_{ds6} and g_{ds13} are negligible. Minimization of the resistance term requires making $g_{m6}[(R_f + R_2) \parallel R_O] \gg 1$ (making the output resistance of the source follower much less than the load resistance). This involves a combination of a large I_{D6} and k_{n6} .

Suppose that $I_{D6} = 400 \mu A$, $W_6 = 500 \mu m$, $KP_n = 100 \mu A/V^2$, and that the gate length is $L = 10 \mu m$. Recalling that $k_n = (KP_n/2)(W/L)$ (Table 3.1), we obtain $k_n = 2500 \mu A/V^2$ and $1/g_{m6} = 500 \Omega$. Thus, for example, for $R_O = 10 k\Omega$, $R_f = 10 k\Omega$, and $R_f \gg R_G$, $a_{vsf} = 0.80$, compared with the limiting value of $a_{vsf} = 0.87$. Thus the source follower functions very well to isolate the load from the high-gain common-source stage.

Finally, we write the overall gain expression, including a general load resistance, R_L , by combining (14.29) and (14.32) as

Equation 14.34

$$a_v = -\frac{g_{m1}}{g_{ds2} + g_{ds5}} \cdot \frac{g_{m3}}{g_{ds12} + g_{ds3}} \cdot \frac{g_{m6}}{g_{m6}(1 + \eta) + g_{ds13} + g_{ds6} + \frac{1}{R_L}}$$

A good approximation for the case of a sufficiently large g_{m6} is

Equation 14.35

$$a_v = -\frac{g_{m1}}{g_{ds2} + g_{ds5}} \cdot \frac{g_{m3}}{g_{ds12} + g_{ds3}} \cdot \frac{1}{1 + \eta}$$

14.7. Output Resistance of the Feedback Amplifier and Effect on Gain from Loading

The output resistance of the resistance feedback amplifier was considered in [Unit 11.4](#). There it was shown to be $R_o = r_o/(1 + T)$ [(11.12)], where r_o is the output resistance of the open-loop amplifier and T is the loop gain of the feedback amplifier. Therefore, with the expectation that $T \gg 1$, the output resistance of the feedback amplifier is very small. For example, consider the amplifier of [Fig. 14.7](#), with no external load on the output and neglecting the effect of the feedback resistors. The open-loop output resistance is

Equation 14.36

$$r_o \approx \frac{1}{(1 + \eta)g_m \epsilon}$$

The output resistance of the feedback amplifier is

Equation 14.37

$$R_o = \frac{r_o}{1 + T} \approx \frac{1}{a_v R_G / R_f (1 + \eta) g_m \epsilon}$$

where the loop gain is $T = a_v R_G / (R_G + R_f)$.

Suppose that in order to get a high gain from the amplifier, we redesign the MOSFETs to have $\lambda_p = \lambda_n = 0.01 \text{ V}^{-1}$ for a new $a_v = 96,000$ [from (14.35), no loading]. Also, let $R_f = 10 \text{ k}\Omega$ and $R_G = 100 \Omega$. Using the same numbers as in the calculation of a sample source-follower stage gain [(14.33)], $r_o = 435 \Omega$ and $R_o = 0.45 \Omega$.

The gain of the feedback amplifier is (11.3), which is

$$A_v = \frac{A_{vNI}}{1 + \frac{A_{vNI}}{a_v}}$$

with (11.4)

$$A_{vNI} = 1 + \frac{R_f}{R_y}$$

Here, gain A_v is for the case of no external load (including the omission of the load from the feedback resistors). The value is $A_v = 100.9$ for $A_{vNI} = 101$.

Now we include an external load of $R_O = 100 \Omega$ and combine this in parallel with $R_f = 10 \text{ k}\Omega$ for load $R_L = 99 \Omega$. The loading effect reduces the open-loop gain to $a_v = 18,000$. The reduction is all attributed to the loading on the source-follower stage. The amplifier gain [from (11.3)] is now $A_{vLoad} = 100.4$. However, with R_O known, an alternative means of obtaining the gain with loading is to use

Equation 14.38

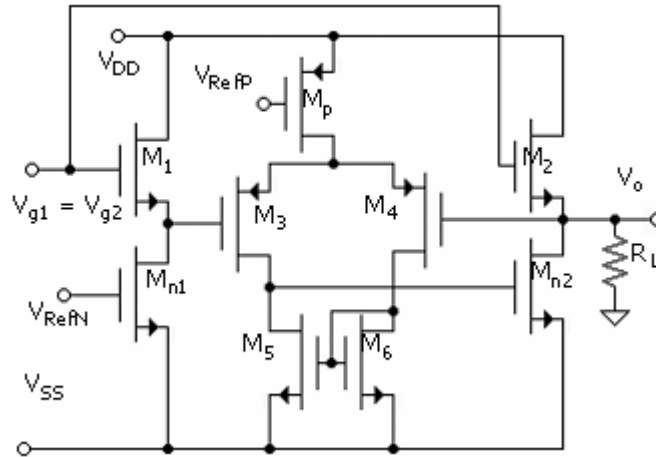
$$A_{vLoad} = A_v \frac{R_L}{R_L + R_O}$$

A_v and R_O are calculated once, using (14.35) and (14.37), respectively. Thus, the effect of loading can be obtained with one calculation using (14.38). As in the sample calculation above for the amplifier gain with loading included, $A_{vLoad} = 100.4$.

14.8. Output Circuit of the TS271 Opamp

The TS271 operational amplifier is designed to operate at very low current levels. This would include the current of the source follower of the output stage. In this case, the output resistance of an amplifier with a simple source-follower stage for an output stage, such as in Fig. 14.7, would be unacceptably high. In the TS271 operational amplifier, the 9-transistor circuit of Fig. 14.8 replaces the simple source-follower stage consisting of M_6 and M_{13} of Fig. 14.7. Note that the circuit of Fig. 14.8 uses different transistor designators. In Fig. 14.7, M_6 and M_{13} are equivalent to M_1 and M_{n1} of Fig. 14.8.

Figure 14.8. The output stage of the TS271 opamp. The new source follower consisting of M_1 and M_{n1} is equivalent to the stage of M_6 and M_{13} of the amplifier of Fig. 14.7.

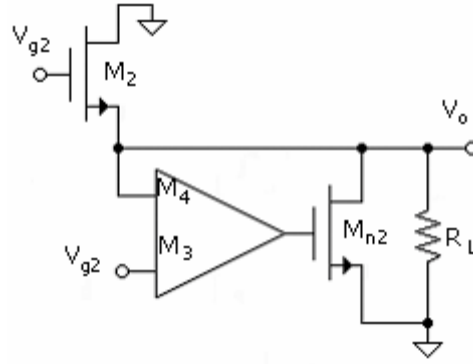


The circuit of [Fig. 14.8](#) has an additional source-follower stage consisting of M_2 and M_{n2} . The output from the common-source stage (drain of M_3 of [Fig. 14.7](#)) is applied to the gates of both source-follower stages. The sources (outputs) of M_1 and M_2 are connected to the inputs of the differential-stage gates of M_3 and M_4 , respectively. There is no load at the source of M_1 such that the gain of the source follower can be considered unity. Thus, $V_{g3} \approx V_{g2} = V_{g1}$ (incremental voltages). By contrast, the source of M_2 is connected to R_L such that, in general, $V_o = V_{s2} < V_{g2}$.

From another perspective, suppose $V_{g2} = 1V$, that is the source-follower gates are raised by 1V above the bias level. With a load, R_L , the current increase will be greater for M_2 compared to M_1 . This will cause the source voltage of M_2 to be slightly lower (than that of M_1) such that the input to the differential amplifier is nonzero and the output voltage (drain of M_3) will drop. The result is a reduction of the gate voltage of M_{n2} and thus a reduction of the drain current in M_{n2} . The incremental drain current of M_{n2} provides most of the current through the load; the load current is dominated by the change in the current of M_{n2} instead of M_2 .

[Fig. 14.9](#) is a reasonable equivalent, incremental, circuit. The differential stage (M_3 , minus, M_4 , plus, and the drain of M_3 as the output) is represented by an opamp symbol.

Figure 14.9. Equivalent circuit of the output stage of [Fig. 14.8](#). The opamp symbol represents the differential-amplifier stage of M_3 and M_4 .



Since the source follower of M_1 has a transfer function of nearly unity, incremental $V_{g2}(=V_{g1})$ is moved directly to gate of M_3 (minus input of the differential amplifier) as in [Fig. 14.9](#). Thus

Equation 14.39

$$V_{gs2} = V_{gsn2} / a_{vda}$$

where a_{vda} is the gain of the opamp and the output of the opamp is V_{gsn2} . The amplifier (output stage) output voltage is therefore

Equation 14.40

$$V_o = (I_{d2} - I_{dn2})R_L = (g_{m2}V_{gs2} + g_{mn2}V_{gsn2})R_L = (1 + a_{vda})g_{m2}R_L V_{gs2}$$

where we now assume that $g_{m2} = g_{mn2}$. It follows that the relation between V_{gs2} and V_o is

Equation 14.41

$$V_{gs2} = \frac{V_o}{(1 + a_{vda})g_{m2}R_L}$$

Combining this with

Equation 14.42

$$V_g = V_{gs2} + V_o$$

gives the transfer function, input to output, as

Equation 14.43

$$\frac{V_o}{V_{g2}} = \frac{G_m R_L}{1 + G_m R_L}$$

where

Equation 14.44

$$G_m = (1 + a_{vda}) g_{m2}$$

The output resistance of the circuit of [Fig. 14.8](#) is $1/G_m$. The output resistance of the circuit of [Fig. 14.8](#) is thus reduced by a factor of about $1/a_{vda}$ compared to the circuit of [Fig. 14.7](#) for the same g_m of the source follower transistors. From ([14.18](#)), the gain of the differential-amplifier stage is

$$a_{vda} = \frac{2}{V_{effp3} (\lambda_n + \lambda_p)}$$

This would typically be on the order of 100. Thus a significant benefit over the simple source-follower stage output is realized with the circuit of [Fig. 14.8](#). It is also significant that there is no body effect associated with M_{n2} .

14.9. Summary of Equations

$\frac{I_{D11}}{I_{D10}} = \frac{W_{11}}{W_{10}} \frac{1 + \lambda_n V_{DS11}}{1 + \lambda_n V_{DS10}} \approx \frac{W_{11}}{W_{10}}$	Bias equation for differential amplifier stage.
$a_{vcm} = - \frac{g_{m1} R_{D2}}{1 + g_{m1} 2r_{ds11}} \approx - \frac{R_{D2}}{2r_{ds11}}$	Differential-stage common-mode gain equation for case of drain bias resistor, R_{D2} .
$CMRR = 1 + g_{m1} 2r_{ds11}$	Common-mode rejection ratio for case of drain bias resistor, R_{D2} .
$\frac{I_{D12}}{I_{D10}} = \frac{W_{12}}{W_{10}} \frac{1 + \lambda_n V_{DS12}}{1 + \lambda_n V_{DS10}} \approx \frac{W_{12}}{W_{10}}$	Bias equation for common-source stage.

$\frac{I_{D11}}{I_{D10}} = \frac{W_{11}}{W_{10}} \frac{1 + \lambda_n V_{DS11}}{1 + \lambda_n V_{DS10}} \approx \frac{W_{11}}{W_{10}}$	Bias equation for differential amplifier stage.
$a_{vcs} = - \frac{g_{m3}}{g_{ds12} + g_{ds3}}$	Common-source stage gain with current-source load.
$a_v = - \frac{I_{D2} R_{D2}}{V_{eff1}} \frac{2}{V_{eff3} (\lambda_n + \lambda_p)}$	Amplifier gain for case of differential-stage drain bias resistor, R_{D2} , and common-source stage gain with current-source load.
$R_{o25} = \frac{1}{g_{ds2} + g_{ds5}}$	Output resistance of differential stage with current-source load.
$a_{v2} = \frac{g_{m1}}{g_{ds2} + g_{ds5} + \frac{1}{R_L}} = \frac{g_{m1} R_L}{1 + (g_{ds2} + g_{ds5}) R_L}$	Differential-stage gain with current-source load and external load resistor, R_L .
$W_3 = 2W_5 \frac{W_{12}}{W_{11}}$	Relation to determine M_3 gate width, W_3 .
$W_3 = 2W_5 \frac{1 + \lambda_p V_{SD5}}{1 + \lambda_p V_{SD3}} \frac{W_{12} (1 + \lambda_n V_{DS12})}{W_{11} (1 + \lambda_n V_{DS11})}$	Precision relation to determine M_3 gate width, R_{D2} .
$a_v = - \frac{g_{m1}}{g_{ds2} + g_{ds5}} \frac{g_{m3}}{g_{ds12} + g_{ds3}}$	Gain for two-stage amplifier, both stages with current-source loads.
$a_{vsf} = \frac{g_{m6}}{g_{m6} (1 + \eta) + g_{ds13} + g_{ds6} + \frac{1}{(R_f + R_G) \parallel R_O}}$	Gain of the source-follower stage for the case of load $R_f + R_G \parallel R_O$ (feedback resistors and external load, R_O).
$r_o \approx \frac{1}{(1 + \eta) g_{m6}}$	Output resistance of source-follower stage.
$W_3 = 2W_5 \frac{W_{12}}{W_{11}}$	Output resistance of resistance feedback amplifier
$W_3 = 2W_5 \frac{1 + \lambda_p V_{SD5}}{1 + \lambda_p V_{SD3}} \frac{W_{12} (1 + \lambda_n V_{DS12})}{W_{11} (1 + \lambda_n V_{DS11})}$	TS271 output resistance, where g_{m2} is for the source-follower stage and a_{vda} is the gain of the differential amplifier stage.

Unit A. Communicating with the Circuit Board: LabVIEW Programming and Measurement Exercises

Communicating between the computer and a circuit board (sending and receiving voltages from the computer) is accomplished with the data acquisition card (DAQ). The DAQ is normally connected from the computer to the circuit board with a 68-pin cable. In general, all 68 pins are used for such things as analog inputs and outputs, triggering,

digital inputs and outputs, a 5-V dc supply, and counters. In our projects, we use, for the most part, only analog inputs and outputs. A digital output could also be used if we required an additional output, which of course is limited to on or off.

Since the analog inputs and outputs involve digital-to-analog and analog-to-digital conversions, special consideration must be given to the discrete nature of the conversion relations. These are discussed in the following along with a series of basic programming and measurement exercises. A sequence of VIs (virtual instruments) is programmed. In [Unit A.2](#) and beyond, preprogrammed VIs are used.

The LabVIEW library, ProjectA.llb, of [Project A](#) contains the preprogrammed VIs, along with samples of all of the VIs in the exercises. [Project A](#) complements this unit. It is recommended that the parts of [Project A](#) be completed at the end of a given programming segment in the unit. The appropriate project part is flagged as with the following:

Project PA.x (Title of Project Segment)

A.1. Basics of Sending and Receiving Circuit Voltages

The following is based on a National Instruments PCI-MIO-16E-4 DAQ card (or equivalent). The sample rate is not critical. The DAQ card has eight differential analog input channels and two analog output channels. We generally use only channels 0, 1, and 2 for receiving in our projects.

LabVIEW provides some preprogrammed VIs for readily sending voltages to the circuit board and sampling a signal for reading into a LabVIEW program. The most important of these for analog circuit measurements are AI Acquire Waveform.vi and AO Update Channel.vi. Some measurements will be made next using these two VIs.

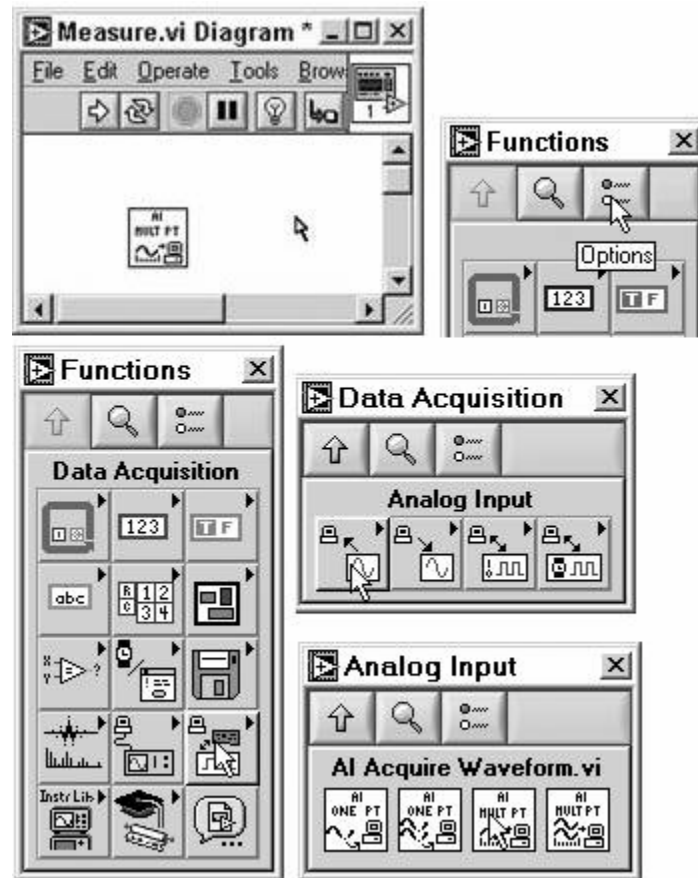
As the name suggests, the VI, AI Acquire Waveform.vi samples the stipulated voltage a number of specified times. For dc measurements, the samples are simply averaged. This provides for a more precision measurement than from a single sample. LabVIEW has a single-point DAQ function, AI Sample Channel.vi. This function does not take advantage of computer-based data acquisition.

A.1.1. Programming Exercise: Installing, Sending, and Receiving VIs in the Program

Open a new VI in LabVIEW and save it with a personal name. Here the VI is referred to as Measure.vi. The VI has a Front Panel and a Diagram. The Diagram is shown in [Fig. A.1](#). Moving from the Diagram to the Front Panel or the opposite can be performed under menu Window>>Show Diagram (or Ctrl/E) and Window>>Show Panel (or Ctrl/E). The inputs and outputs, including graphics, appear in the Front Panel and the program is set up in the Diagram. For programming, three palettes are used: Tools Palette, Controls

Palette, and Functions Palette. The methods for getting these palettes are outlined in [Table A.1](#).

Figure A.1. LabVIEW Diagram with icon for AI Acquire Waveform.vi. To place the icon in the Diagram, Right Click on the Diagram to get Functions. Click the stickpin. Then follow through the sequence:>>Data Acquisition>>Analog Input>> AI Acquire Waveform.vi. Go to Options in the Functions Tool Bar to Change Palette Set if desired.



Place AI Acquire Waveform.vi in the Diagram of Measure.vi. It is located in the Functions Palette and under Data Acquisition>>Analog Input ([Fig. A.1](#)). Note that the Functions Palette has various Options, that is, more or less features. To get the version shown in [Fig. A.1](#) (if is not already in this form), go through Options in the Functions Tool Bar ([Fig. A.1](#)).

TABLE A.1

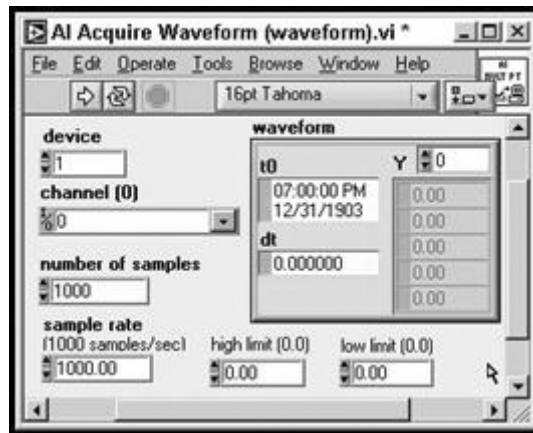
Diagram	Tools Palette	Shift/Right Click
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TABLE A.1

	Functions Palette	Right Click – Stickpin to hold
	Or Menu Sequence:	Window>>Show Functions Palette
Front Panel	Tools Palette	Shift/Right Click
	Controls Palette	Right Click – Stickpin to hold
	Or Menu Sequence:	Window>>Show Controls Palette

When installed, AI Acquire Waveform.vi appears as in [Fig. A.1](#). Now Double Click on the icon (or Right Click and Open Front Panel) to bring up the Front Panel of AI Acquire Waveform.vi. It will appear similar to the example shown in [Fig. A.2](#), which has some cosmetic changes.

Figure A.2. Front Panel of LabVIEW VI AI Acquire Waveform.vi. Digital Controls provide for various inputs including the number of samples to be taken, in this example, 1000. The output is the designated waveform and is an array of sample values of index 0 through 999 in this case.



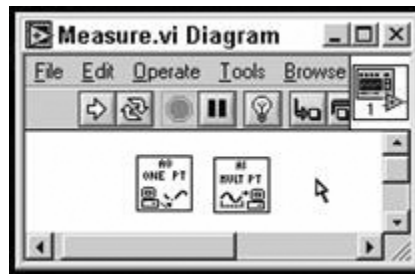
The Digital Controls in the Front Panel of AI Acquire Waveform.vi provide for installing the machine device number (e.g., 1), the input channel, number of samples, and sample rate in addition to the ADC (analog-to-digital converter) Digital Controls high limit and low limit. The default, 0 V, as in the example, is equivalent to 10 V and –10 V for high limit and low limit, respectively.

The device number of the DAQ card is set using the National Instruments Measurement & Automation utility. Go to Start>>Programs>>National Instruments>>Measurement &

Automation. This opens a window called Max. Then under My System, locate Devices and Interfaces and Right Click on the name of the DAQ card to bring up Properties. The device number can be set from the Properties window. The sample programs in ProjectA.llb use Device 4 (or as set on your PC). Close Max.

We will want to send out some voltages to be read by the input channel. For this, add output channel function AO Update Channel.vi to the Diagram of your VI. It is under Functions>>Data Acquisition>>Analog Output. Install this VI in your VI diagram as shown in [Fig. A.3](#). Position the icons with the Positioning Tool (arrow). It is obtained by cycling through the Tools by striking the Tab key or Space Bar, or, again, get the Tools Palette with Shift/Right Click from the Front Panel or Diagram.

Figure A.3. LabVIEW VI Diagram with the addition of output channel VI AO Update Channel.vi. The two VIs represented by the icons can be opened with Right Click and open Front Panel or with Double Click.



[Project PA.1](#) Sending and Receiving Voltages with the Sending and Receiving VIs (Measure.vi, version 1)

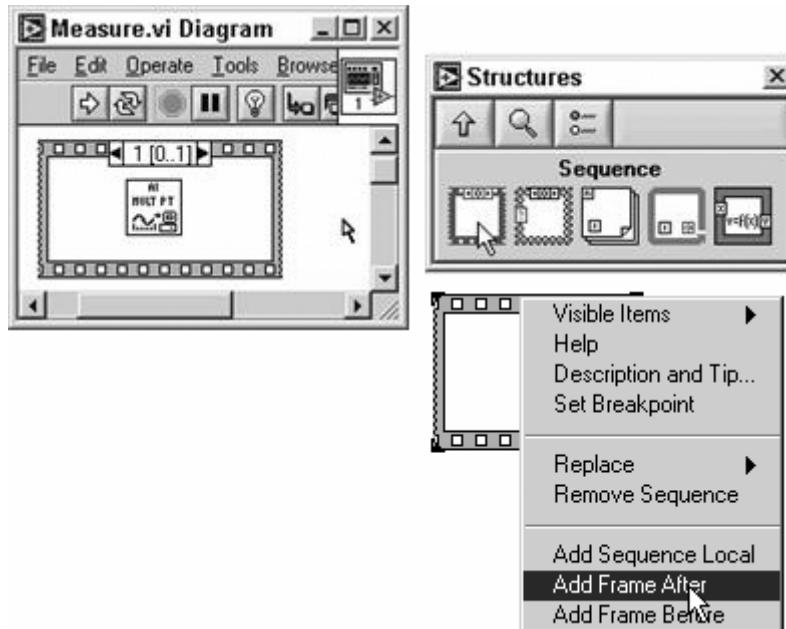
A.1.2. Programming Exercise: Using a Sequence Structure

The two VIs represented by the icons of the Diagram are subVIs of Measure.vi. That is, they will run when we run Measure.vi. In this case, it is necessary that they execute in the proper order. For example, we would want to send out the voltage before reading the response.

For ensuring proper execution order, install a Sequence Structure as shown in [Fig. A.4](#). Obtain the Sequence Structure from the Diagram under the menu series Functions>>Structures>>Sequence. Recall that to obtain the Functions Palette, Right Click on the Diagram.

Figure A.4. LabVIEW VI Diagram with Sequence Structure. This Structure executes in order of index at the top of the Frames. Frame 1 shown contains the VI for reading voltage. Frame 0 contains the VI for sending voltage. Click on

arrows to move through Frames. Also shown is the Structures Palette under Functions and the process of adding a frame.



Initially, the Sequence Structure is too small. To enlarge, get the Positioning Tool (arrow) by pressing the Space Bar, by sequencing through the Tools with Tab, or by using Shift/Right Click to bring up the Tools Palette. Position the Tool at the lower-right-hand corner of the Sequence Structure, Left Click, and drag to enlarge the Structure.

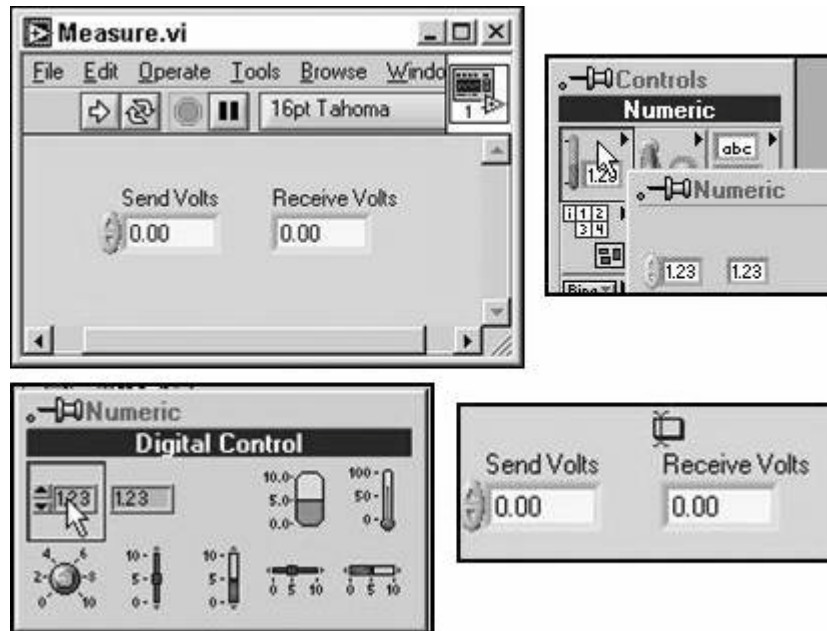
Right Click on the edge of the Frame of the Sequence Structure and go to Add Frame After ([Fig. A.4](#)). The result will be two Frames with index 0 and index 1. Now place (e.g., drag) the send and receive VIs into Frame 0 and Frame 1, respectively. Left Click on the left and right arrows at the top of the Sequence Structure to move between Frames. The result will be as shown in [Fig. A.4](#). The Diagram shows Frame 1. The program will execute the VI in numerical Frame order. Thus, the output voltage will always be sent before the input channel attempts to read.

A.1.3. Programming Exercise: Digital Controls and Digital Indicators

The Front Panel of Measure.vi will now be configured to permit assigning the voltage to be sent out and reading the input voltage from the Front Panel. The Front Panel of the new configuration is shown in [Fig. A.5](#). From the Front Panel and then from the Controls Palette, go to Controls>>Numeric ([Fig. A.5](#)), and from this selection, get a Digital Control and a Digital Indicator. You might want to color the Front Panel. For this, Shift/Right Click and get the Coloring Tool from the bottom of the Palette. Then, with the Coloring Tool, Right Click on the Front Panel and pick a color. You can also color the Diagram. Change the labels over the Digital Control and Digital Indicator using the

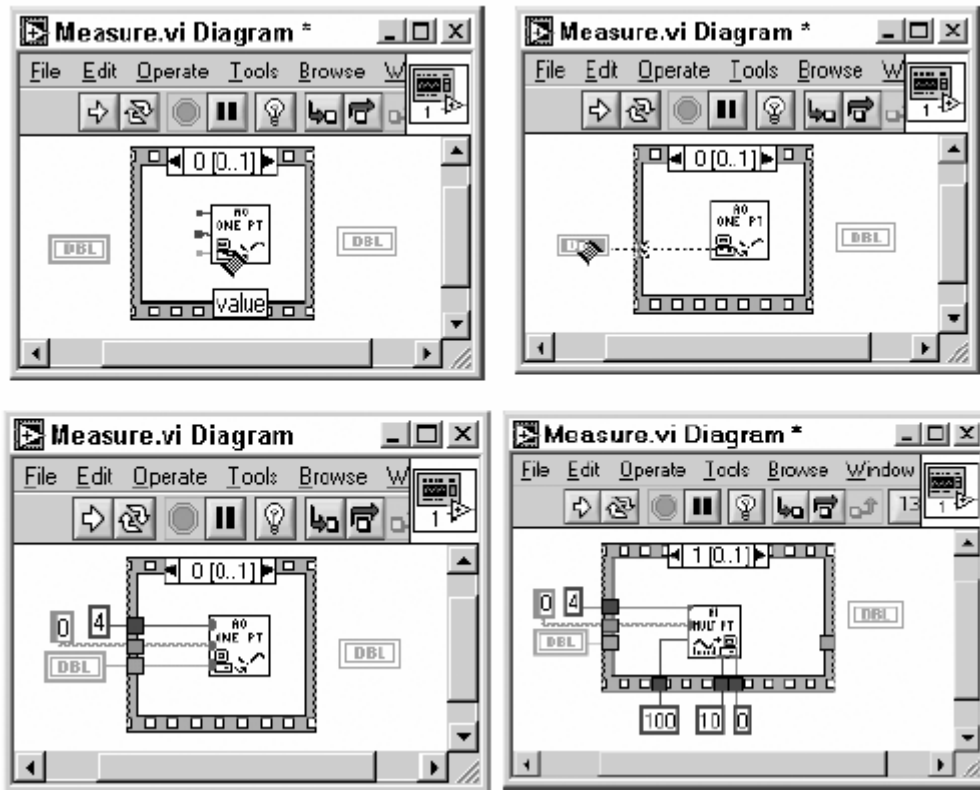
Edit Text Tool ([Fig. A.5](#)). Obtain the Tools Palette with Shift/Right Click and the Edit Text Tool. The labels are defaulted as Numeric. Use the Edit Text Tool from the Tools Palette to relabel as in the example (or similar).

Figure A.5. Front Panel with addition of Digital Control (send) and Digital Indicator (receive). These are obtained with a Right Click on Front Panel for Controls Palette and then Numeric and Digital Control and Digital Indicator. Also shown is the Edit Text Tool for changing the labels on the Controls from the default Numeric.



Now go to the Diagram (under Menu Window>>Show Diagram or use Ctrl E) for wiring the program in the diagram. This will include Terminals and Numerics and Strings. Start by connecting the sending VI icon (Frame 0) value to the Digital Control terminal ([Fig. A.6](#)). To locate the terminal for the Digital Control, go back to the Front Panel, Right Click the Digital Control for Send Volts, and move the pointer to Find Terminal. The Diagram will open and the proper terminal will be highlighted. Note that the terminal for Digital Control is darker than those for Digital Indicator. This provides for an alternative means of locating the proper terminal.

Figure A.6. Top Diagrams: Locating the value terminal on the icon and moving the Wiring Tool to the Digital Control terminal. Bottom Diagrams: Completed wiring for Frame 0 and Frame 1. Completion of Frame 1 is discussed below.



Get the Wiring Tool (appears like a roll of solder) by striking the Space Bar. To wire, first find the correct terminal on the icon. There are three in this case. The terminal for this connection, value, is the bottom third of the icon. To locate the region of the connection, sweep the Wiring Tool over the icon and find the value. A given terminal displays a black-and-white blinking while the Wiring Tool is in contact with this terminal. Now, with the Wiring Tool, Left Click on the value connection region, Release, and move the Wiring Tool to the terminal of the Digital Control. Left Click again to finish.

For an alternative view of where the connections are made, you can Right Click on the icon, and get Visible Items>>Terminals. Recall again that the Digital Control terminal is darker than the Digital Indicator terminal. This adds additional clarification as to the correct terminal.

Complete the connections to the icon with a Numeric Constant for the device (Right Click, then Functions>>Numeric>>Numeric Constant) and a String Constant for the channel (Right Click, then Functions>>String>>String Constant). Use the Pointing Tool (finger) to write in the constant values. Obtain the various Tools by striking the Tab key.

Now sequence to Frame 1 to connect to the AI Acquire Waveform.vi icon. (Click on the arrows at the top of the Sequence Structure.) Complete all wiring, as shown in [Fig. A.6](#), except for the waveform terminal (connection on the right side of the icon in Frame 1). This includes device (numeric constant, 4), channel (string constant, 0), number of

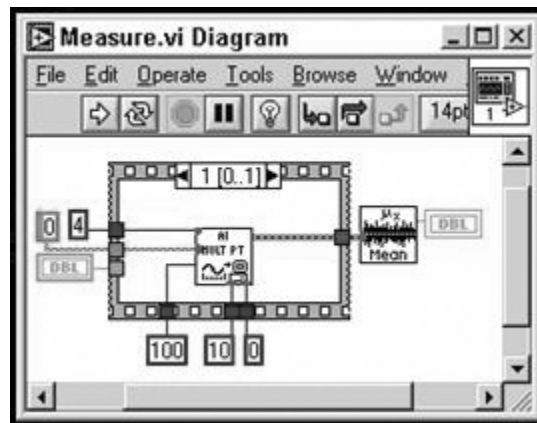
samples (numeric constant, 100), high limit (numeric constant, 10), and low limit (numeric constant, 0). The low limit can be set to 0 because no negative numbers are to be read.

As a reminder for using the Wiring Tool: In the Diagram, use Shift/Right Click to get the Tools Palette and obtain the Wiring Tool. The Wiring Tool is also obtainable by pressing the Space Bar, while the Diagram is the active window, to sequence between the Wiring Tool and the Positioning Tool. Left Click on the terminal to be wired (do not hold) and move the Tool to the other termination and Left Click again.

A.1.4. Programming Exercise: Using the Array Average Value VI from Probability and Statistics for Averaging the Samples Received

To complete connections to the terminals associated with Frame 1, we must add the VI Mean.vi ([Fig. A.7](#)). It is used to obtain the average of all of the number of samples (as selected) in the reading (receiving) function. Note that Mean.vi is connected between the waveform terminal of the AI Acquire Waveform.vi and the Digital Indicator terminal. The output from AI Acquire Waveform.vi (waveform) in this case is a cluster of information including a data array. In the Diagram, a much broader line than that for a single data point represents the connection.

Figure A.7. Diagram showing Frame 1. The VI Mean.vi has been added to average the data samples.

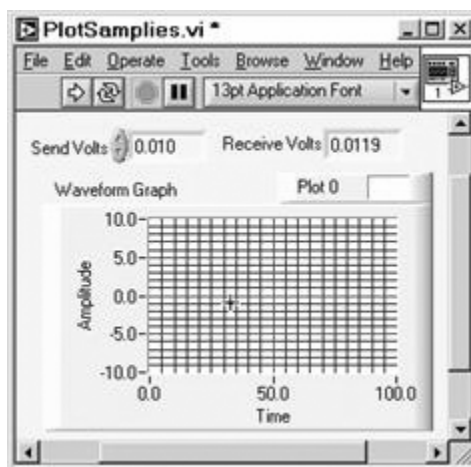


Mean.vi is obtainable from two different locations in the Functions palette: Functions>>Analyze>>Mathematics>>Probability and Statistics>>Mean.vi; Functions>>Mathematics>>Probability and Statistics>> Mean.vi. Connect the waveform of the Frame 1 icon to Mean.vi to the Digital Indicator Terminal. Connect the output in Mean.vi, mean, to the Terminal of the Digital Indicator. This measurement, which is the average of a large number of samples, is superior to obtaining a single point that might very well fluctuate due to noise.

A.1.5. Programming Exercise: Installing and Using the Waveform Graph

A given dc voltage is sent out via an output channel (e.g., frame 0 of [Fig. A.7](#)). A circuit node response is measured, in our projects, by sampling the voltage a number of times. With LabVIEW, we can easily obtain a graphical presentation of the sample data points to provide an indication of the noise that exists on the dc voltage being measured and in the measurement system. For this, obtain a new VI by saving a Save As copy and giving it a new name. Here it is named PlotSamples.vi. The new VI will initially appear something like that in [Fig. A.8](#).

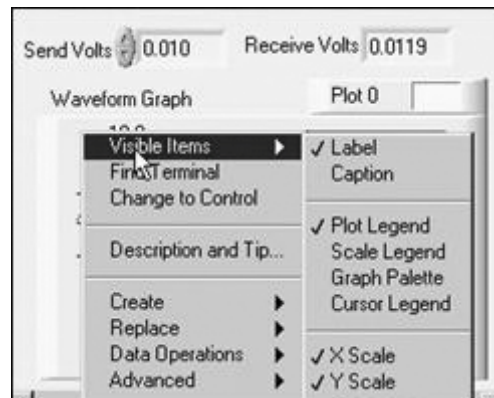
Figure A.8. Front Panel with addition of a Waveform graph. The graph is obtainable from the Front Panel under Controls>>Graph>>Waveform Graph.



To install the Waveform graph, in the Front Panel, Right Click and get Controls>>Graphs>>Waveform Graph and place it on the Front Panel. Shift/Right Click on the face of the graph to get the Coloring Tool and pick a color for your graph background field and Frame. [Figure A.8](#) shows a new graph installation with some color changes.

A new installation of a graph has the Plot Legend open (Plot 0 in the example). In general, this feature and others can be hidden or made to appear by a Right Click anywhere on the graph and then selecting Visible Items as shown in the example of [Fig. A.9](#).

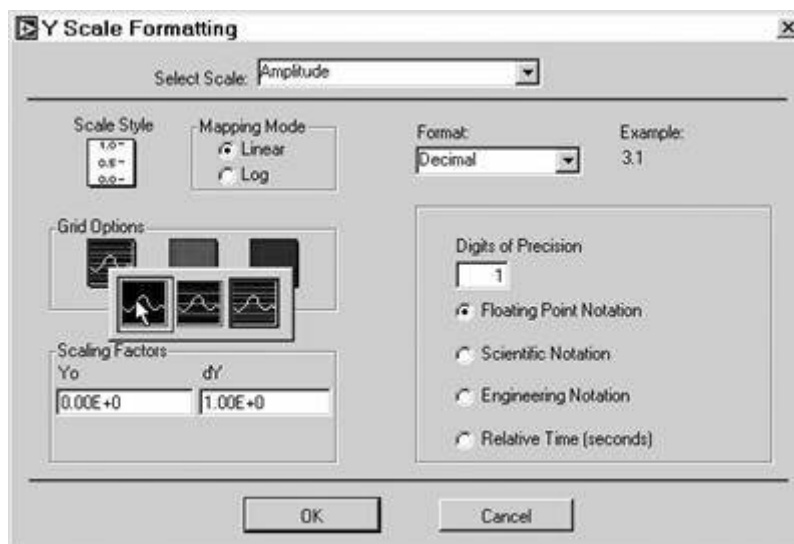
Figure A.9. Various graph features are obtained under Visible Items as shown. This includes the Plot Legend for setting up the plot style.



In the box of the Plot Legend, Right Click, go to Color and pick a color for the plotting points or lines. Also, select Points under Common Plots. Hiding the Plot Legend might be in order, now although this is optional. (Right Click on the graph face and select Visible Items>>Plot Legend.)

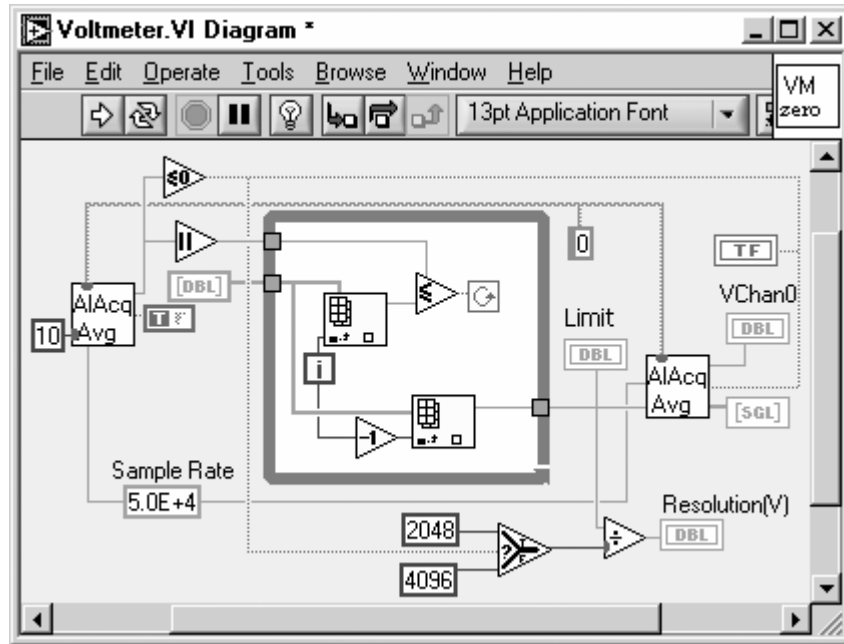
For the present application, it is preferred to have no grid. To accomplish this, Right Click on a number on the Y-scale of the Frame of the graph, go to Formatting... and Grid Options ([Fig. A.10](#)). Select the no grid (Left Click) condition as illustrated in [Fig. A.10](#). Repeat for the X-axis. Note that a given axis can be selected from the Formatting window at the top under Select Scale.

Figure A.10. Y-scale Formatting showing the section of no grid from Grid Options. Selection of Digits of Precision is also made in this window.

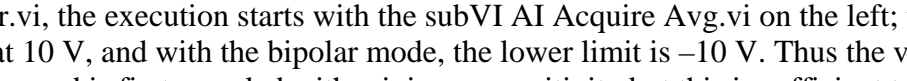


Waveform.vi, for example, 10 V (high limit) and 0 V (low limit) in the diagram of our autoranging voltmeter as shown in [Fig. A.12](#).

Figure A.12. Diagram of autoranging dc voltmeter, Voltmeter.vi. The While Loop runs until the comparison finds the measured value less than the lowest possible limit.



The autoranging voltmeter is programmed as follows. First, a copy of AI Acquire Waveform.vi is Saved As..., modified for dc only and renamed as AI Acquire Avg.vi. The modification consists of taking the average of the array of samples and providing for a single-point output of the average value of the samples. AI Acquire Avg.vi is programmed to operate in selectable bipolar or unipolar mode. Thus, this feature is included in Voltmeter.vi.

This VI is then used as a subVI in the autoranging dc voltmeter. Upon running Voltmeter.vi, the execution starts with the subVI AI Acquire Avg.vi on the left; the high limit set at 10 V, and with the bipolar mode, the lower limit is -10 V. Thus the voltage being measured is first sampled with minimum sensitivity but this is sufficient to obtain an approximate value of the voltage. The initial measurement value is then processed through a selection scheme to decide which limit should be set into the final measurement subVI that is again AI Acquire Avg.vi. The limit choices are given in [Table A.2](#). These are stored in an Array Digital Control as seen on the left in the diagram . The While Loop compares the measured value with each limit value starting at the top. The loop halts when the comparison finds the smallest possible limit. Bipolar or unipolar are set according to the sign of the initially measured value.

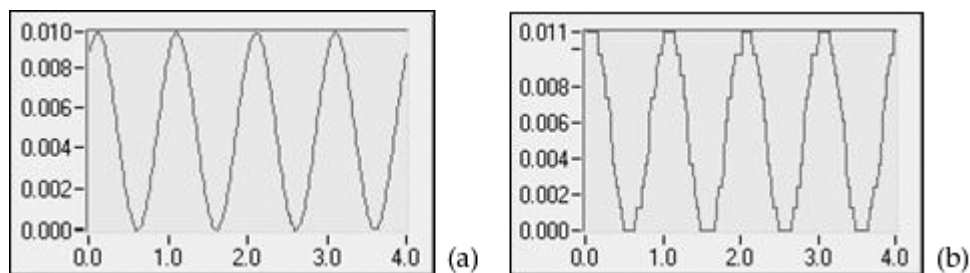
A factor of 2 of improved resolution is available with the unipolar case. As noted, the valid limit settings and associated resolutions are shown in [Table A.1](#). With the autoranging voltmeter, during a sweep, the resolution is maintained, through the sweep, at roughly a fixed percentage of the value of the numbers being measured.

TABLE A.2

Unipolar	Range	Resolution	Bipolar	Range	Resolution
	0 to +10 V	2.44 mV		-10 to +10 V	4.88 mV
	0 to +5 V	1.22 mV		-5 to +5 V	2.44 mV
	0 to +2 V	488 μ V		-2 to +2 V	1.22 mV
	0 to +1 V	244 μ V		-1 to +1 V	488 μ V
	0 to +500 mV	122 μ V		-500 to +500 mV	244 μ V
	0 to +200 mV	48.8 μ V		-200 to +200 mV	122 μ V
	0 to +100 mV	24.4 μ V		-100 to +100 mV	48.8 μ V
				-50 to +50 mV	24.4 μ V

[Figure A.13](#) shows a measurement of a sine-wave from a function generator. This was obtained in LabVIEW using the ac voltmeter that is discussed in a following unit. The voltage waveform is sampled at the two limit settings, 100 mV (a) and 2 V (b). The peak of the sine-wave is 10 mV. It is clear that the resolution is not adequate at the higher limit setting. In an autoranging version of the ac voltmeter, the best limit setting is automatically assigned.

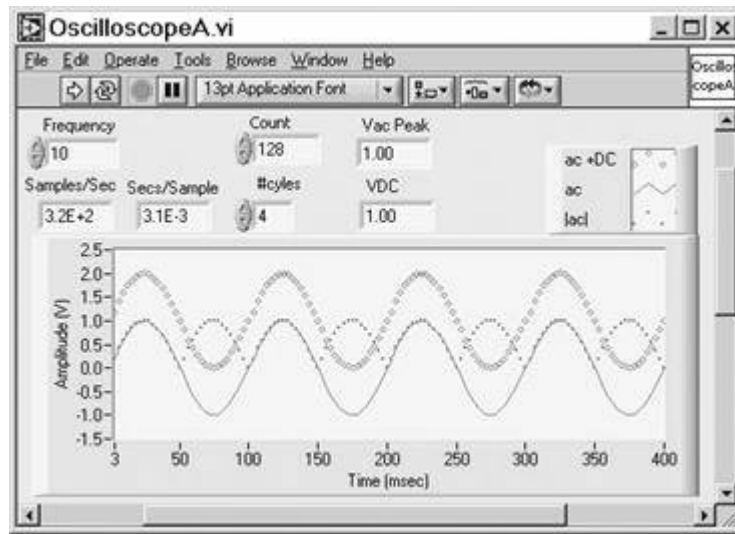
Figure A.13. LabVIEW measurement of a sine-wave voltage with peak of 10 mV. The sinewave is sampled at two limit settings of (a) 100 mV and (b) 2 V. The resolutions are (a) 48.8 μ V and (b) 1.22 mV.



A.3. LabVIEW Oscilloscope and Voltmeter (ac)

An example of an oscilloscope is shown in [Fig. A.14](#). This special version displays three waveforms. These are the total waveform (ac + D), the sine-wave minus the dc (average value) and the absolute value of the sine-wave (|ac|).

Figure A.14. Front Panel for a possible LabVIEW oscilloscope. Digital Controls provide for inputs of Frequency, Count (number of samples), and #cycles. Digital Indicators show Samples/Sec, Sec/Sample, VDC, and Vac Peak. The limits are set in the Diagram.



The voltmeter is designed to measure peak or RMS values for periodic waveforms, thus, to be an ac voltmeter. Total sampling time must occur in exact multiples of cycles. This provides for obtaining the waveform (sine-wave) peak or RMS value based on an average of all the samples.

The required sample rate, SR, is obtained from

$$SR = \left[\frac{\#Samples}{\#Cycles} (Frequency) \right] (Samples/Sec)$$

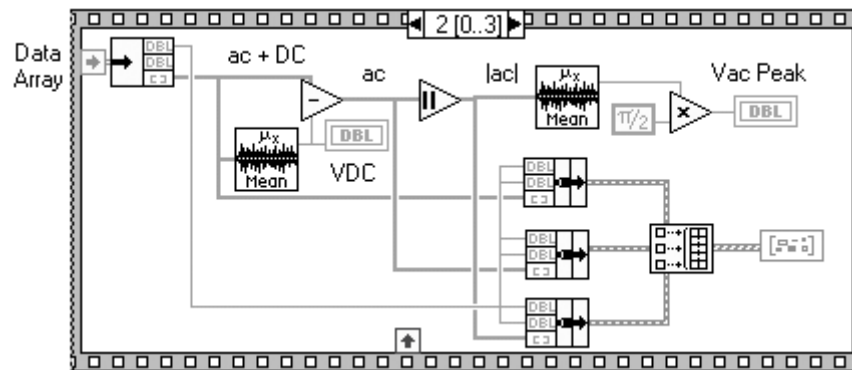
The values for all the numbers on the right-hand side are set in Digital Controls on the Front Panel ([Fig. A.14](#)). The #Samples is also referred to as the Count.

SR and #Samples are then sent to the subVI, AI Acquire Waveform.vi. For example, assume that the #Cycles is 4 and the #Samples is 256. Once cycle will be sampled 64

times. Assume also that $f = 1000$ Hz. Sample rate SR will be 64000 samples/sec or about 16 $\mu\text{sec/sample}$. The total sampling time is $T \cdot \text{\#Cycles}$, where T is the period and is $T = 1/f$. In this example this is 4msec.

Digital Indicators in the Front Panel display the dc voltage, VDC, and the peak, Vac Peak. Vac peak is calculated as the average value of $|ac|$ multiplied by $\pi/2$. The programming sequence for obtaining the three waveforms is shown in [Fig. A.15](#). In the Sequence Frame is also shown the programming configuration for connecting to the graph on the Front Panel and the computation of the peak value, Vac Peak.

Figure A.15. Frame of Sequence Structure showing series of events for obtaining ac peak value. The mean is subtracted from the array with the result converted to the absolute value, and the average is multiplied by $\pi/2$.



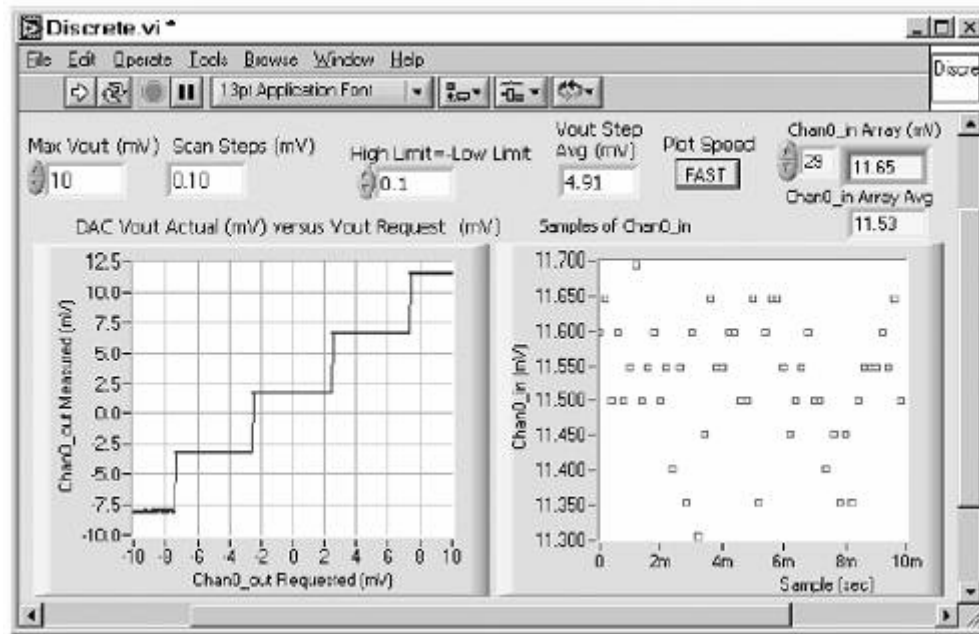
[Project PA.5](#) Observing the Oscilloscope Output Graph

A.4. Measuring the Discrete Characteristics of Sending and Receiving Voltages

In the discussion above of the autoranging voltmeter, the discrete nature of the ADC and DAC was demonstrated. LabVIEW VIs that plot the input voltage versus output voltages for both ADC and DAC demonstrate the effect further.

[Figure A.16](#) shows the Front Panel of the VI Discrete.vi. This VI is for demonstrating the discrete nature of the sending and receiving functions. We first consider that of the sending function, as in the case of Discrete.vi in [Fig. A.16](#).

Figure A.16. DAC in the bipolar mode. Plot of the programmed voltage sent out (0.1-mV steps) versus measured actual output voltage. The graph on the right is a plot of the input voltage samples for a single output sweep.

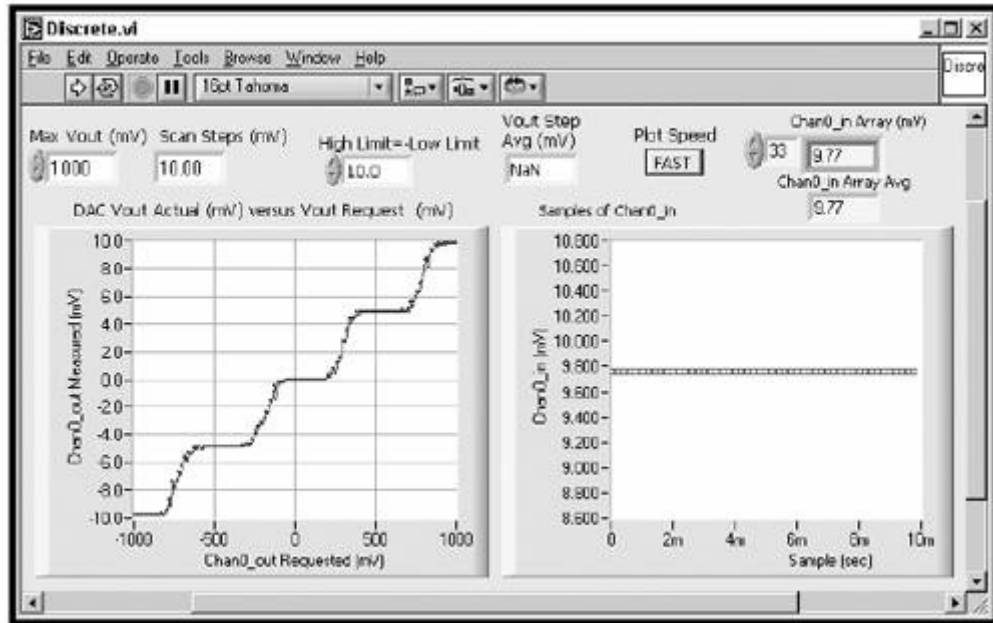


The graph on the left shows the voltage actually sent out (Y-axis) for a quasi-continuous range of requested (programmed) voltage to be sent out (X-axis). In the example, the program called for the output voltage to be swept over a range from -10 to 10 mV in steps of $100\text{ }\mu\text{V}$. The actual voltage sent out is discrete in steps of 4.88 mV; thus only four steps over the entire range are sent out. The graph on the right is a plot of the input voltage samples from one (the last) measurement. The plot illustrates the inherent scatter in this type of measurement.

[Project PA.6 Discrete Output Voltage from The DAQ](#)

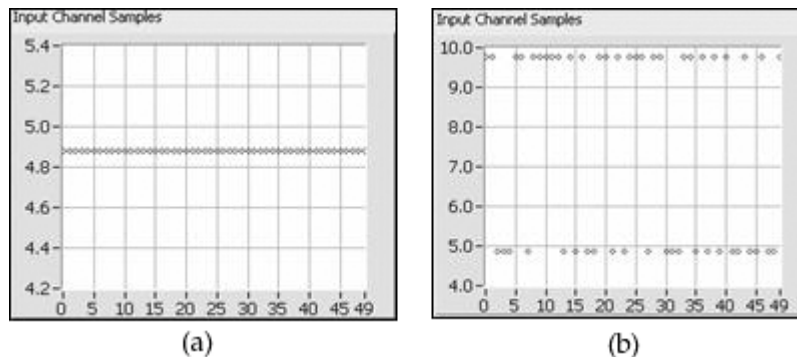
In [Fig. A.17](#) is shown the result of measuring a high-resolution output sweep with a less precision input mode of measurement. The high resolution is obtained with a resistance voltage divider. The graph on the left shows the discrete nature of the receiving function for a low-resolution mode. For the limit settings, the receiving resolution is 4.88 mV.

Figure A.17. The full range of the output sweep is 10 mV. (The X-axis is before the 100:1 voltage division.) The input channel has low resolution, as revealed by the steps in the plot.



The transition regions are nonabrupt due, to the bit uncertainty (due to noise) in these regions. In [Fig. A.18](#) is shown input voltage samples (for one output voltage) for a point in a plateau region (a) and for a point in the transition region (b). Note that the plot in the graph on the right in [Fig. A.17](#) is for a plateau region (last point in the sweep in the plot on the left) and shows no scatter.

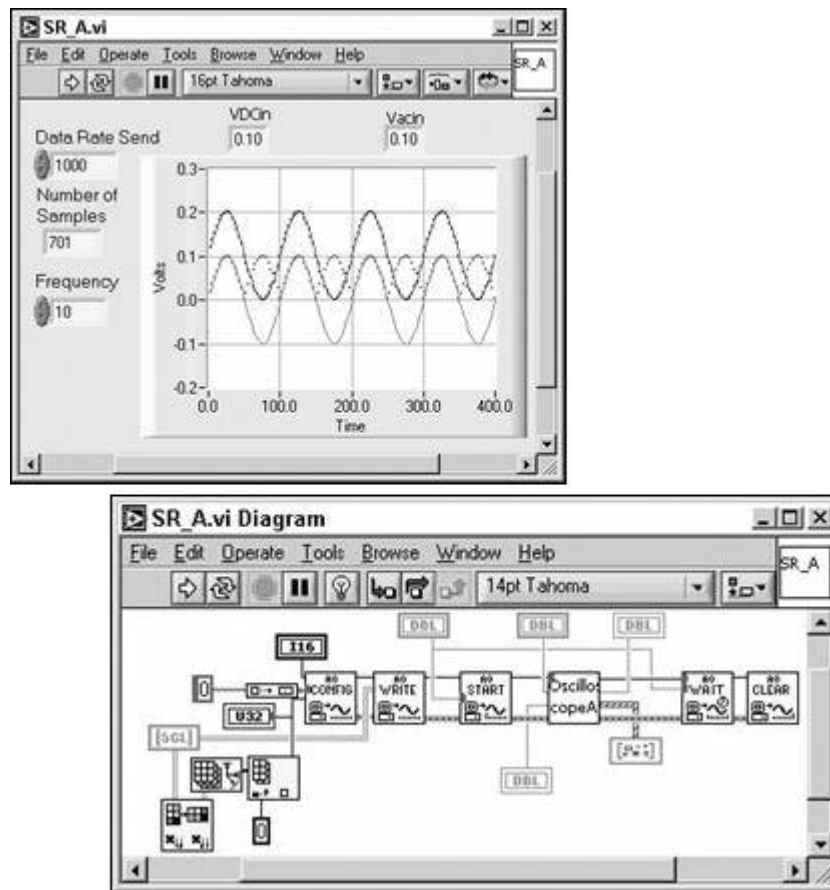
Figure A.18. (a) Chan0_in samples for a voltage in a plateau region in the plot of [Fig. A.17](#). (b) Samples for a voltage from the transition region.



A.5. Sending and Receiving Waveforms

Measuring circuit response to waveform input signals requires simultaneous sending and receiving data with the DAQ. An example was demonstrated above with the oscilloscope measurement project ([Fig. A.14](#)). The function generator VI used for that measurement is FG_A.vi. The main subVI of the function generator VI is SR_A.vi. The Front Panel and Diagram of the VI are shown in [Fig. A.19](#). The VI sends and receives voltage arrays from one output channel and one input channel.

Figure A.19. The Front Panel and Diagram of SR_A.vi. LabVIEW AO Write receives the array of the numerical values of the waveform and sends them out on the output channel. The oscilloscope is launched in sequence and measures the response voltage of the input channel while the output array is still being executed.



Information supplied from the top VI, FG_A.vi, includes sine-wave (or square-wave) frequency (Frequency), number of cycles (Cycles), and number of samples per cycle (Samples/Cycle). The information is used to facilitate the computation $\text{Sample Rate} =$

Frequency · Sample/Cycle and the size of the waveform data array (Cycle · Samples/Cycle).

The subVI, SR_A.vi, receives this information along with the waveform data array, as required by the functions in the Diagram of SR_A.vi ([Fig. A.19](#)). The waveform data array is calculated by a subVI, Sine-wave.vi (on the same level as SR_A.vi). (An alternative waveform, SquareWave.vi, is selectable from FG_A.vi). The execution sequence is AO Config.vi, AO Write.vi, AO Start.vi, OscilloscopeA.vi, AO Wait.vi, and AO Clear.vi.

The Number of Cycles is adjusted automatically in the top VI and increases for higher frequencies. The increase provides assurance that the outgoing waveform is still executing during the oscilloscope measurement phase. The duration of the waveform sent out is Cycles · T.

Project PA.8 Using the Simultaneous Sending/Receiving Function
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A.6. Summary of Programming Projects

- PA.1 [Sending and Receiving Voltages with the Sending and Receiving VIs](#)
- PA.2 [Sending and Receiving Voltages from the Front Panel](#)
- PA.3 [Plotting Measured Samples](#)
- PA.4 [Using the Autoranging Voltmeter](#)
- PA.5 [Observing the Oscilloscope Output Graph](#)
- PA.6 [Discrete Output Voltage from the DAQ](#)
- PA.7 [Discrete Input Voltage from the Circuit Board](#)
- PA.8 [Using the Simultaneous Sending/Receiving Function](#)

Unit B. Characterization of the Bipolar Junction Transistor for Circuit Simulation

The inclusion of transistors in circuit simulation requires a model for the transistor. That is, the simulator needs to know how the transistor behaves. Specifically, the simulator requires knowledge of the terminal characteristics of the transistor. For example, for a given dc voltage V_{BE} applied between terminals B (base) and E (emitter), what will be the current into B (I_B)? The mathematical relations are obtained from theory or experimental observation. Often, the theory is very complicated, but much more simple relations serve very well as good approximations.

TABLE B.1

SPICE Name	Math Symbol	Description
IS	I_S	Description
ISE	I_{SE}	Saturation current.
BF	β_F	Base – emitter leakage saturation current.
VAF	V_{AF}	Ideal maximum forward current gain.
BR	β_R	Forward Early voltage.
NF	n_F	Reverse-current gain.
NE	n_E	Forward-current emission coefficient.

The parameters for the BJT, which are to be determined in the projects as described in the following, are given in [Table B.1](#). The parameters are obtained by fitting the mathematical expressions as used by SPICE to measured current – voltage relations.

B.1. Fundamentals of Bipolar Junction Transistor Action

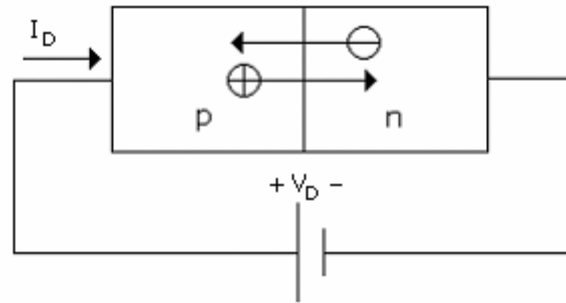
A bipolar junction transistor is made up of a sandwich of two semiconductor pn junctions. Transistors are either npn or pnp. In the active-mode transistor state, one junction, the input junction, is forward biased, and the opposite junction, the output junction, is reverse biased. The behavior of the individual pn junction diode will first be considered to clarify forward and reverse bias.

A semiconductor pn-junction diode in diagrammatic form is shown in [Fig. B.1](#). The current and voltage of an ideal pn-junction diode are related by

Equation B.1

$$I_D = I_{Sd} \left(e^{V_D/V_T} - 1 \right)$$

Figure B.1. Diagrammatic pn-junction diode. Applied voltage V_D is shown for forward bias for which current freely flows. Opposite polarity is reverse bias, where the diode is essentially cut off and $I_D = -I_S$.



where V_D is the voltage applied between the p and n regions (positive at the p terminal), I_D is the responding current, and I_{Sd} is the saturation current. The thermal voltage, V_T , is defined as $V_T = kT/q$, where T is the temperature, k is the Boltzmann constant and q is the electron charge. At 27°C , $V_T \approx 26 \text{ mV}$. V_D is positive for forward bias and negative for reverse bias.

As illustrated by the arrows in the junction of [Fig. B.1](#), for forward bias, the diode current consists of injection of holes from the p region (positive free carriers) into the n region and injection of electrons from the n region (negative free carriers) into the p region. The exponential factor in [\(B.1\)](#) is associated with the statistics of the free carriers, holes and electrons, and the effect on the lowering of the barrier to free-carrier flow of the application of the positive applied voltage.

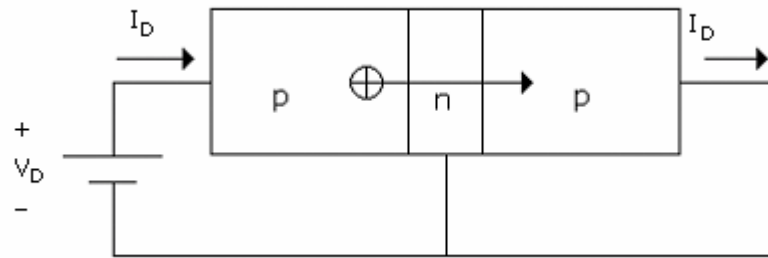
The magnitude of I_{Sd} is dependent on semiconductor electronic properties such as doping levels and free-carrier lifetime and mobility. The magnitude of current flow for a given V_D is dictated by a combination of the barrier-lowering factor (in the exponential) and the rate of free-carrier recombination; that is, carriers are annihilated by recombining with the opposite type of carrier on the opposite side of the junction from which they are injected.

A typical value for saturation current, I_{Sd} , is 10^{-11} mA , such that, for example, for $V_D = 0.6 \text{ V}$, $I_D \approx 0.1 \text{ mA}$ and increases by an order of magnitude for each additional increment of $\delta V_D \approx 60 \text{ mV}$ (at room temperature). The equation would suggest that for negative V_D and $|V_D| \gg V_T$, $I_D = -I_{Sd}$ and therefore is very small compared to the current under forward bias. In real pn-junction diodes, the current is larger and is not independent of the value of the reverse applied voltage. Nonetheless, it is still very small compared to the value of current for normal forward bias.

As mentioned above, the bipolar junction transistor is made up to two junctions, which share a common region. A pnp example is shown in [Fig. B.2](#). The individual pn junctions of a transistor also exhibit diode characteristics, and the junction currents are related to the junction voltages by equations similar to [\(B.1\)](#). There is an essential difference that can be explained for the case of one junction being forward biased while the other junction is zero or reverse biased. Specifically, suppose that the transistor is a pnp (the

other possibility being the npn) and the pn junction on the left (input junction) is forward biased and the pn junction on the right (output junction), is made to have zero bias by connecting a wire across this junction, as shown in [Fig. B.2](#).

Figure B.2. Diagrammatic semiconductor pnp transistor. The input junction on the left has forward bias voltage V_D , and the output junction on the right is shorted for zero bias. The input pn-junction diode current I_D couples with the output pn junction to flow into the output p region.



Even though the forward-biased pn junction on the left shares the n region with another pn junction (on the right), it behaves like a pn-junction diode such that the current – voltage relation for the junction still has the form of [\(B.1\)](#) and is

Equation B.2

$$I_D = I_S \left(e^{V_D/V_T} - 1 \right)$$

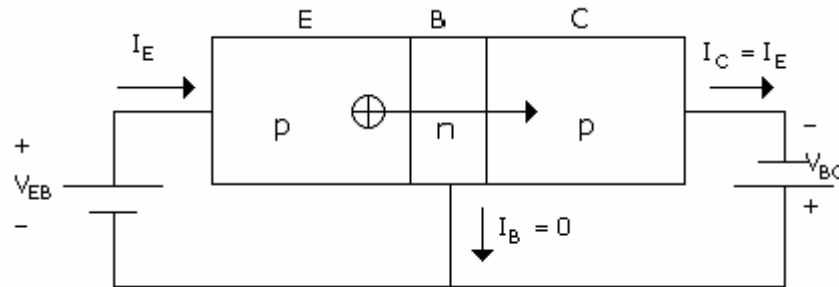
The saturation current is now designated I_S , since $I_S \neq I_{Sd}$. [If the width of the n region is very large, the holes injected into the n region will recombine with electrons in the n region, and the associated current will flow entirely out through the contact to the n region. In this limiting case, the current – voltage relation for the junction on the left reverts to [\(B.1\)](#). The presence of the p region on the right will play no role.]

In a transistor, the n-region width is made small and the statistical chance of an injected hole surviving the transit across this region, without recombining with an electron, is very high. Upon entering the p region on the right, the holes are accommodated by the exit of an equal number of holes moving out at the contact to this p region. (Actually, they are converted to electrons at the interface of the contact and semiconductor to be consistent with having only electron flow in the metal contact and device connecting metal.)

In an ideal transistor, this is the only current mechanism. The various other current components of the real transistor are discussed below. These include electron injection that would be expected from the n region into the p region of the forward-biased junction on the left (as in [Fig. B.2](#)). In practice, special fabrication techniques are employed to make this component of current very small.

In the application of a transistor in the active mode, as for example, in an analog amplifier stage, the output junction is reverse biased, as shown in [Fig. B.3](#). The junction configuration now represents a transistor; the regions are accordingly designated as emitter (E), base (B), and collector (C). Consistent with these assignments, the current into the transistor on the left is the emitter current, I_E , and the output current on the right is the collector current, I_C , and according to the mechanism described above for the ideal transistor, $I_C = I_E$.

Figure B.3. Diagrammatic semiconductor pnp transistor in active-mode operation. Regions are now designated emitter, base, and collector. The emitter – base junction is forward biased and the collector – base junction is reverse biased.



With forward bias (voltage) across the emitter – base junction and reverse bias across the collector – base junction, the emitter – base current equation, ([B.2](#)), is slightly altered, to become

Equation B.3

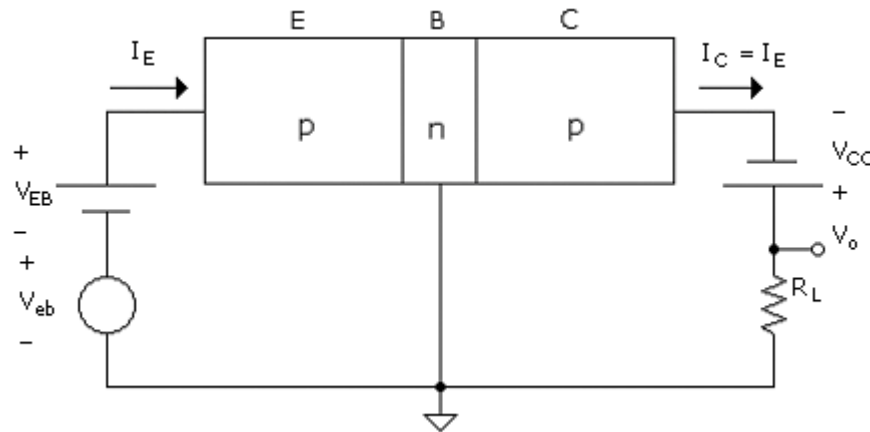
$$I_E = I_C = I_S e^{V_{EB}/V_T}$$

Variable subscripts have been changed to reflect the fact that the pnp structure is now specifically a transistor. (The parameter I_S is the same in SPICE for the diode and the transistor models. I_{Sd} was used above for this discussion alone to differentiate between the two.) For reverse bias of the collector – base junction, the -1 of ([B.2](#)) is eliminated.

In this simplified version of the transistor, the output current is independent of the collector – base voltage. The possibility of infinite voltage gain is thus provided, because the output circuit behaves like a pure current source; that is, it will force current into an external resistance, which can be large without limit. This conclusion remains subject to additional alterations for the real transistor. However, in principle, the transistor mechanism as discussed is applicable short of certain limitations.

An example of obtaining gain from a transistor in an amplifier circuit is shown in [Fig. B.4](#). A ground has been added at the n-region terminal to establish a reference. Therefore, the output voltage, V_O , is defined with respect to this reference. To the transistor circuit of [Fig. B.3](#), we have added an input signal V_{eb} and a load resistor, R_L . Note that since the input and output are referred to the base, the circuit is a common-base amplifier.

Figure B.4. Amplifier circuit incorporating a pnp transistor. The output voltage is with respect to ground at the n-region terminal. The base is common to the input and output and hence, this is the common-base amplifier configuration.



Suppose that initially, $V_{eb} = 0$ and V_{EB} is selected to give $I_C = 1 \text{ mA}$. This is, in an amplifier, the bias current. The output supply voltage is chosen to be $V_{CC} = 12 \text{ V}$. We select $R_L = 5 \text{ k}\Omega$ giving a bias output voltage $V_O = I_C R_L = 5 \text{ V}$. Note now that the collector-base bias voltage is $V_{BC} = V_{CC} - V_{RL} = 7 \text{ V}$ and the junction is reverse biased.

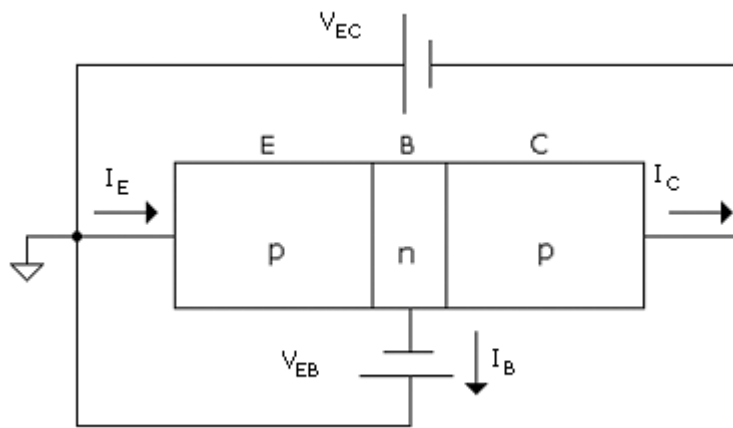
We now apply an input voltage (signal, ac, or incremental voltage) $V_{eb} = 18 \text{ mV}$. This increment added to V_{EB} will cause I_C to double (i.e., $I_C = 2 \text{ mA}$). The result is a new output voltage of $v_O(\text{sig}) = 10 \text{ V}$. Thus the incremental (or signal) output voltage is $\delta V_O = v_O(\text{sig}) - V_O = V_o = 5 \text{ V}$. Base – collector voltage is now $v_{BC} = V_{RL} = 2 \text{ V}$ and the junction remains in a reverse-bias state.

The incremental (ac or signal) ratio of output and input voltage is $V_o/V_{be} = 5\text{V}/18 \text{ mV} = 278$. Note that the gain is power-supply limited. Employment of a larger power supply permits the use of a larger R_L , which results in a higher gain. We will later show that the

gain is in fact V_{RL}/V_T based on an approximate linear representation between the input and output voltage. Note that in this case, the linear approximation is $V_{RL}/V_T = 192$, which indicates, for the example above, that the linear relation provides a fair estimate.

An alternative bias arrangement is shown in [Fig. B.5](#). The input terminal is now that of the n region and the p terminal is a common node to the input and the output. This is the common-emitter configuration. Note that the voltage applied to the input junction has not changed, and the output junction now has a bias voltage of $V_{EC} - V_{EB}$ and remains reverse biased as long as $V_{EC} > V_{EB}$.

Figure B.5. Common-emitter transistor amplifier configuration. The common terminal is now the p-emitter region and the input is at the base (n region). The output is between the common emitter and the collector. This diagram includes a possible I_B and thus $I_C < I_E$, as is discussed in [Unit B.3](#).



In electronic amplifiers, all three terminal configurations are possible: common base, common emitter, and common collector (emitter follower). The most frequently employed amplifier stage is based on the common-emitter mode, but the other two serve important roles in electronic amplifiers. In all three, the emitter – base voltage is the input control-terminal voltage. In the common-emitter and common-base, the output current is the collector current and in the common collector, it is the emitter current. The common-collector configuration is usually referred to as the emitter follower.

In the subunits that follow, the detailed SPICE relations that generally relate the branch currents to the terminal voltages are developed. This includes the addition of many aspects of the real transistor which were not included in the discussion above of the highly idealized transistor.

Initially, the forward-active mode is explored, and this is followed by a discussion of the reverse-active mode. The reverse-active mode is where the situation in [Fig. B.3](#) is reversed. Thus, V_{CB} is positive (base – collector junction forward biased), while V_{EB} is

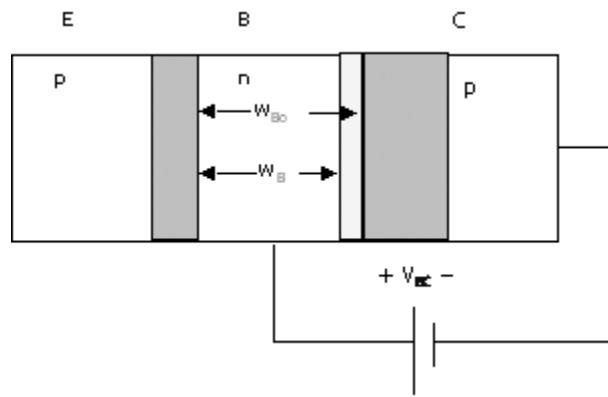
zero or negative. In the reverse-active mode, V_{CB} becomes the input voltage, and the output current is the emitter current, I_E .

Although the transistor is not operated in the reverse-active mode, the relationships developed for this case can be combined with those from the forward-active mode. The combination produces the general equations for relating currents and terminal voltages in transistors for all bias possibilities (i.e., where the transistor is biased out of the active mode).

B.2. Base-Width Dependence on Junction Voltage

In real transistors, the base width depends on the voltage applied across a pn junction. The effect is known as base-width modulation. A diagrammatic representation of the effect is illustrated in [Fig. B.6](#). Not shown in the diagrams in [Unit B.1](#) of the pnp structure are the depletion regions, which are shown shaded in [Fig. B.6](#). These are the transition regions that are depleted of free carriers, and in which the barrier is formed that impedes electron and hole flow to the p regions and n region, respectively. The base width (n region) is actually the width between the depletion regions. In the diagram of [Fig. B.6](#), we define w_{B0} for $V_{BC} = 0$ and w_B for $V_{BC} > 0$.

Figure B.6. Diagrammatic pnp structure showing the effect of base – collector voltage on base width.



The magnitude of saturation current, I_s , in [\(B.3\)](#) is specifically defined for $V_{BC} = 0$ or w_{B0} . In a circuit application, though, V_{BC} will in general be nonzero and the base width can vary as shown; in the example of [Fig. B.6](#), the base width is w_B for a given applied V_{BC} . To a reasonable approximation, the relationship between the effective saturation current with a nonzero V_{BC} can be accounted for with the form.

Equation B.4

$$I'_S = \frac{I_S}{1 - \frac{V_{BC}}{V_{AF}}} \approx I_S \left(1 + \frac{V_{BC}}{V_{AF}} \right)$$

where I'_S is the effective saturation current for a reverse-applied base – collector voltage. The approximate form assumes that $V_{BC} \ll V_{AF}$. The expression in the denominator, $1 - V_{BC}/V_{AF}$, comes from the fact that I'_S is approximately inversely proportional to the base width, and the base width is roughly

Equation B.5

$$w_B = w_{B0} \left(1 - \frac{V_{BC}}{V_{AF}} \right)$$

The parameter V_{AF} is called the Early voltage for forward-active operation. It has a counterpart for reverse-active operation, V_{AR} . The base-width dependence on junction voltage is included in the nonideal factors taken up in [Unit B.3](#).

B.3. BJT Base, Emitter, and Collector Currents in the Active Mode

The discussion of the transistor mechanism of [Unit B.1](#) is extended here to include base-current mechanisms, which exist in the real transistor, and the effect of base-width modulation as discussed in [Unit B.2](#). Initially, the case of active-mode operation is discussed, and this is followed by the general-bias case, which includes the possibility of both pn junctions becoming forward biased.

In the device model for the bipolar junction transistor, the parameter β_{DC} is

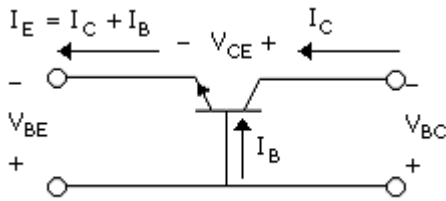
Equation B.6

$$\beta_{DC} = \frac{I_C}{I_B}$$

where I_B is the composite of all contributions to the base current. (The use of "dc" is consistent with SPICE.) This relates the output current, I_C , to the input current, I_B , for the transistor operated in the common-emitter terminal configuration.

In the following, the discussion is based on the npn transistor, as shown now with the schematic symbol in [Fig. B.7](#). The npn is chosen over the pnp, as it is substantially more basic to BJTs than the pnp. This probably has to do mostly with the fact that a common-emitter stage is consistent with a positive power supply (a holdover from the vacuum-tube days), that the npn is superior in terms of frequency response, and that it has consistently been the device of BJT digital switches including the TTL. The pnp was used in the discussion above on the fundamentals of transistor action, as it is somewhat more intuitively satisfying to have the current and particle flow (where the illustration is with the hole) in the same direction.

Figure B.7. BJT (nnp) in the common-base configuration, showing terminal voltages and branch currents.



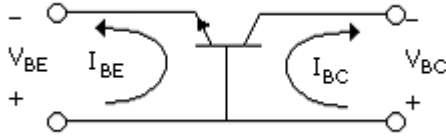
The assignment of polarities of voltages V_{BE} and V_{BC} is consistent with forward bias for both the emitter – base and collector – base junctions. V_{BC} will be negative in active-mode operation. The assignment of V_{CE} is standard, as it will be positive in active-mode operation. Current directions are assigned to correspond to the actual directions of the currents in the forward-active mode.

Base currents do not couple between junctions. Rather, as suggested in [Fig. B.8](#), a given base current is associated with a given pn junction. In the forward-active mode, a base-current component is added to the emitter current as indicated in [Fig. B.7](#). However, since the base – collector junction is reverse biased, the base-current contribution to the collector current is negligible and (B.3) still represents the total collector current. The collector-current relation with the base-width-modulation effect ([Unit B.2](#)) now included becomes

Equation B.7

$$I_C = I_S e^{V_{BE}/n_F V_T} \left(1 - \frac{V_{BC}}{V_{AF}} \right)$$

Figure B.8. Diodelike characteristics of the base currents. They are separately associated separately with their respective pn junctions. These currents do not couple across the base.



The equation has also been generalized to include the SPICE parameter n_F , the forward current emission coefficient (which is usually assumed to be 1).

The base current, for the forward-biased base – emitter junction, is made up of two terms; these are the ideal base current (because it has the same V_{BE} dependence as I_C) and the leakage base current. The leakage component is typically small enough to be neglected and often is, for simplicity, in practice.

In the absence of leakage base current, the base current is

Equation B.8

$$I_B = I_{SBE} \left(e^{V_{BE}/n_F V_T} - 1 \right)$$

where I_{SBE} is the base – emitter saturation current. In the active mode, V_{BE} is positive, and typically, $V_{BE} \approx 0.6V$. The exponential term is, for this case, $\exp(V_{BE}/n_F V_T) \approx 10^{10}$ such that the -1 is quite negligible. The -1 is necessary for the current to go to zero for $V_{BE} = 0$. This current is due to (npn) injection and recombination of minority-carrier electrons in the base and injection and recombination of minority-carrier holes in the emitter. Base current could also be connected to injection of holes into the n-type emitter, which recombine at the emitter-contact metal – semiconductor interface. In any event, base – emitter junction base current is predominantly from injection into the emitter, as opposed to injection into the base.

For this case of no leakage current and for $V_{BC} = 0$, the forward-active current gain (or current ratio) is defined as $\beta_{DC} \equiv \beta_F$. Using (B.6), (B.7), and (B.8) (with -1 neglected), this is

Equation B.9

$$\beta_F = \frac{I_S e^{V_{BE}/n_F V_T}}{I_{SBE} e^{V_{BE}/n_F V_T}} = \frac{I_S}{I_{SBE}}$$

and $I_{SBE} = I_S/\beta_F$. Consequently, the ideal base current is normally written as

Equation B.10

$$I_B = \frac{I_S}{\beta_F} e^{V_{BE}/n_F V_T}$$

Note that the base current does not have V_{BC} dependence as exhibited by the collector current. This is demonstrated to be valid in [Project B](#).

The parameter β_F is the SPICE transistor-model β . Implicit in the assignment of I_S (I_S) and β_F (β_F) in the SPICE device model is the assignment of the ideal base-current saturation $I_{SBE} = I_S/\beta_F$. That is, there is no I_{SBE} (SPICE parameter) in the model. If the general active-region relation for I_C , ([B.7](#)), is now used in the β_{DC} definition, we obtain the β_{DC} relation for nonzero V_{BC} , namely,

Equation B.11

$$\beta_{DC} = \frac{I_C}{I_B} = \beta_F \left(1 - \frac{V_{BC}}{V_{AF}} \right)$$

where β_F is a constant (SPICE β_F) and $\beta_{DC} = \beta_F$ for $V_{BC} = 0$.

In the real transistor, there is, in general, a component of leakage current. It is associated with recombination of holes and electrons in the depletion region of the base – emitter junction. In SPICE, the leakage-current component is characterized with parameters n_E (base – emitter leakage coefficient) and I_{SE} (base – emitter leakage saturation current). When added to the ideal component, the total base current is

Equation B.12

$$g_m = \frac{\partial i_c}{\partial V_{BE}} = \frac{\partial \left[I_S e^{V_{BE}/V_T} \left(1 + \frac{V_{CE} - V_{BE}}{V_{AF}} \right) \right]}{\partial V_{BE}} \approx \frac{I_C}{V_T}$$

(These are both positive into the transistor for the npn.) The parameter n_E is ideally 2 (according to early, simplified theories), but in practice is typically about 1.5. When transistors are operated in or above the midrange of their rated current capacity, the leakage component becomes negligible.

For a small power transistor, the midrange starts at about 100 mA, such that the leakage term would be very significant at, for example, 1 mA. In the project on the transistor, which is designed to determine the SPICE parameters discussed here, a low-current range of less than about 1 mA is selected. The low current also avoids inadvertent rise in the temperature of the device during evaluation.

Similar equations that apply to the reverse-active mode can be obtained by direct substitution of the equivalent variables. The output current and input voltage, are, respectively, I_E and V_{BC} , while, for the reverse-active mode, $V_{BE} = 0$ or negative. Based on the equivalent substitution

Equation B.13

$$I_E = -I_S e^{V_{BC}/n_R V_T} \left(1 - \frac{V_{BE}}{V_{AR}} \right)$$

The minus sign comes from having assigned the current direction of the emitter current out of the transistor ([Fig. B.7](#)). The parameter I_S is common to ([B.7](#)) and ([B.13](#)). Equations ([B.7](#)) and ([B.13](#)) are combined to obtain the general case of V_{BE} and V_{BC} both positive (forward bias) in a following unit.

The reverse-operation base current is [the reverse-operation equivalent of ([B.12](#))]

Equation B.14

$$I_B = \frac{I_S}{\beta_R} \left(e^{V_{BC}/n_R V_T} - 1 \right) + I_{SC} \left(e^{V_{BC}/n_C V_T} - 1 \right) \approx \frac{I_S}{\beta_R} e^{V_{BC}/n_R V_T} + I_{SC} e^{V_{BC}/n_C V_T}$$

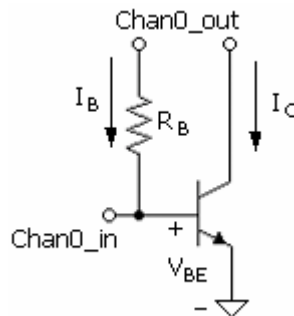
where β_R is the dc current ratio (reverse-active mode) for the case of no leakage component of base current, and is given by $\beta_R = I_E/I_B$. In practice, the reverse ideal current gain is $0.1 < \beta_R < 10$, and therefore, the reverse-operated transistor is not a useful configuration in, for example, analog amplifiers. However, the reverse-active mode equations are essential, as is shown below, for developing the general equations for the forward mode. The general form includes the possibility that the transistor will be out of the active mode.

Although V_{AR} (reverse Early voltage), I_{SC} (reverse leakage saturation current) and n_c (base – collector leakage emission coefficient) are officially SPICE parameters, we will not be concerned with these as they would rarely be a factor in the study of analog circuits.

B.4. Diode-Connected Transistor Circuits for Measuring Base and Collector Current

The circuit shown in [Fig. B.9](#) is used in the parameter-determination project for measuring the base current with the collector open. The collector is tied to the output channel voltage, which holds the collector – base junction in reverse bias, and therefore in the active mode. The configuration is effectively a diode (open collector) with two terminals, base and emitter, and the diode current is the base current. The base current is that of the active-mode transistor, because the collector – base junction is maintained in a reverse-bias state. Note that according to [\(B.12\)](#), base-current magnitude does not otherwise depend on the value of the base – collector voltage. This is shown to be valid in the project. (The base current is much different when the collector terminal is tied to ground and the base – collector junction is forward biased.)

Figure B.9. Circuit for measuring base current in the open-collector diode-connected circuit.



The measurement circuit includes a current sensing resistor, R_B . The relation between I_B and V_{BE} for this terminal configuration is [\(B.12\)](#), repeated here

$$I_B = \frac{I_S}{\beta_F} e^{V_{BE}/n_F V_T} + I_{SE} e^{V_{BE}/n_E V_T}$$

As noted above, the base current of (B.12) exhibits no base-collector voltage dependence other than the requirement that the transistor be in the active mode.

This is assured in this circuit since V_{BC} is

Equation B.15

$$V_{BC} = V_B - V_C = V_{CC} - I_B R_B - V_{CC} = -I_B R_B$$

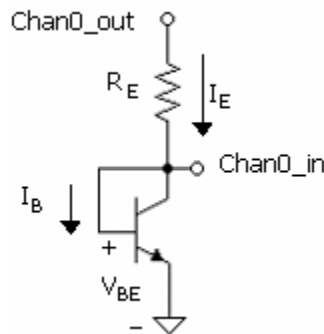
where $V_{CC} \equiv \text{Chan0_out}$.

In the alternative diode circuit of Fig. B.10, the base and collector are connected such that $V_{BC} = 0$ and the equation for the collector current is (B.7) with $V_{BC} = 0$, or, simply,

Equation B.16

$$I_C = I_S e^{V_{BE}/n_F V_T}$$

Figure B.10. Circuit for measuring emitter current. In this circuit, $V_{BC} = 0$ and (B.16) applies.



The measured current in the circuit, that is, sensed by the sensing resistor, R_E , is the emitter current, I_E . This is the sum of (B.12) and (B.16), which is

Equation B.17

$$I_E = I_C + I_B = I_S e^{V_{BE}/n_F V_T} + \frac{I_S}{\beta_F} e^{V_{BE}/n_F V_T} + I_{SE} e^{V_{BE}/n_E V_T}$$

We note that (B.12) remains valid for $V_{BC} = 0$. Current measurements from the two diode circuits provide, therefore, I_C , from $I_C = I_E - I_B$. In the parameter measurement project, it is verified that the base current is the same for both measurement circuits. This is a means by which the independence of base current on reverse voltage is demonstrated.

In the BJT parameter determination project, parameter extraction from the exponential relations is obtained from plots of current – voltage characteristics. The form of the log of the current versus voltage provides information on the two parameters of the exponential from a straight-line curve fit. For example combining the currents from the two circuits gives, as noted, $I_C = I_E - I_B$ and [from (B.16)]

Equation B.18

$$\log(I_C) = \log(I_S) + V_{BE} / \ln(10) n_F V_T$$

The zero voltage intercept thus reveals I_S and the slope will show that $n_F \approx 1$.

A simple exponential represents the base current only approximately, as it is a combination of two exponentials; that is,

Equation B.19

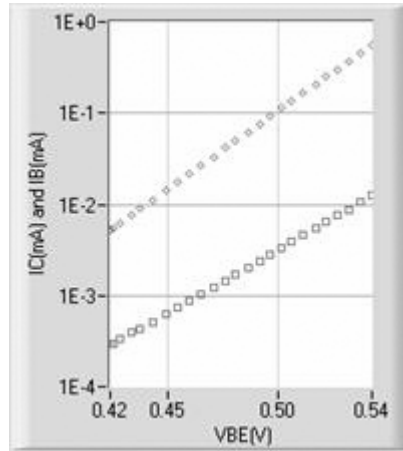
$$a_v = \frac{V_{CE}}{V_S} = \frac{V_{BE}}{V_S} \times \frac{V_{CE}}{V_{BE}} = - \frac{R_b}{R_b + R_S} g_m R_C$$

In the measurement of the base current, using the circuit of Fig. B.9, the measured semilog plot will appear to be a straight line. This would suggest that the current – voltage relation is a simple exponential, as in the case of the collector current. However, parameters n_{Eprime} and $I_{SEprime}$ vary, depending on the given narrow range of base – emitter voltage of the measurement. Note that since $n_F = 1$ and $n_E > 1$, n_{Eprime} will be in the range $1 < n_{Eprime} < n_E$.

Sample measured results are shown in Fig. B.11, where the current is plotted on a log scale. For this plot, I_C has been obtained from $I_C = I_E - I_B$. Both plots are straight lines,

indicating the exponential relationship between current and voltage for the two cases. Careful scrutiny of the lower curve (I_B) should show a slight upward curvature since it is a mixture of two exponential terms, as noted. The steeper slope for emitter current is a result of the fact that $n_F = 1$ and $n_{Eprime} > 1$.

Figure B.11. LabVIEW plot of measured currents (mA) versus base – emitter voltage (V). The ratio of the two is the β_{DC} of the transistor.

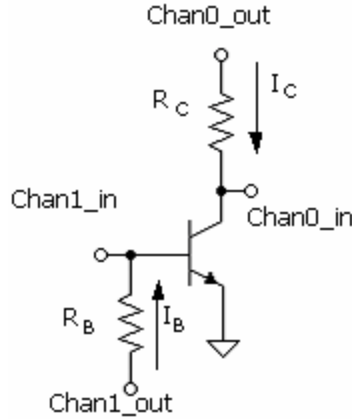


B.5. Output Characteristics of BJT in the Common-Emitter Mode

The common-emitter terminal configuration can be considered the fundamental building block of BJT analog amplifier circuits. The common base and common collector are applied to some extent in more special-purpose roles, such as for high output resistance (common base) and low output resistance (common collector). Therefore, a study of the transistor in the common-emitter mode is basic to a study of analog circuits.

The output characteristic of the common-emitter transistor is defined as the output current (I_C) as a function of the output terminal voltage (V_{CE}) for I_B (or V_{BE}) held constant. It is very important to understand in the design of both analog and digital circuits. The output characteristic is experimentally explored in the project on parameter determination. The measurement circuit for obtaining the output characteristic is shown in [Fig. B.12](#). This is also the circuit for measuring β_{DC} versus I_C , which is discussed below.

Figure B.12. Circuit for measuring transistor output characteristic and β_{DC} versus I_C .



The equations presented in [Unit B.2](#) are in terms of terminal voltages V_{BE} and V_{BC} . The common-emitter configuration has input voltage V_{BE} but output voltage V_{CE} . Therefore, it is preferable to eliminate V_{BC} in the equations in favor of V_{CE} , using $V_{BC} = V_{BE} - V_{CE}$.

When plotting the output characteristic, for example, from $0 < V_{CE} < 5 \text{ V}$, the range of V_{BC} is $V_{BE} < V_{BC} < -(5 - V_{BE})$. Since $V_{BE} \approx 0.5 \text{ V}$, V_{BC} covers the full range from strongly forward biased (out of the forward-active mode) to clearly reverse biased (forward-active mode). This requires that the I_C [(B.7)] and I_B [(B.12)] equations be modified to include all possibilities.

To obtain an equation for I_C under general biasing conditions, we start with the fundamental equation of transistor action of the BJT. This is

Equation B.20

$$I_C = I_S \left(e^{V_{BE}/V_T} - e^{V_{BC}/V_T} \right) = I_E$$

This is a composite of (B.7) and (B.13) in which the base-modulation effects are neglected. Both of these equations are for I_C and I_E in the ideal transistor. (In this unit and beyond, we assume that $n_F = 1$.) Sometimes referred to as the linking-current equation, this is symmetrical in I_C and I_E and neglects all components of base current and dependence of the base width on V_{BC} and V_{BE} .

To add an additional degree of applicability to the real transistor, base-width modulation must be added. We are interested in a relation between currents and voltages for the transistor in the forward-active mode. In this case, V_{BE} is fixed at $0.4 < V_{BE} < 0.6 \text{ V}$. On this basis, base-width modulation of the emitter – base junction can be neglected. On the other hand, the relation must apply for a V_{BC} range, which includes relatively large

negative values. Therefore, the effect of base-width modulation of the collector – base junction must be included in a modification of (B.20). If base-width modulation due to the dependence of base width on V_{BC} is added and the substitution $V_{BC} = V_{BE} - V_{CE}$ is now made, (B.20) becomes:

Equation B.21

$$I_C = I_S \left(e^{V_{BE}/V_T} - e^{(V_{BE}-V_{CE})/V_T} \right) \left(1 - \frac{V_{BE} - V_{CE}}{V_{AF}} \right)$$

The general equation must include any significant base current that contributes to collector current, that is, for when the base – collector junction becomes forward biased. For this we use (B.14) with base leakage current neglected. (Leakage current is neglected throughout this unit.) This is

Equation B.22

$$I_B = \frac{I_S}{\beta_R} \left(e^{(V_{BE}-V_{CE})/V_T} - 1 \right)$$

The -1 term has been retained in order for the equation to apply at all possible polarities and values for $V_{BC} = V_{BE} - V_{CE}$, including zero where this contribution of collector current is zero.

We note that this base current is in a loop (Fig. B.8) through the base – collector junction and is positive out of the collector terminal; I_C of (B.21) is positive into the collector terminal as in Fig. B.7. Thus, the component of base current from (B.22) subtracts from the collector current to give

Equation B.23

$$|A_v| = \frac{V_{ce}}{V_s} = \frac{I_c}{I_s} \frac{R_c}{R_s + R_b} \approx \frac{I_c}{I_b} \frac{R_c}{R_s + r_n} \approx \beta_{ac} \frac{R_c}{R_s + r_n} \approx \beta_{ac} \frac{R_c}{R_s}$$

Again, for simplicity, this equation neglects the leakage current associated with the collector – base junction [I_{SC} term (B.14)], which is only a fair approximation for the level of I_C at which we will obtain an output characteristic in the parameter extraction project on the BJT.

A plot of this equation for I_C as a function of V_{CE} can be obtained for constant V_{BE} or constant I_B . For the latter, V_{BE} is also a variable such that the plot requires, in addition, a solution for V_{BE} as a function of V_{CE} for constant I_B . In the project on the output characteristic, the measured data and a SPICE solution are compared during the measurement. The input circuit provides a more or less constant I_B . However, the SPICE solution, which is computed by LabVIEW, is exact. It allows for a variable I_B and associated variable V_{BE} . The SPICE solution formulation used by LabVIEW is outlined below in [Unit B.7](#). The SPICE solution is also explored in the project Mathcad file.

After measuring the output characteristic, the segment of the resulting data array from the active region ($V_{CE} > V_{BE}$) is extracted. From these data, a straight-line curve fit is obtained by LabVIEW, which produces a number for the slope. From [\(B.23\)](#), we obtain an equation for the active region from letting $V_{CE} > V_{BE}$, with the result

Equation B.24

$$I_{C_{act}} = I_S e^{V_{BE}/V_T} \left(1 - \frac{V_{BE} - V_{CE}}{V_{AF}} \right)$$

The slope for the active-region equation is $\text{slope} = dI_{C_{act}}/dV_{CE} = I_C/V_{AF}$. Thus, V_{AF} can be calculated from $V_{AF} = I_C/\text{slope}$, where I_C is the value taken from the data array for $V_{CE} = V_{BE}$.

For the special case of $V_{CE} = V_{BE}$, one can obtain a value for I_S from the relation

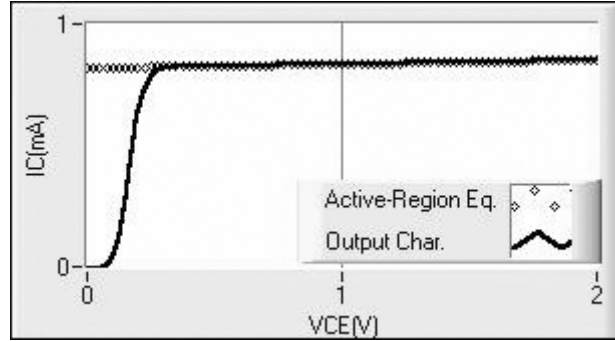
Equation B.25

$$I_S = I_C e^{\frac{-V_{BE}}{V_T}}$$

After V_{AF} and I_S have been obtained, an iteration on β_R will produce a final curve fit to the output characteristic, thereby providing a number for β_R . In this manner, all three parameters of this unit are obtained: I_S , β_R , and V_{AF} .

An example of a SPICE output characteristic (calculated in Mathcad), which uses the measured transistor-model parameters, is shown in [Fig. B.13](#). Also shown is the calculated active-region plot.

Figure B.13. Mathcad calculated plot of the complete output characteristic, (B.23), and the active-region segment, (B.24). The saturation region [(B.34)] is graphically revealed where the plots separate.



The low-voltage region where $I_C < I_{C\text{sat}}$ is called the saturation region. The voltage and current in this region are $V_{CE\text{sat}}$ and $I_{C\text{sat}}$. In the derivation of the general equation for I_C , (B.23), the leakage current of the base – collector junction was neglected. If this is not valid, the reverse β_R , which is determined through curve fitting, is somewhat artificial. For example, suppose that the leakage current totally dominates the collector – base current. For this case, the general I_C equation is

Equation B.26

$$V_{be} = V_T \ln \left(1 \pm \frac{I_c}{I_r} \right) \rightarrow \pm V_T \left| \frac{I_c}{I_r} \right| \left| \frac{I_c}{I_r} \right| < 1$$

where a new definition of a sort of β_R is defined in the relation $I_{SC} = I_S / \beta_{R\text{leak}}$. A curve fit to the measured output characteristic would give $\beta_{R\text{leak}}$. However, because $n_C > 1$, the result depends on the level of collector current at which the measurement is made. If, in a given measurement, the $\beta_{R\text{leak}}$ is interpreted as β_R , it would function as β_R in SPICE (when assigning BR to the model), but it would only be valid in a simulation for collector currents close to that of the measurement.

B.6. SPICE Solution for I_C versus V_{CE} of the Measurement Circuit

All components of base current are those from (B.12) (forward mode) and (B.14) (reverse mode). These are summed together to obtain the following expression for the general bias case:

Equation B.27

$$a_v = \frac{V_o}{V_{be}} = \frac{-I_c R_c}{V_T \ln \left(1 \pm \left| \frac{I_c}{I_c} \right| \right)} = -g_m R_c \frac{\left| \frac{I_c}{I_c} \right|}{\ln \left(1 \pm \left| \frac{I_c}{I_c} \right| \right)}$$

In the measurement circuit of the project on the output characteristic of the BJT, the circuit will provide a current source at the input and I_B (to a good approximation) will be a constant. However, in the event that a precision solution for I_B and V_{BE} is required, it can be obtained by equating (B.27) to I_B from the input circuit equation (Fig. B.12), which is

Equation B.28

$$I_B = (V_{BB} - V_{BE}) / R_B$$

In the Mathcad project file, a solution for V_{BE} as a function of V_{CE} is obtained (using a root finder) for a given V_{BB} and R_B . The result for V_{BE} (at a given V_{CE}) is used in (B.23) for a solution for I_C , and hence the output characteristic is obtained. For this, the leakage components of base current are neglected.

In the LabVIEW output characteristic measurement project, a SPICE solution is obtained along with the measurement. The solution is obtained with LabVIEW using an iterative solution. The equations are (B.28) and (B.27) (without the leakage terms) solved for V_{BE} . This is

Equation B.29

$$V_{BE} = V_T \ln \left(\frac{\frac{\beta_F I_B}{I_S}}{1 + \frac{\beta_F}{\beta_R} e^{-V_{CE}/V_T}} \right)$$

The iterative method consists of guessing an initial V_{BE} and solving for I_B from (B.28). This is used in (B.29), with the given V_{CE} , to obtain a better value for V_{BE} . The new V_{BE} is then put back into the circuit equation, (B.28), and so on, until V_{BE} stops changing significantly. This V_{BE} is then used in (B.23) for a solution for I_C at the specified V_{CE} . For the complete output characteristic, the solution is repeated in increments of V_{CE} for a

range from zero up to a specified maximum. This is accomplished in LabVIEW with a Formula Node in a While Loop. The analytical formulation is explored in the project Mathcad file.

From (B.29), we note that over the full range of $0 < V_{CE} < 5 \text{ V}$, for example, the limits of V_{BE} are for $V_{CE} = 0$,

Equation B.30

$$V_{BE(lo)} \approx V_T \ln \left(\frac{\beta_R I_B}{I_S} \right)$$

while for large V_{CE} ,

Equation B.31

$$V_{BE(hi)} = V_T \ln \left(\frac{\beta_F I_B}{I_S} \right)$$

The case of (B.30) uses $\beta_F \gg \beta_R$. For example, for $\beta_F/\beta_R = 100$, the difference $V_{BE(hi)} - V_{BE(lo)} \approx V_T \ln(\beta_F/\beta_R)$, which is about 120 mV. In the base circuit equation (B.28), this change would be minor compared, for example, with a base bias voltage, V_{BB} , of 5 to 10 V. Thus, base current is close to a constant over the full range of V_{CE} , based on $\text{Chan1_out} \equiv V_{BB} = 5 \text{ V}$ or greater, as dictated by the design goal. That is, V_{BB} is much greater than the change in V_{BE} as V_{CE} moves from (B.30) to (B.31).

We note that, consistent with the limit of (B.31), the solution for I_B is from

Equation B.32

$$V_{BB} = V_T \ln \left(\frac{\beta_F I_B}{I_S} \right) + I_B R_B$$

Therefore, I_B and V_{BE} are constant and I_C only varies with V_{CE} due to the V_{CE} dependence in (B.24). The value of the limit V_{CE} is quantified in the next unit.

B.7. Collector-Emitter Voltage and Collector Current in the Saturation Region

An equation for V_{CEsat} (V_{CE} in the saturation region) can be obtained using the equations for I_B , (B.27) (with leakage components neglected), and I_C , (B.23), to eliminate V_{BE} . The approximate equation for I_B is again

$$I_B = \frac{I_S}{\beta_F} e^{V_{BE}/V_T} + \frac{I_S}{\beta_R} e^{(V_{BE}-V_{CE})/V_T}$$

The equation for I_C , (B.23), which neglects the V_{AF} term (well justified at low voltage of the saturation region), and the -1 is

Equation B.33

$$I_C = I_S \left[\left(e^{V_{BE}/V_T} - e^{(V_{BE}-V_{CE})/V_T} \right) - \frac{1}{\beta_R} e^{(V_{BE}-V_{CE})/V_T} \right]$$

Eliminating V_{BE} between the approximate form of (B.27) and (B.33) gives

Equation B.34

$$V_{CEsat} = V_T \ln \left(\frac{1 + \frac{1}{\beta_R} + \frac{\beta_{forced}}{\beta_R}}{1 - \frac{\beta_{forced}}{\beta_F}} \right)$$

where

Equation B.35

$$\beta_{forced} = \frac{I_{Csat}}{I_B} < \beta_F$$

The subscript comes from "beta forced," the conventional way to define I_C/I_B in the saturation region. Note that by definition, $\beta_{\text{forced}} < \beta_F$, as $I_C < I_{C\text{act}}$ defines the saturation region.

For example, we compute $V_{CE\text{sat}}$ at $I_{C\text{sat}} = I_C/2$. Assume that $\beta_F = 100$ and $\beta_R = 1$ such that $\beta_{\text{forced}} = \beta_F/2 = 50$. For this case, $V_{CE\text{sat}} = 120$ mV. Note that this is the midrange for the validity of (B.30).

It should be noted that $V_{CE\text{sat}}$ is not defined for $\beta_{\text{forced}} \rightarrow \beta_F$. This is because of the neglect of the -1 from the equations. Technically, the maximum $V_{CE\text{sat}}$ is $V_{CE\text{sat}} \approx V_{BE}$, as this is the maximum V_{CE} at which the base – collector junction is forward biased, the condition for the onset of the saturation region. At V_{CE} values very near V_{BE} , the -1 terms in the current equations are not mathematically negligible. On practical grounds, though, (B.34) is valid for when the output characteristic is clearly in the saturation region.

A value for β_R can be obtained, using (B.34), from one set of data points, $I_{C\text{sat}}$ and $V_{CE\text{sat}}$ from a measurement, with β_F known. If this proved to vary with the $I_{C\text{sat}}$ chosen for the calculation, it would suggest that the discussion at the end of Unit B.5 applies. That is, the leakage current is not negligible and the effective β_R , $\beta_{R\text{leak}}$, is a variable throughout the saturation region.

B.8. SPICE BJT β_{DC} as a Function of Collector Current

SPICE calculates a dc beta, β_{DC} , when ISE and NE are included in the transistor model. The equation for the calculation uses $\beta_{DC} = I_C/I_B$, where I_B (forward-active mode) includes the ideal and leakage terms, (B.12),

$$I_B = \frac{I_S}{\beta_F} e^{V_{BE}/n_F V_T} + I_{SE} e^{V_{BE}/n_E V_T}$$

and I_C is the collector current for the forward-active mode, (B.7),

$$I_C = I_S e^{\frac{V_{BE}}{V_T}} \left(1 - \frac{V_{BC}}{V_{AF}} \right)$$

Using (B.7) and (B.12) in the definition of β_{DC} , we obtain

Equation B.36

$$\beta_{DC} = \frac{I_C}{I_B} = \frac{I_S e^{V_{BE}/V_T} \left(1 - \frac{V_{BC}}{V_{AF}}\right)}{\frac{I_S}{\beta_F} e^{V_{BE}/V_T} + I_{SE} e^{V_{BE}/n_E V_T}}$$

which is

Equation B.37

$$\beta_{DC} = \frac{\beta_F \left(1 - \frac{V_{BC}}{V_{AF}}\right)}{1 + \frac{\beta_F I_{SE}}{I_S} e^{\frac{V_{BE}}{V_T} \left(\frac{1}{n_E} - 1\right)}}$$

The V_{BE} variable can be eliminated with

Equation B.38

$$e^{V_{BE}/V_T} = \frac{I_{CO}}{I_S}$$

giving

Equation B.39

$$\beta_{DC} = \frac{\beta_F \left(1 - \frac{V_{BC}}{V_{AF}}\right)}{1 + X \left(\frac{I_{CO}}{I_S}\right) \left(\frac{1}{n_E} - 1\right)}$$

where

Equation B.40

$$I_{C0} = \frac{I_C}{1 - \frac{V_{BC}}{V_{AF}}}$$

and

Equation B.41

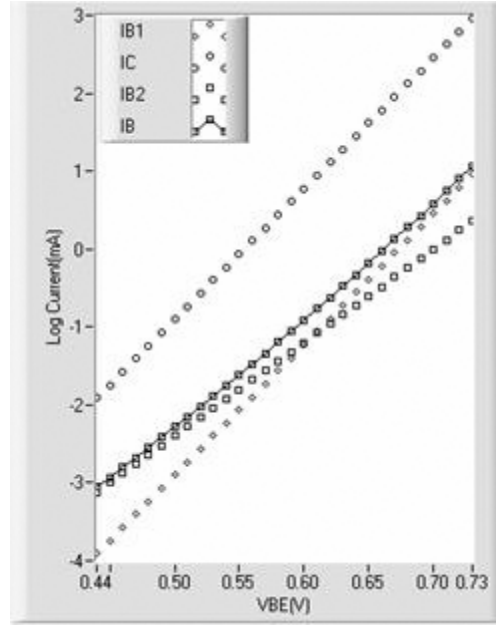
$$X = \frac{\beta_F I_{SE}}{I_S}$$

Note that for a given transistor, X is just a collection of parameters and the final result is a relatively simple function of I_C if the V_{AF} effect is neglected such that $I_{C0} \approx I_C$. This is a very good approximation for the project transistors in which V_{AF} is about 200 V and the maximum $|V_{BC}|$ is 3 or 4 V. In the project on the investigation of the β_{DC} dependence on I_C , β_{DC} is specifically measured for $V_{BC} = 0$ (as set automatically by LabVIEW) such that the V_{AF} effect is eliminated.

The form of (B.39) indicates that β_{DC} is an increasing function of collector current, I_C . The high-current asymptote is β_F . The simple SPICE model treats β_{DC} as a constant β_F , which therefore neglects leakage current. When parameters ISE and NE are included in the transistor model, SPICE uses (B.39) to compute β_{DC} . Note that the circuit solution of Unit B.6 is substantially more complicated if (B.39) is included to account for the variable nature of β_{DC} . The project Mathcad file performs this SPICE solution.

Mathcad-generated plots of collector current and the components of base current (mA) are shown in Fig. B.14. $I_{B1}(V_{BE})$ is the component with the same exponent as I_C (ideal transistor base current) and $I_{B2}(V_{BE})$ is the leakage component. That is [(B.12)]

Figure B.14. LabVIEW computed plots of $I_C(\text{mA})$ and $I_B(\text{mA})$ as a function of $V_{BE}(\text{V})$, including components I_{B1} (ideal) and I_{B2} (leakage).



$$I_B = \frac{I_S}{\beta_F} e^{V_{BE}/n_F V_T} + I_{SE} e^{V_{BE}/n_E V_T} = I_{B1}(V_{BE}) + I_{B2}(V_{BE})$$

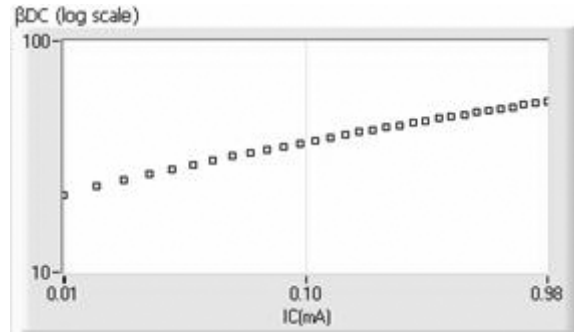
The total $I_B(V_{BE})$ is seen to develop at high currents into a plot that is parallel to I_C .

The plot was made with numbers that are representative of the transistor of the project in which the parameters are measured. The diode-connected transistor measurements are made for the range $0.01 < I_C < 1$ mA ($-2 < \log I_C < 1$). It is apparent from the plots that the base current for these measurements is made up of a mixture of the two components, or that the base current is in the transition region between leakage-current domination and ideal-current domination.

The pure ideal base current does not develop until $I_C > 100$ mA. This mode cannot be observed experimentally (even if means to avoid heating are implemented, for example, using pulse measurements). Certain high-level effects, which have not been included in the discussion, would come into play at this level, rendering the theories presented here as marginally valid.

An experimental plot of $I_C/I_B = \beta_{DC}$ as a function of I_C is shown in [Fig. B.15](#). A curve fit to the data yields n_E , I_{SE} , and $\beta_F(104)$. We note that the maximum $\beta_{DC} \ll \beta_F$.

Figure B.15. LabVIEW measurement of ratio $I_C/I_B = \beta_{DC}$ as a function of I_C .



B.9. Signal or Incremental Common-Emitter Current Gain

Since β_{DC}/I_B is a variable function of I_C , the incremental β_{ac} is, in general, different from the dc β_{DC} . (The incremental β_{ac} is often referred to as β_o , but β_{ac} is consistent with SPICE.) To get an approximate value, we could calculate β_{ac} from

Equation B.42

$$\beta_{ac} = \frac{I_c}{I_b} = \frac{I_{C2} - I_{C1}}{I_{B2} - I_{B1}} = \frac{I_{C2} - I_{C1}}{\frac{I_{C2}}{\beta_{DC2}} - \frac{I_{C1}}{\beta_{DC1}}}$$

with $I_{C1} = I_{C2}$ and where (B.39) is used to obtain β_{DC} at the two currents. The definition of β_{ac} is from the limiting case of $I_{C2} \rightarrow I_{C1}$, that is,

Equation B.43

$$\beta_{ac} = \frac{dI_C}{d\left(\frac{I_C}{\beta_{DC}}\right)}$$

Performing this operation with the use of (B.39) leads to

Equation B.44

$$\beta_{ac} = \frac{\beta_F \left(1 - \frac{V_{BC}}{V_{AF}}\right)}{1 + \frac{\chi}{n_E} \left(\frac{I_{C0}}{I_S}\right)^{\left(\frac{1}{n_E} - 1\right)}}$$

The result indicates that $\beta_{ac} > \beta_{DC}$ ($n_E > 1$) and that the two converge in the limit for high I_C . This is the expression used in SPICE for the incremental β_{ac} , and it is used by SPICE when NE and ISE are included in the model. Otherwise, SPICE uses $\beta_{ac} = \beta_{DC} = \beta_F$. (If IS and BF are not specified in the model, SPICE will use default values, typically, BF = 100 and IS = 10^{-16} A. For the project transistors, IS is about 10^{-13} A.)

B.10. Summary of Equations

$\beta_{DC} = \frac{I_C}{I_B}$	Common-emitter dc current gain.
$I_C = I_S e^{V_{BE}/n_F V_T} \left(1 - \frac{V_{BC}}{V_{AF}} \right)$	Forward active-mode output characteristic relation.
$I_B = \frac{I_S}{\beta_F} \left(e^{V_{BE}/n_F V_T} - 1 \right) + I_{SE} \left(e^{V_{BE}/n_E V_T} - 1 \right)$	Forward base current.
$I_B = \frac{I_S}{\beta_R} \left(e^{V_{BC}/n_R V_T} - 1 \right) + I_{SC} \left(e^{V_{BC}/n_C V_T} - 1 \right)$	Reverse base current.
$I_C = I_S \left[\left(e^{V_{BE}/V_T} - e^{(V_{BE}-V_{CE})/V_T} \right) \left(1 - \frac{V_{BE}-V_{CE}}{V_{AF}} \right) - \frac{1}{\beta_R} \left(e^{(V_{BE}-V_{CE})/V_T} - 1 \right) \right]$	Output characteristic relation.
$V_{CEsat} = V_T \ln \left(\frac{1 + \frac{1}{\beta_R} + \frac{\beta_{Forced}}{\beta_R}}{1 - \frac{\beta_{Forced}}{\beta_F}} \right)$	Collector-emitter voltage in saturation region.
$\beta_{DC} = \frac{\beta_F \left(1 - \frac{V_{BC}}{V_{AF}} \right)}{1 + X \left(\frac{I_{Co}}{I_S} \right)^{\left(\frac{1}{n_E} - 1 \right)}}$	Collector-current dependence of dc common-emitter current gain.
$\beta_{ac} = \frac{\beta_F \left(1 - \frac{V_{BC}}{V_{AF}} \right)}{1 + \frac{X}{n_E} \left(\frac{I_{Co}}{I_S} \right)^{\left(\frac{1}{n_E} - 1 \right)}}$	Collector-current dependence of incremental (ac) common-emitter current gain.

B.11. Exercises and Projects

Project Mathcad Files	ExerciseB.mcd - ProjectB1.mcd - ProjectB.2.mcd - ProjectB3.mcd
Laboratory Project B	Characterization of the Bipolar Junction Transistor for Circuit Simulation
PB.3	Diode-Connected Transistor Measurements
PB.4	Measurement of β_{DC} versus the Collector Current
PB.5	BJT Output Characteristic Measurement
PB.6	Simulation of the Output Characteristic Measurement

Unit C. Common-Emitter Amplifier Stage

Analog amplifier stages (BJT) generally comprise three possible terminal configurations: common emitter, common base, and common collector (emitter follower). As the names suggest, in each case, the input and output are referred to the common terminal. The MOSFET equivalents are common source, common gate, and common drain (source follower), respectively.

As the most fundamental of transistor amplifier building blocks, the common-emitter stage is the logical configuration to begin a study of signal amplifiers. The common-emitter considered initially in this unit is obtainable with a simple extension of the dc measurement circuit from [Unit B \(Fig. B.12\)](#). Based on this circuit various aspects germane to signal amplifiers in general are discussed. In [Unit C.7](#), the common-emitter stage with active load is explored. Both types of amplifiers, with resistive and active loads, are investigated extensively in projects.

The emitter-follower stage is discussed in [Unit C.9](#) along with a general treatment of the various effects on the common-emitter stage with an emitter-branch resistor. An entire unit ([Unit C.7](#)) is devoted to the source-follower stage, the MOSFET equivalent of the emitter-follower stage. The common-gate stage, the MOSFET equivalent of the BJT common-base stage, is considered extensively in conjunction with the role it plays in the differential amplifier stage ([Unit 8](#)).

C.1. DC (Bias) Analysis

The two common-emitter amplifier-stage configurations studied in projects are shown in [Fig. C.1](#). We note that the dc circuit of [Fig. C.1\(a\)](#) is that of [Fig. B.12](#). Amplifier performance analysis can be performed with the two output channels available from the DAQ in the following manner: [Fig. C.1\(a\)](#) uses separate channels for the input and output bias circuits and superimposes the input signal on the input bias. In [Fig. C.1\(b\)](#) we add a signal-source resistor and coupling capacitor and use one channel for the input signal and one channel for bias. The capacitor is required to prevent the connection of the signal

source from affecting the dc (bias) operation of the circuit. The latter represents the classical practical common-emitter amplifier stage. The signal-source resistor provides for a current-source input signal and hence linear amplification.

Figure C.1. (a) Base current bias (dc) and input signal applied to the same node. Collector circuit has a separate power supply. (b) Amplifier with signal coupling capacitor and single power supply for bias.

In the experimental project on the common-emitter amplifiers, we measure the voltage gain as a function of the collector current and compare the results with SPICE. The bias collector current is swept from 0.1 to 1 mA. The measurement uses the circuit in [Fig. C.1\(a\)](#). A DAQ output channel (V_{BB}) sets the bias currents. Another output channel (V_{CC}) sets the design bias V_{CE} at each bias current. The gain of the circuit of [Fig. C.1\(b\)](#) is also measured with particular emphasis on the bias solution and the frequency response.

C.1.1. DC (bias) Formulation

A formulation for obtaining the collector current for a given V_{BB} [[Fig. C.1\(a\)](#)] is based on the following: From the input loop equation,

Equation C.1

$$I_B = \frac{V_{BB} - V_{BE}}{R_B}$$

The two relevant transistor characterization equations are ([B.6](#)),

$$I_C = \beta_{DC} I_B$$

and base – emitter dc voltage equation, from (B.7), setting $V_{BC} = 0$ for simplicity,

Equation C.2

$$V_{BE} \approx V_T \ln\left(\frac{I_C}{I_S}\right)$$

The equation set above has three unknowns, I_C , I_B , and V_{BE} . The equations can be combined to obtain a function for I_C ,

Equation C.3

$$I_C = \frac{V_{BB} - V_T \ln\left(\frac{I_C}{I_S}\right)}{R_B} \beta_{DC}$$

A good estimate can be obtained with using $V_{BE} \approx 0.6$ V, giving simply

Equation C.4

$$I_C = \frac{V_{BB} - 0.6V}{R_B} \beta_{DC}$$

The simple expression is sufficient, for example, for selecting a value for R_B in the circuit of Fig. C.1(a) in the BJT amplifier project. We have also neglected the dependence of β_{DC} on V_{BC} of (B.11). This is a reasonable approximation for establishing the nominal values for the measurement circuit components.

The collector – emitter voltage, V_{CE} , is then established with

Equation C.5

$$V_{CE} = V_{CC} - I_C R_C$$

In designing a project circuit for a current sweep, R_B and R_C are selected for the highest currents and highest DAQ output voltage. In the amplifier project, the circuit of Fig. C.1(b) uses the resistors from the circuit of Fig. C.1(a). It thus has a unique bias variable

and V_{CC} solution for a given design V_{CE} requirement. This is based on (C.3) and (C.5) and is

Equation C.6

$$\frac{V_{CC} - V_{CE}}{R_C} = \frac{V_{CC} - V_{BE}}{R_B} \beta_{DC}$$

In the amplifier project, the project Mathcad file is used to find the solution for V_{CC} [with an educated guess for V_{BE} , as in (C.4)]. If the result has $V_{CC} > 10$ V, the limit from the DAQ, it is necessary to decrease R_C or increase R_B . This may be necessary, as the transistor β_{DC} is not known with precision at the point of the selection of the resistor values. After measuring the actual V_{CC} , the result is used in the Mathcad file to compute a value for β_{DC} from a circuit solution.

C.2. Linear or Signal Model for the BJT

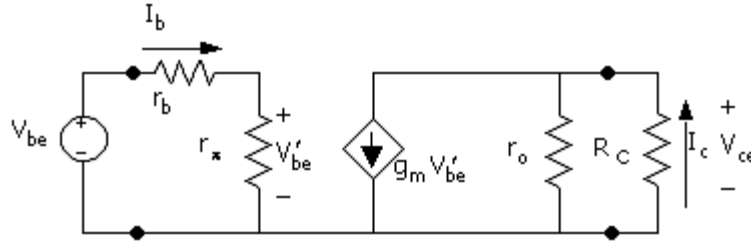
The primary function of a transistor in analog circuits is to produce a signal output current in response to a signal input voltage. In the case of the common-emitter circuits of Fig. C.1, the transistor input voltage is V_{be} and the responding output current is the collector current I_c . (Variable subscript conventions are covered in Unit 2. Uppercase symbols with lowercase subscripts denote RMS or peak magnitude of periodic signals.) The linear relation between the two variables is the transconductance, g_m . By definition, for the BJT

Equation C.7

$$I_c = g_m V_{be}$$

In some simple amplifier circuits, this would be all that would have to be known about the transistor to perform a design or analysis. More generally, however, the model also includes input and output resistances, $r_i = r_b + r_\pi$ and r_o , respectively. The transistor linear model, which includes these components and a load resistor (in this case, bias resistor, R_C), is shown in Fig. C.2. Applied input voltage V_{be} and responding I_c are indicated.

Figure C.2. Linear signal model for the BJT. Model parameters are r_b , r_π , g_m , and r_o . Added to the model are circuit components R_C and applied voltage V_{be} .



The input resistance relates the input signal base current I_b to the signal emitter – base voltage, V_{be} , that is

Equation C.8

$$I_b = \frac{V_{be}}{r_b + r_{\pi}}$$

Parameter r_b is the actual physical resistance through which the base current must flow to arrive at the true, internal physical base – emitter junction. The signal voltage across the internal base – emitter junction is V'_{be} . Parameter r_{π} is the linear relation between I_b and V'_{be} and is not a physical resistance.

We assume in the following discussion that $r_{\pi} \gg r_b$. This is especially true in the low current range of our BJT transistor projects. As will be seen below, r_{π} is inversely proportional to bias collector current, I_C , whereas r_b is close to a constant and could be significant at currents corresponding to midrange or higher for the transistor. (As a rule, r_b must be taken into consideration when the model is applied in very high frequency applications.) Note that neglecting r_b compared to r_{π} is equivalent to $V'_{be} \approx V_{be}$, as assumed below.

The output-resistance parameter, r_o , accounts for the fact that total collector current, i_C , increases with increasing total collector – emitter voltage, v_{CE} . According to the output resistance parameter relationship, the current through this resistance is

Equation C.9

$$I_{r_o} = \frac{V_{ce}}{r_o}$$

Including r_o , the current through the load, in this case bias resistor, R_C , is

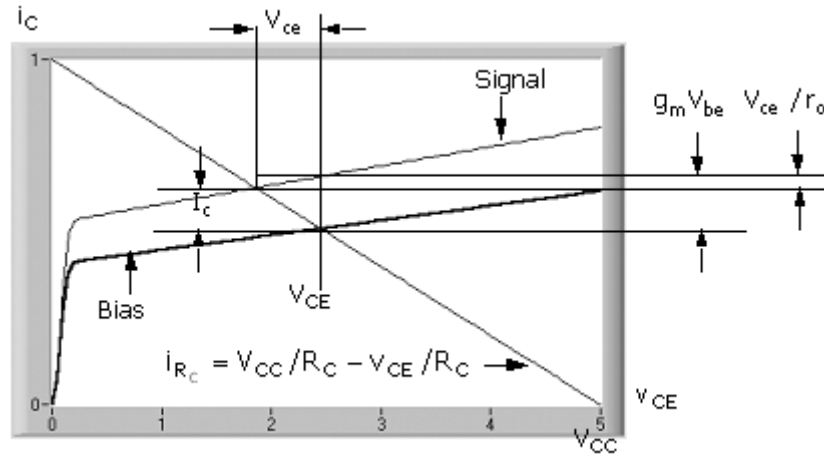
Equation C.10

$$I_c = g_m V_{be}' + \frac{V_{ce}}{r_o} \approx g_m V_{be} + \frac{V_{ce}}{r_o}$$

This current flows up through R_C such that V_{ce} is negative for positive V_{be} . Thus, the current associated with r_o subtracts from $g_m V_{be}$ to reduce the current through R_C . For a positive V_{be} , there is an increase in the total v_{BE} , thus causing the total collector current to increase. The result is a decrease in the total v_{CE} and hence a negative incremental V_{ce} .

The two components of (C.10) are illustrated graphically in Fig. C.3. The output characteristics are for two values of base – emitter voltage: bias only, V_{BE} , and bias plus base – emitter signal voltage, $V_{BE} + V_{be}$. They are designated Bias and Signal. The solution to i_C and v_{CE} is constrained to the "load line," which is $i_{R_C} = i_C = V_{CC}/R_C - v_{CE}/R_C$ [(C.5)].

Figure C.3. Transistor output characteristic with no signal and signal. Also plotted is the R_C load line. The solution for i_C and v_{CE} is always the intersection.



At a constant $v_{CE} = V_{CE}$, the change in the current-source current for the applied V_{be} is $g_m V_{be}$ [(C.7)]. The net collector current change (signal current), I_c , though, is as given by (C.10); that is, it includes the component associated with r_o . Since the output characteristic slopes downward for decreasing v_{CE} , the actual transconductance decreases, but the linear model treats this effect with a constant g_m combined with the effect of the output resistance, r_o .

C.2.1. Determination of the Linear Model Parameters

We can relate the values of the two parameters in (C.10) to the SPICE model parameters using (B.7) with the substitution $v_{BC} = -(v_{CE} - v_{BE})$. This is

Equation C.11

$$i_C = I_S e^{v_{BE}/V_T} \left(1 + \frac{v_{CE} - v_{BE}}{V_{AF}} \right)$$

Differentiating (C.11) with respect to v_{BE} (with $v_{CE} = V_{CE}$, $V_{ce} = 0$), g_m is found to be

Equation C.12

$$g_m = \frac{\partial i_C}{\partial v_{BE}} = \frac{\partial \left[I_S e^{v_{BE}/V_T} \left(1 + \frac{v_{CE} - v_{BE}}{V_{AF}} \right) \right]}{\partial v_{BE}} \approx \frac{I_C}{V_T}$$

The approximate form only ignores a term on the order of I_C/V_{AF} , where $V_{AF} \gg V_T$.

The output resistance is obtained with $v_{BE} = V_{BE}$ (bias value) or $V_{be} = 0$. This is

Equation C.13

$$1/r_o = \frac{\partial i_C}{\partial v_{CE}} = \frac{I_C (v_{CE} = V_{BE})}{V_{AF}}$$

where, from (C.11), $I_C(v_{CE} = V_{BE}) = I_S \exp(V_{BE}/V_T)$. For simplicity, the bias collector current, from (C.11), $I_C = i_C(V_{CE})$, is usually used for the calculation for r_o . Finally, a relation for r_π comes directly from (C.7), $I_C = g_m V_{be}$, and (B.41), $I_C = \beta_{ac} I_b$. Equating the two gives

Equation C.14

$$g_m V_{be} = \beta_{ac} I_b$$

Hence, from the definition $r_{\pi} = V_{be}/I_b$ (with $V_{be} \approx V'_{be}$)

Equation C.15

$$r_{\pi} = \frac{\beta_{ac}}{g_m} = \beta_{ac} \frac{V_T}{I_C}$$

Note that the right-hand side of (C.14) is the alternative current-dependent current source of the linear model of Fig. C.2.

As discussed in Unit B.9, β_{ac} can be slightly different from β_{DC} , but the distinction usually need not be made in analysis or design. This is due to the fact that β_{DC} tends to be quite variable among devices and that most analog designs are based on making the results as independent of β_{DC} as possible. Signal parameter β_{ac} will be used in the following, but it is understood that β_{DC} can be used in the calculations without serious penalty in precision.

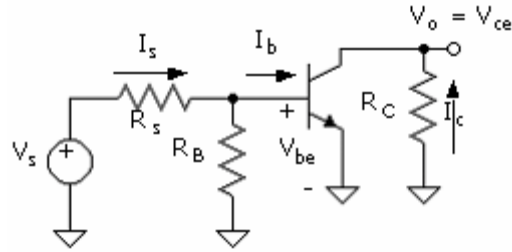
C.3. Amplifier Voltage Gain

Any transistor amplifier stage has a gain from the input terminal to its output terminal (base and collector, respectively, for this case). But the circuit gain, from the source to the output, takes into consideration the possible finite input resistance at the transistor input terminal. Due to the finite signal-source resistance, an attenuation results from the signal-source to the transistor input terminal. The example of this case of the common-emitter amplifier stage is considered here.

C.3.1. Gain from Base of Transistor to Output at Collector

The midband (frequency-independent) signal version of the circuit of Fig. C.1(b) is shown in Fig. C.4. It is obtained from the general circuit by converting dc voltages to zero. This includes the capacitor voltage, which ideally, remains at its constant dc (bias) value. The rule followed here is that if there is no incremental variation between any two nodes with a signal applied at the input, then the component between the two nodes is superfluous. In Fig. C.4, the output voltage is $V_o = V_{ce}$ and the signal source is designated V_s along with its source resistance, R_s . In the case of an actual signal source, R_s is probably an equivalent rather than an actual resistor. Thus, it is assigned a lowercase subscript.

Figure C.4. Signal equivalent circuit for the amplifier. dc nodes have been grounded and the capacitor has been shorted.



Neglecting the output resistance, the signal collector current for a signal voltage applied at the base reverts to (C.7), which is

$$I_c = g_m V_{be} = \frac{I_C}{V_T} V_{be}$$

with $g_m = I_C/V_T$ [(C.12)]. I_C is the bias current with uppercase subscript, not the signal, with lowercase subscript. (Recall that signal voltage and currents can be, for example, periodic peak or RMS values or instantaneous values, as these are all proportional throughout the linear circuit. To be specific, as in the experiment on the common-emitter amplifier, we consider them to be periodic-signal peak values.)

The signal voltage developed at the collector is (still assuming that $r_o \gg R_C$)

Equation C.16

$$V_o = V_{ce} = -I_c R_C$$

From (C.16) and (C.7), the gain of the transistor in the circuit (input at the base of the transistor and output at the collector) is

Equation C.17

$$a_{vb} = \frac{V_o}{V_{be}} = -g_m R_C = -\frac{I_C}{V_T} R_C$$

We note that the magnitude of the result is the dc voltage drop across the bias resistor divided by V_T . For example, for $V_{RC} = 5$ V and $V_T = 26$ mV (room temperature), the gain

magnitude is about 200. The BJT circuit is capable of providing very substantial voltage gains.

C.3.2. Overall Gain Magnitude from Signal Source Voltage to Output

The circuit input resistance looking into the base of the transistor, R_b , is the transistor input resistance, r_π (still neglecting r_b), in parallel with the bias resistor R_B , that is,

Equation C.18

$$R_b = \frac{r_\pi R_B}{r_\pi + R_B}$$

The gain from the signal source to the output at the collector of the transistor is thus

Equation C.19

$$a_v = \frac{V_{ce}}{V_s} = \frac{V_{be}}{V_s} \cdot \frac{V_{ce}}{V_{be}} = - \frac{R_b}{R_b + R_s} g_m R_C$$

When the input-signal source resistance is large ($R_s \gg R_b$), a good approximation for a_v is

Equation C.20

$$a_v \approx - \frac{R_b}{R_s} g_m R_C$$

Further approximation can be made using $R_B \gg r_\pi$, to obtain

Equation C.21

$$a_v = - \frac{r_\pi}{R_s} g_m R_C$$

Finally, using $\beta_{ac} = g_m r_\pi$ [(C.15)],

Equation C.22

$$a_v = -\beta_{ac} \frac{R_C}{R_s}$$

This result is intuitive on the basis of $I_b \approx I_s$, $I_s \approx V_s/R_s$, and $I_c = \beta_{ac}I_b$. The sequence of approximations for the gain magnitude is

Equation C.23

$$|a_v| = \frac{V_{ce}}{V_s} = \frac{I_c}{I_s} \frac{R_C}{R_s + R_b} \approx \frac{I_c}{I_b} \frac{R_C}{R_s + r_\pi} \approx \beta_{ac} \frac{R_C}{R_s + r_\pi} \approx \beta_{ac} \frac{R_C}{R_s}$$

An additional approximation is with $\beta_{ac} \approx \beta_{DC}$.

Note that the requirement for the voltage gain to be greater than unity is that $R_s < \beta_{ac}R_C$. Thus, for sources with a large R_s , an amplifier design should have a high input resistance stage such as an emitter-follower stage. The emitter-follower stage is discussed in [Unit C.9](#).

In the [Project C1](#) the gain of the amplifier as a function of bias current, I_C , is measured using the circuit of [Fig. C.1\(a\)](#). This is made possible with the use of LabVIEW and the DAQ, with two output channels, to provide a signal source superimposed on the input bias voltage. In this case, the overall gain from the signal source is restricted (input bias and signal source resistor are the same) and $a_v \approx -\beta_{ac}R_C/R_B$. Since R_B of the circuit is roughly $\beta_{DC}R_C$, the gain is on the order of unity.

C.4. Accuracy of Transistor Gain Measurements

We want now to consider the measurement of the gain of the transistor amplifier ([Fig. C.1](#)) based on the linear model. This model is not valid if the signals are too large, such as to cause an unacceptable degree of nonlinearity. If the linear model is valid, the signal input voltage magnitude, between the base and emitter of the transistor, is related to the fraction I_c/I_C according to [combining [\(C.7\)](#) and [\(C.12\)](#)]

Equation C.24

$$v_{be} = \frac{I_c}{I_C} V_T$$

This voltage must be large enough for a good measurement using the DAQ board in the computer (i.e., at least a few millivolts), yet small enough so as not to cause substantial nonlinearity, which would invalidate the measurement of signal gain. Again, the signal-gain concept is based on the linear model. In the following, we will find the conditions under which the linear model is valid and to what extent.

The general expression (active region) between total collector current and total base – emitter voltage, $v_{BE} = V_{BE} + V_{be}$, is (neglecting the V_{AF} factor) [from (B.7)]

Equation C.25

$$i_C = I_S e^{V_{BE}/V_T} e^{V_{be}/V_T} = I_C e^{V_{be}/V_T} = I_C + I_C$$

or

Equation C.26

$$v_{be} = V_T \ln \left(1 \pm \left| \frac{I_c}{I_C} \right| \right) \rightarrow \pm V_T \left| \frac{I_c}{I_C} \right| \left| \frac{I_c}{I_C} \right| \ll 1$$

where the limit form for $|I_c/I_C| \ll 1$ gives (C.24). The plus sign is for I_c and V_{be} positive, and vice versa.

Since the signal output voltage is $V_o = -I_c R_C$, the signal "gain" is

Equation C.27

$$a_v = \frac{V_o}{V_{be}} = \frac{-I_c R_C}{V_T \ln \left(1 \pm \left| \frac{I_c}{I_C} \right| \right)} = -g_m R_C \frac{\left| \frac{I_c}{I_C} \right|}{\ln \left(1 \pm \left| \frac{I_c}{I_C} \right| \right)}$$

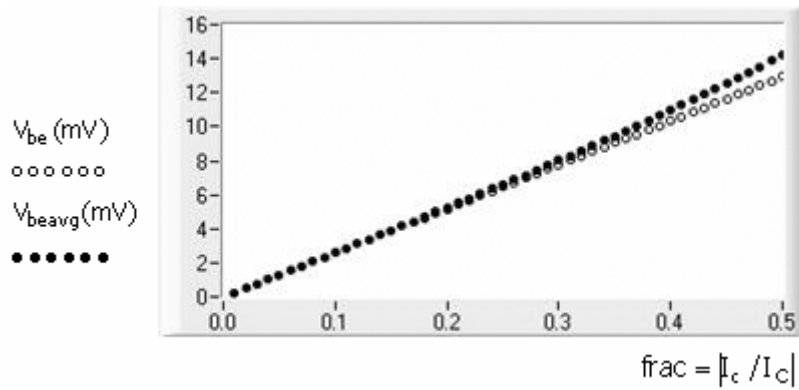
which reduces to the linear form, (C.17), when $|I_c/I_C| \ll 1$.

In the amplifier project, the gain is obtained by dividing the measured signal V_o by the measured signal V_{be} . The circuit has $R_s (=R_B) \gg r_\pi$, such that $I_c \approx \beta_{ac} V_s / R_s$ [as in (C.23)]. As a result, for the positive and negative peaks of V_s , the positive and negative peak

magnitudes of I_c are equal. Thus, the fraction $|I_c/I_C|$ is the same for both signal polarities. Incremental voltage V_{be} will respond nonsymmetrically according to (C.26). We note that this does not constitute a form of distortion at the output, as is evident from the approximate form of (C.23) (neglecting a very small effect due to variation of β_{ac}).

For output currents on the order of, for example, $|I_c/I_C| = 0.5$, the plus and minus peak values of V_{be} are significantly different. In the amplifier-gain measurement project, the ac voltmeter indicates a peak voltage, which is the average of the plus and minus peak values. To some extent, in this manner, the error is canceled. A comparison is made of the average value of the peaks, V_{beavg} , with the distortion-free V_{be} , as shown in Fig. C.5.

Figure C.5. Plot of calculated measured V_{beavg} as a function of $frac = I_c/I_C$ and distortion-free V_{be} . The measured value exhibits a 10% error at $|I_c/I_C| = 0.5$ and $V_{be} = 12.9 \text{ mV}$ and $V_{beavg} = 14.2 \text{ mV}$.



In the amplifier project, the measurement base – emitter voltage is limited to about $|I_c/I_C| = 0.2$. This corresponds to $V_{be} \approx 5 \text{ mV}$. The measurement error, due to the distortion discussed here, is only about 1% for this case.

The measurement circuit is configured to be able to measure the signal voltage with the dc base – emitter voltage removed. Therefore, the DAQ conversion limit can be set at the minimum of 50 mV, where the resolution is much less than $V_{be} \approx 5 \text{ mV}$.

As noted, a possible nonlinearity in β_{ac} could be a contributor to nonlinearity in the gain function (C.22). The effect from including the β_{ac} nonlinearity in the development of (C.27) is also, to a degree, canceled in the averaging process of measuring the gain.

C.5. Effect of Finite Slope of the Transistor Output Characteristic

As discussed in Unit C.2, the signal model contains an output resistance, which reflects the fact that the output characteristic of the transistor has a finite slope. This is characterized with the SPICE parameter VAF. It was noted that the effect of the finite

slope is to place a resistance effectively across the output of the amplifier. It is calculated from $r_o = V_{AF}/I_C$, (C.13).

With increasing bias current, r_o may not be negligible compared to R_C . In the project on the gain of the amplifier, we will read the experimental data into a Mathcad file and adjust V_{AF} to make the SPICE calculation and measured data match. The gain expression that includes transistor output resistance is

Equation C.28

$$a_{vb} = -g_m \frac{R_C r_o}{R_C + r_o} = -g_m \frac{R_C}{1 + \frac{I_C}{V_{AF}} R_C}$$

In a representative transistor, $V_{AF} \approx 100$. The circuit design could call for $I_C R_C \approx 5$ V for a 10-V supply voltage. In this case, the output resistance has about a 5% effect on the gain value.

C.6. Selection of Coupling Capacitors

External capacitors are added to the circuits of this unit for two purposes. One, shown in Fig. C.1(b), is to connect the signal input source to the amplifier. The other is a special-purpose capacitor of the amplifier project. It is attached to facilitate measurement of the base – emitter voltage with high resolution. Design considerations for selection of the capacitor values are discussed in the following.

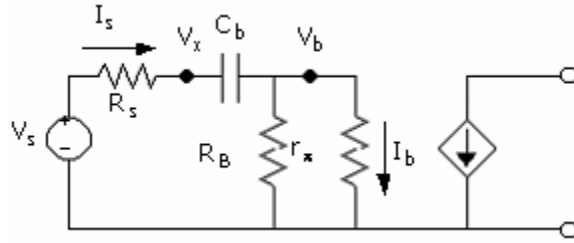
C.6.1. Coupling Capacitor for the Common-Emitter Amplifier

The linear equivalent circuit of the amplifier of Fig. C.1(b) is shown in Fig. C.6. The selection of the value of the capacitor C_b is based on the requirement that it has negligible effect on the signal current at any frequency at which the amplifier will be operated. Including the reactance of the capacitor, the input signal current is

Equation C.29

$$i_s(f) = \frac{V_s}{R_s + \frac{1}{j2\pi f C_c} + R_b} = \frac{V_s}{R_s + R_b} \cdot \frac{1}{1 + \frac{1}{j2\pi f (R_s + R_b) C_c}}$$

Figure C.6. Signal model of the circuit for the determination the characteristic frequency of the frequency response associated with the coupling capacitor.



The input signal current magnitude is

Equation C.30

$$|I_s(f)| = \frac{V_s}{R_s + R_b} \frac{1}{\sqrt{1 + \left(\frac{f_b}{f}\right)^2}}$$

where

Equation C.31

$$f_b = \frac{1}{2\pi(R_s + R_b)C_b}$$

Frequency f_{3dB} is defined as the frequency where the response magnitude is

Equation C.32

$$|I_s(f_{3dB})| = \frac{V_s}{R_s + R_b} \frac{1}{\sqrt{2}}$$

It follows that for this case, $f_{3dB} = f_b$. Note that at $f = 10f_{3dB}$,

$$|I_s(10f_{3dB})| = \frac{V_s}{R_s + R_b} 0.995$$

C.6.2. Coupling Capacitor for Measuring the Base Input Voltage

In [Project C1](#) measurement of the base input voltage is made at the signal side of the coupling capacitor. This is the node designated by V_x in [Fig. C.6](#). Good measurement precision is provided, as the dc component of the base voltage is blocked by the coupling capacitor. The maximum voltage sensed by the input channel is only the signal voltage $V_b \approx V_x \approx 5 \text{ mV}$, as discussed in [Unit C.4](#). This value is much smaller than dc $V_{BE} \approx 0.5 \text{ V}$. In this configuration, the limit setting for the input channel is set at 0.1 V , for a resolution of about $48 \mu\text{V}$ with the input channel in the bipolar mode. If the peak signal voltage is, for example 5 mV , the resolution is about 1% of the measured voltage

The required capacitor, C_b , for this case and for same $f_{3\text{dB}}$, is substantially larger than that obtained from [\(C.31\)](#). As will be shown, at $f \approx f_{3\text{dB}}$, the requirement is that $|X_{Cb}| \approx r_\pi$. It follows that at f near $f_{3\text{dB}}$, $R_s \gg |X_{Cb}|$ since $R_s \gg R_b \approx r_\pi$. The input signal current is thus given approximately by

Equation C.33

$$I_s = \frac{V_s}{R_s + R_b + |X_{Cb}|} \approx \frac{V_s}{R_s}$$

This includes the good assumption that $R_s \gg R_b$ and that, by design, $|X_{Cb}| \approx R_b$ for $f \approx f_{3\text{dB}}$. (Technically, the pole of the transfer function is ignored.)

The signal voltage, $V_x(f)$, at the input signal source side of the capacitor is the sum of the voltage at the base plus the voltage across the capacitor, that is, with [\(C.33\)](#),

Equation C.34

$$V_x(f) = \frac{I_s}{j2\pi f C_b} + I_s R_b$$

The ratio $V_x(f)/V_b$ is thus

Equation C.35

$$\frac{V_x(f)}{V_b} = \frac{V_x}{I_s R_b} = \frac{1}{j2\pi f R_b C_b} + 1$$

Note that the form of $V_x(f)$ is falling, for increasing frequencies, to a plateau (V_b). We will define f_{3dB} as the frequency where the magnitude of this ratio is $\sqrt{2}$. This could qualify as a type of corner frequency, as it represents the frequency where the frequency response function is $\sqrt{2}$ times the asymptotic value. A solution for f_{3dB} then comes from

Equation C.36

$$\sqrt{1 + \left(\frac{1}{2\pi f_{3dB} R_b C_b} \right)^2} = \sqrt{2}$$

giving

Equation C.37

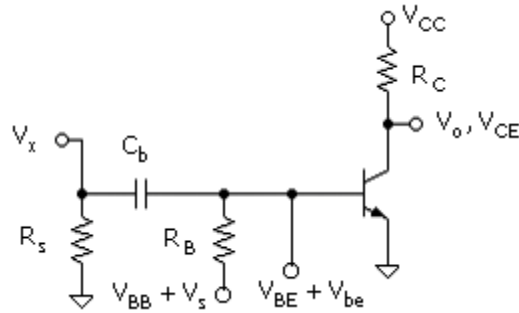
$$f_{3dB} = \frac{1}{2\pi R_b C_b} \approx \frac{1}{2\pi r_\pi C_b}$$

At $f=10f_{3dB}$, $V_x = 1.005V_b$. Note that for this case of a current source [(C.33)], the value of the capacitor can be obtained simply to satisfy $|X_{Cb}| = r_\pi$.

C.6.3. Coupling Capacitor for the Base Voltage Measurement of the DC Sweep Circuit

The project circuit for making a precision measurement of the gain of the amplifier of [Fig. C.1\(a\)](#) is shown in [Fig. C.7](#). The circuit has the addition of C_b and R_s for measuring the base signal voltage without the dc component. The selection of R_s in the amplifier project is made to satisfy if $R_s \gg r_\pi$ such that the effect on the gain referred to V_s is small. The choice is, on the other side, $R_s < R_B$, such that the charging time of C_b is not prohibitively long during the bias sweeps. The f_{3dB} frequency at V_x for this case is (C.31) with $f_b = f_{3dB}$. In the amplifier project, the capacitor is selected from (C.37) to satisfy the requirement for the amplifier of [Fig. C.1\(b\)](#) using (C.37). The capacitor will thus certainly be adequate for the amplifier of [Figs. C.1\(a\)](#) and [C.7](#).

Figure C.7. Common-emitter amplifier for measuring the amplifier gain as a function of bias current. The signal base – emitter voltage is measured at V_x .

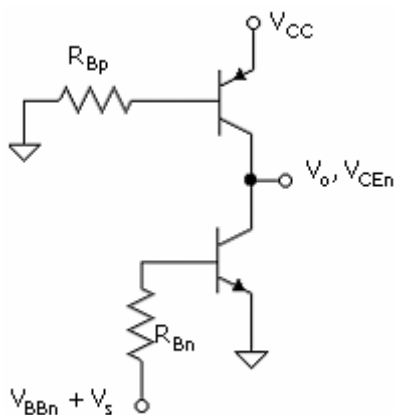


C.7. Common-Emitter Amplifier with Active Load

In the early days of electronic amplifiers, the voltage amplification device was, of course, the vacuum tube. It existed in only one polarity configuration, that is, with positive plate (bus or rail) voltage. With the appearance of semiconductor transistors came the availability of the dual set of devices with opposite terminal voltage polarities. This is the case for BJTs, JFETs, and MOSFETs. (Vacuum tubes were implemented in class B amplifiers. The application required a pair of inputs, one 180° out of phase with the other, normally derived from a transformer.)

The dual set has provided the versatility for a wide range of electronic system applications, including the amplifier with active load shown in [Fig. C.8](#). This can be compared with the circuit of [Fig. C.1](#), which in place of the pnp transistor, has a bias collector resistor, R_C .

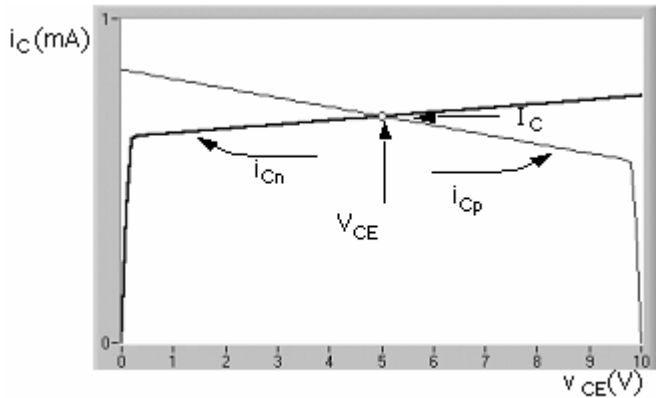
Figure C.8. Common-emitter amplifier with active (transistor) load. The npn is the driver transistor and the pnp is the load transistor. The input signal source could be moved to the base of the pnp, in which case the two transistors play opposite rolls.



Note that either transistor base could serve as the input such that the opposite transistor becomes the load. In [Fig. C.8](#), the npn is chosen as the *driver* transistor and the pnp as the *load* transistor.

A dc output characteristic plot is shown in [Fig. C.9](#). Since the collector current of the individual transistors is the same, the solution to bias voltage V_{CE} for the npn is the intersection of the two curves, that is, 5 V. This would be a good choice for a bias output voltage for the power supply voltage of this case, which is 10 V. For the plots, $V_{AFn} = 100$ V and $V_{AFp} = 20$ V were used. (In the discussion of the npn – pnp amplifier, the added subscript n or p will denote npn or pnp, respectively.)

Figure C.9. Output characteristics for the npn and pnp transistors. The pnp emitter – collector voltage is $v_{EC} = V_{CC} - v_{CE}$, where v_{CE} is the collector – emitter voltage of the npn. The bias variables V_{CE} , V_{EC} , and I_C are at the intersection of the two plots.



A signal impressed at the base of the npn causes the npn curve to move up or down while the pnp curve remains in place. Note that the pnp acts like a load line of a resistive load; however, an extension of the active-region characteristic of the pnp intersects the zero-current axis at $V_{CC} + V_{AFp} = 30$ V. The pnp active-load transistor thus provides the equivalent of a bias resistor with a power supply of 30 V instead of the actual 10 V.

C.7.1. Gain of the NPN – PNP Common-Emitter Amplifier with Active Load

The gain benefit for the case of the active load is apparent from the following. The equation for the gain of the BJT common-emitter amplifier with resistor R_C , which includes the output resistance of the driver transistor (in this case, npn), is a form of ([C.28](#))

Equation C.38

$$a_{vb} = -g_m \frac{R_C r_{on}}{R_C + r_{on}}$$

where r_{on} is the output resistance of the npn and is given by [generalization of (C.13)]

Equation C.39

$$r_{on} = \frac{V_{AFn} + V_{CE} - V_{BE}}{I_C(V_{CE})}$$

where V_{CE} and V_{BE} are the transistor bias voltage variables and I_C is the collect bias current of the amplifier. Parameters V_{AFn} and V_{AFp} are used in this unit for the slope parameter for the npn and pnp, respectively.

The output resistance expression is generalized here to emphasize that strictly speaking, the collector currents and voltages must match as suggested in the expression. Voltages V_{BE} and V_{CE} are bias values. In the following, as is standard in electronics circuit analysis, we approximate the output resistance, for example, for the npn as follows:

Equation C.40

$$r_{on} = \frac{V_{AFn}}{I_C}$$

It follows that for the pnp

Equation C.41

$$r_{op} = \frac{V_{AFp}}{I_C}$$

where $I_C \equiv I_C(V_{CE})$, that is, the actual bias collector current.

For the pnp active load, the resistor R_C is now replaced with the output resistance of the pnp, to obtain

Equation C.42

$$a_{vb} = -g_m \frac{r_{on} r_{op}}{r_{on} + r_{op}} = -g_m \frac{1}{I_C} \frac{V_{AFn} V_{AFp}}{V_{AFn} + V_{AFp}} = -\frac{1}{V_T} \frac{V_{AFn} V_{AFp}}{V_{AFn} + V_{AFp}}$$

where the far right-hand side uses $g_m = I_C/V_T$.

The gains for the resistive load and active load cases can readily be compared with the substitution of $g_m = I_C/V_T$ in (C.38) (gain with load R_C) and (C.40) for r_{on} to obtain

Equation C.43

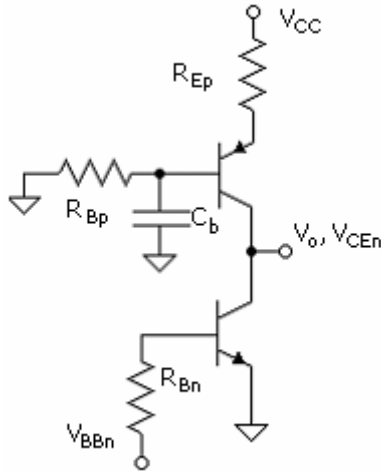
$$a_{vbR_C} = -\frac{1}{V_T} \frac{V_{R_C} V_{AFn}}{V_{R_C} + V_{AFn}}$$

For the example of $V_{CC} = 10$ V and bias $V_{CE} = 5$ V, $V_{R_C} = 5$ V. Using $V_{AFn} = 100$ V and $V_{AFp} = 20$ V, the room-temperature gain magnitudes are $|a_{vbR_C}| = 4.76$ V/0.026 V=183 for the amplifier with R_C load compared to an npn – pnp amplifier gain magnitude of $|a_{vb}| = 16.7$ V/0.026 V=641. In practice, the advantage will be considerably more, as the value for V_{AFp} used here is smaller than normal for BJTs. The small number was used above in the plot (Fig. C.9) to exaggerate the effect of the slope.

C.7.2. Output Resistance at the Collector with an Emitter Resistor

In [Project C2](#) the effect of an emitter resistor in the emitter branch of the pnp, R_{Ep} , on the output resistance of the pnp will be explored. The circuit is shown in [Fig. C.10](#). The effect of the emitter resistor is to increase the output resistance, due to the negative feedback effect, at the collector of the pnp. This increase can be made to be substantial; in fact, to a good approximation, the load on the amplifier is only r_{on} of the npn.

Figure C.10. Amplifier with an emitter resistor in emitter branch of pnp to increase the output resistance at the collector of the pnp. Also included is a capacitor, C_b , for grounding (signal) the base voltage of the pnp.



The generalized gain expression, which includes the effect of the emitter resistor, is

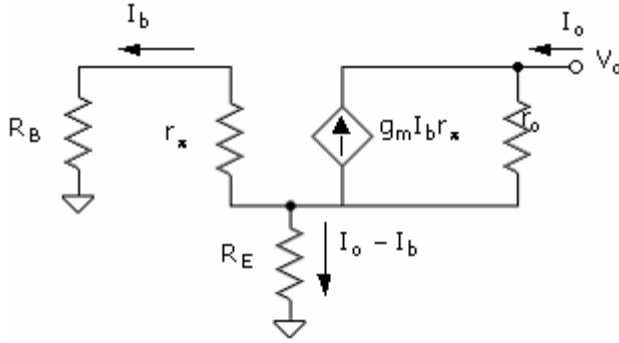
Equation C.44

$$a_{vb} = -g_m \frac{r_{on} R_{op}}{r_{on} + R_{op}}$$

where R_{op} is the output resistance at the collector of pnp, for the circuit with R_{Ep} .

Here, we develop an expression for the output resistance of a BJT with the emitter resistor. This is a function of both R_{Ep} and R_{Bp} . In the [Project C2](#) the amplifier gain is measured with and without a base shunt capacitor, C_b . With the capacitor in place, the base resistance in the signal circuit is effectively zero, and this alters R_{op} significantly. The linear circuit for the general case of a BJT common-emitter amplifier with emitter resistor is shown in [Fig. C.11](#). The circuit includes a base resistor, R_B .

Figure C.11. Linear circuit for the determination of the output resistance at the collector for a circuit with emitter and base resistor.



A test voltage, V_o , is applied at the collector with the base resistor and emitter resistor at signal ground. In response, a current I_o flows from V_o through R_E in parallel with $R_B + r_\pi$. This induces a voltage $V_{RE} = (I_o - I_b)R_E$ across R_E that is applied to r_π in series with R_B . Base current I_b is a fraction of I_o as given by

Equation C.45

$$I_b = -\frac{R_E}{r_\pi + R_B + R_E} I_o$$

Applied voltage V_o sums up to

Equation C.46

$$V_o = (I_o + g_m I_b r_\pi) r_o + (I_o - I_b) R_E \approx (I_o + g_m I_b r_\pi) r_o$$

The approximation is based on $r_o \gg R_E$ and is consistent with the fact that the signal voltage drop across the output of the transistor is much greater than across the emitter resistor.

Eliminating I_b in (C.46) using (C.45) results in the solution for R_o , which is

Equation C.47

$$R_o \equiv \frac{V_o}{I_o} = \left(1 + g_m \frac{R_E}{r_\pi + R_B + R_E} r_\pi \right) r_o$$

The result has two limiting forms based on the relative value of R_B : When $R_B \gg r_\pi + R_E$,

Equation C.48

$$R_O = \left(1 + \frac{g_m r_\pi R_E}{R_B}\right) r_O = \left(1 + \frac{\beta_{ac} R_E}{R_B}\right) r_O \approx \left(1 + \frac{\beta_{ac} V_{R_E}}{\beta_{DC} V_{R_B}}\right) r_O \approx r_O$$

The approximate form uses $I_C \approx I_E$.

Intuitively, for $R_B \rightarrow \infty$, the feedback current, $g_m I_b r_\pi$, goes to zero.

For $R_B \rightarrow 0$,

Equation C.49

$$R_O = \left(1 + g_m \frac{r_\pi R_E}{r_\pi + R_E}\right) r_O = \left(1 + \beta_{ac} \frac{R_E}{r_\pi + R_E}\right) r_O \approx \frac{\beta_{ac}}{1 + \frac{\beta_{ac} V_T}{V_{R_E}}} r_O \gg r_O$$

The alternative forms on the right in (C.48) and (C.49) use (C.15), $\beta_{ac} = g_m r_\pi$. Again, the approximate form uses $I_C \approx I_E$. Note that the solution corresponds to the maximum fraction of I_o that can flow through r_π ($R_B = 0$), to induce a feedback current.

When applied specifically to the npn – pnp circuit of Fig. C.10, the limiting case is, for R_{Bp} zero,

Equation C.50

$$R_{Op} = \frac{\beta_{acp}}{1 + \frac{\beta_{acp} V_T}{V_{R_{Ep}}}} r_{Op}$$

where R_{op} is the signal resistance looking up into the collector of the pnp. For example, if R_{Ep} is selected to produce $V_{REp} = 1$ V and $\beta_{acp} = 50$, the denominator is roughly 2, such that $R_{op} \approx (\beta_{acp}/2)r_{op}$. In this case, the load on the amplifier is due almost entirely to the output resistance looking into the collector of the npn ($R_{on} = r_{on}$), with the result that the gain is [from [C.44](#)]

Equation C.51

$$a_{vb} \approx -\frac{V_{AFn}}{V_T}$$

The other extreme is for $R_{Bp} \gg r_{\pi p} + R_{Ep}$, where $R_{op} \approx r_{op}$ [[C.48](#)]. The gain reverts to [\(C.42\)](#), repeated here

$$a_{vb} \approx -\frac{1}{V_T} \frac{V_{AFn}V_{AFp}}{V_{AFn} + V_{AFp}} = -\frac{1}{V_T} V_{AFnp}$$

The npn – pnp amplifier circuit of [Fig. C.10](#) is used in the project on the amplifier to investigate the signal-derived magnitude of the slope parameters of npn and pnp transistors. Using a bypass capacitor at the base of the npn, as discussed below, the signal circuit will effectively have $R_B = 0$, and a gain measurement along with [\(C.51\)](#) yields V_{AFn} . The gain will also be measured for the circuit without the capacitor and with $R_{Bp} \gg r_{\pi p} + R_{Ep}$ (by design). For this case, [\(C.42\)](#) applies and the gain measurement provides information on the combination output resistance parameter, V_{AFnp} . Between the two measurements, values for both parameters are determined.

In the measurement circuit of the npn – pnp amplifier, the gain referred to the signal source (amplifier circuit gain) is

Equation C.52

$$a_v = -\beta_{acn} \frac{R_{onp}}{R_{Bn} + r_{\pi n}} \approx -\beta_{acn} \frac{R_{onp}}{R_{Bn}}$$

where R_{onp} is the signal resistance at the output node ([Fig. C.10](#)), that is, the combined resistance looking back into the collectors of the npn and pnp transistors. In general, this is

Equation C.53

$$R_{onp} = \frac{r_{on} R_{op}}{r_{on} + R_{op}}$$

With R_{Bp} effectively made zero with the shunting capacitor, $R_{onp} \approx r_{on} = V_{AFn}/I_C$. Without the capacitor, R_{op} is obtainable from (C.47) for use in (C.53).

In the amplifier project, the circuit-gain equation (C.52) is used to convert measured gain into R_{onp} and then information on V_{AFn} and V_{AFp} . With the availability of these numbers, we can then calculate the gain produced by the transistor (base to collector), using (C.44).

C.7.3. DC (Bias) of the NPN – PNP Amplifier

The circuit equations for the circuit of Fig. C.10 are (collector power supply through pnp base)

Equation C.54

$$V_{CC} = I_C \frac{1 + \beta_{DCp}}{\beta_{DCp}} R_{Ep} + V_{EBp} + \frac{I_C}{\beta_{DCp}} R_{Bp}$$

and (npn base)

Equation C.55

$$V_{BBn} = \frac{I_C}{\beta_{DCn}} R_{Bn} + V_{BE n}$$

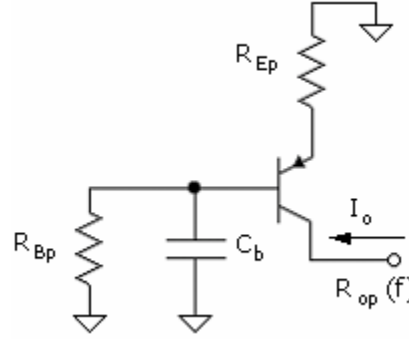
In the project on the amplifier, R_{Bp} is determined for a design collector current. The selection uses (C.54) with, for example, $V_{CC} \approx 9$ V, that is, less than the maximum available from the DAQ output. Then with $R_{Bn} = R_{Bp}$, V_{BBn} will be less than V_{CC} by the amount of the drop across R_{Ep} (for example, 1 V). This makes the good assumption that $\beta_{DCn} \approx \beta_{DCp}$. A LabVIEW program then sets up the circuit for the design collector current by adjusting two supply voltages (DAQ output channels).

C.8. Frequency Response of NPN – PNP Amplifier Due to the Base Shunt Capacitor

In the npn – pnp amplifier project, we determine V_{AFn} directly through a gain measurement for a circuit configuration in which (C.51) is valid. The requirement that

$R_{Bp} \approx 0$ will be implemented by shunting the base of the pnp transistor to ground with a capacitor, C_b , as shown in [Fig. C.12](#). The capacitor must be sufficiently large to hold the base at ground at the frequency of the gain measurements.

Figure C.12. Segment of the npn – pnp amplifier of [Fig. C.10](#) showing the addition of a pnp base-bypass capacitor, C_b .



The expression to determine the required value of C_b can be obtained as follows: We start with the expression for R_o without C_b , which is ([C.47](#)) and repeated here (referring to the generalized circuit [Fig. C.11](#)):

$$R_o \equiv \frac{V_o}{I_o} = \left(1 + g_m \frac{R_E}{r_\pi + R_B + R_E} r_\pi \right) r_o$$

With the capacitor in parallel with R_{Bp} , R_B in ([C.47](#)) is replaced with the impedance of the parallel combination of R_{Bp} and C_b . The resulting R_{op} is frequency dependent and is given by

Equation C.56

$$R_{op}(f) = \left(1 + \frac{r_{\pi p}}{r_{\pi p} + R_{Ep} + \frac{R_{Bp}}{1 + j2\pi f R_{Bp} C_b}} g_m R_{Ep} \right) r_{op}$$

This can be manipulated into the form

Equation C.57

$$R_{Op}(f) = \left(1 + \frac{g_m r_{\pi} R_{Ep}}{r_{\pi p} + R_{Ep} + R_{Bp}} \frac{1 + j f / f_1}{1 + j f / f_2} \right) r_o$$

where

Equation C.58

$$f_1 = \frac{1}{2\pi R_{Bp} C_b}$$

and

Equation C.59

$$f_2 = \frac{r_{\pi p} + R_{Bp} + R_{Ep}}{r_{\pi p} + R_{Ep}} f_1 \approx \frac{1}{2\pi (r_{\pi p} + R_{Ep}) C_b}$$

The approximate form uses $R_{Bp} \gg r_{\pi p} + R_{Ep}$. At $f \approx f_1$, $R_{Op}(f_1) \approx r_o$, such that a good approximation is obtained with dropping the "1" in the numerator. The approximate form is then

Equation C.60

$$R_{Op}(f) = \left(1 + \frac{g_m r_{\pi p} R_{Ep}}{r_{\pi p} + R_{Ep} + R_{Bp}} \frac{j f / f_1}{1 + j f / f_2} \right) r_o$$

Further rearranging leads to

Equation C.61

$$R_{Op}(f) = R_{Opmax} \frac{f_2 + jf}{f_2 + jf}$$

where

Equation C.62

$$R_{opmax} = \left(1 + \frac{\beta_{acp} R_{Ep}}{R_{Ep} + r_{\pi p}} \right) r_{op}$$

and

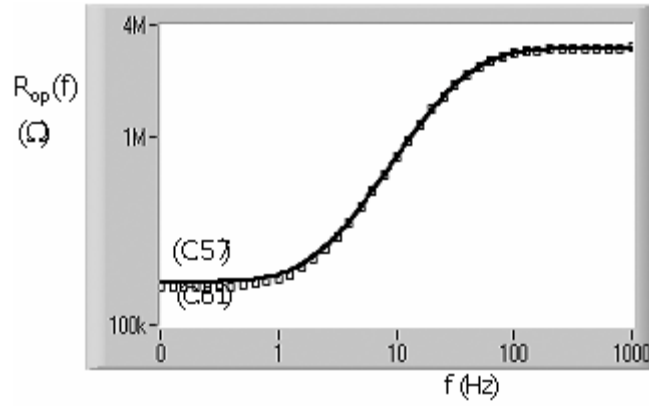
Equation C.63

$$f_z = \frac{r_{\pi p} + R_{Bp} + R_{Ep}}{r_{\pi p} + R_{Ep} + \beta_{acp} R_{Ep}} f_1 \approx \frac{1}{2\pi \beta_{acp} R_{Ep} C_b}$$

The approximate form again uses $R_{Bp} \gg r_{\pi p} + R_{Ep}$.

Mathcad-generated plots of the magnitude of $R_{op}(f)$ using (C.57) (and f_2 and f_z exact) and (C.61) (with the approximate forms for f_2 and f_z) are shown in Fig. C.13. These are for $V_{AFp} = 150$ V, $\beta_{acp} = 50$, $R_{Bp} = 330$ K Ω , $R_{Ep} = 800$ Ω , $C_b = 2$ μ F and $I_C = 1$ mA. Characteristic frequencies are $f_z = 1.9$ Hz and $f_2 = 38.2$ Hz. The approximate form is very close to the exact form except in the lowest frequency range. For $f \rightarrow 0$, $R_{op}(f) \approx r_{op}$ in both cases. For example, in the exact and approximate cases, $R_{op}(f) = 1.12r_{op}$ and $R_{op}(f) = 1.05r_{op}$, respectively, for $f = 0$.

Figure C.13. Mathcad-generated plots of the magnitude of $R_{op}(f)$ using (57) and (61). The calculation made with the approximate form, (61), also uses the approximate forms for f_2 and f_z .



Substituting $R_{op}(f)$ for R_{op} in (C.52), we obtain the frequency-dependent amplifier-circuit gain

Equation C.64

$$a_v(f) = -\frac{\beta_{acn}}{R_{Bn}} \cdot \frac{r_{on}}{1 + \frac{r_{on}}{R_{op}(f)}}$$

Then using (C.61) for $R_{op}(f)$ in (C.64) results in

Equation C.65

$$a_v(f) = \frac{\beta_{acn}}{R_{Bn}} \cdot \frac{R_{opmax} r_{on}}{R_{opmax} + r_{on}} \frac{f_z + jf}{f_p + jf}$$

where

Equation C.66

$$f_p = \frac{R_{opmax} f_z + r_{on} f_z}{R_{opmax} + r_{on}} \approx \left(1 + \frac{r_{on}}{R_{op}}\right) \frac{1}{2\pi\beta_{acp} R_{Ep} C_b}$$

The approximate form uses $R_{opmax} \gg r_{on}$ and the approximate f_z and f_z .

The design frequency f_{3dB} is obtained from $f_{3dB} = \sqrt{f_p^2 - 2f_z^2}$ [(6.8)]. Utilizing the approximate forms of f_2 , f_z , and f_p , f_{3dB} simplifies to

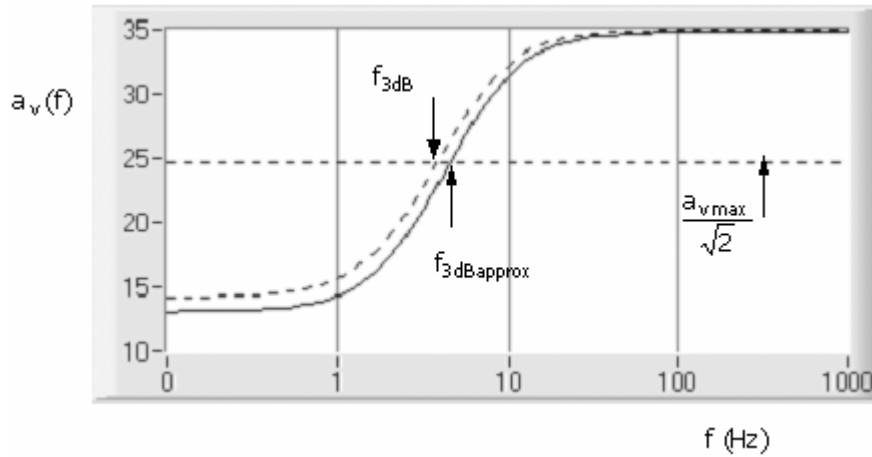
Equation C.67

$$f_{3dB\text{approx}} = \frac{1}{2\pi\beta_{acp}R_{Ep}C_b} \sqrt{\left(\frac{r_{on} + r_{op}}{r_{op}}\right)^2 - 2}$$

The result is significantly lower than f_2 in (C.61) because $R_{op}(f)$ is in parallel with r_{on} , which is much smaller than the high-frequency value of $R_{op}(f)$. The parameter β_{acp} appears from the association $\beta_{acp} = g_m r_{\pi p}$ [(C.15)].

Mathcad-generated plots of (C.65) for the exact and approximate values for f_p and f_z are shown in Fig. C.14. The parameter and component values are from the plots of Fig. C.13 with the addition of $V_{AFn} = 250$ V, $\beta_{acn} = \beta_{acp} = 50$, and $R_{Bn} = R_{Bp} = 330$ k Ω . With capacitor $C_b = 2$ μ F, $f_{3dB} = 3.8$ Hz (exact f_p and f_z) and $f_{3dB\text{approx}} = 4.5$ Hz [(C.67)].

Figure C.14. Plots of (C.65) with approximate and exact values for f_z (C.63), and f_p (C.66). The approximate form is sufficiently close for selecting C_b .



C.9. Common-Emitter Stage with Emitter Resistor and the Emitter-Follower Amplifier Stage

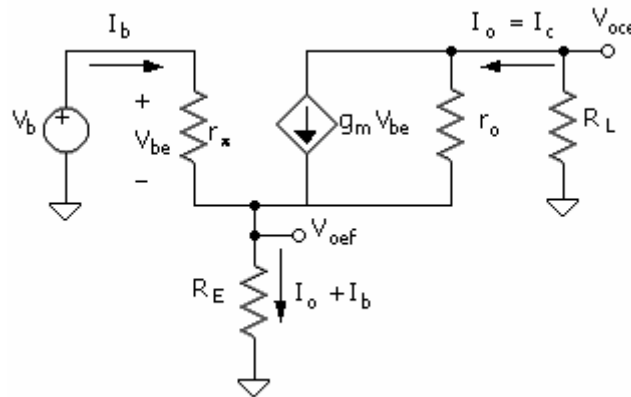
It was shown that the emitter resistor of the measurement circuit of Fig. C.10 can have a significant effect on output resistance. The emitter resistor has the effect of increasing the input resistance as well, such that it is compatible with high-resistance sources and the resistor adds to the bias stability. The common-emitter stage with an emitter resistor is, in fact, a very common configuration in BJT electronics.

Here, we analyze the effect of the emitter resistor on the input resistance (at the base) and on the gain of the common-emitter stage. The development leads directly to an assessment of the signal performance of the emitter-follower (common-collector) stage. The two aspects of the circuit with the emitter resistor are discussed in the following.

C.9.1. Input Resistance in the Common-Emitter Emitter-Resistor Circuit

The discussion is applicable to the circuit of [Fig. C.10](#), where the input resistance is at the base of the npn. In this circuit, $R_E \equiv R_{Ep}$ and, for the analysis, a base signal voltage, V_b , is applied directly to the base of the npn. A general signal circuit for a common-emitter stage (output, V_{oce}) with emitter resistor is shown in [Fig. C.15](#). Also indicated is the output for the emitter-follower stage (V_{oef}).

Figure C.15. Linear circuit for deriving the input resistance of the common-emitter stage with emitter resistor. The amplifier output nodes for the common-emitter stage (V_{oce}) and the emitter-follower (V_{oef}) stage are indicated.



Assume that $r_o \gg R_L$, such that we can neglect the current through r_o . (This assumption is violated in the circuit of [Fig. C.10](#), as the load is the output resistance of the npn, r_{on} .) With this simplification, the loop equation at the input of the circuit is

Equation C.68

$$V_b = V_{be} + g_m V_{be} R_E = I_b r_{\pi} + I_b g_m r_{\pi} R_E$$

having used $V_{be} = I_b r_{\pi}$. The input resistance at the base is thus (with respect to signal ground)

Equation C.69

$$R_{ib} = \frac{V_b}{I_b} = r_{\pi} + (1 + g_m r_{\pi}) R_E = r_{\pi} + (1 + \beta_{ac}) R_E$$

The following form of the result provides a convenient way of assessing the effect of the resistor on the input resistance:

Equation C.70

$$R_{ib} = \frac{\beta_{ac}}{g_m} + (1 + \beta_{ac}) R_E = \beta_{ac} \frac{V_T}{I_C} + (1 + \beta_{ac}) \frac{V_{R_E}}{I_C}$$

The result shows that with R_E in the circuit, the input resistance is increased over that of the circuit without R_E by a factor of approximately V_{R_E}/V_T . For example, with $V_{R_E} = 1$ V, V_{R_E}/V_T is about 40 at room temperature.

For the npn – pnp circuit of [Fig. C.10](#), R_{ib} at the base of the pnp is somewhat less than given by (C.70) because the effective load, that is, r_{on} , is so large and the simplification made above, $r_o \gg R_L$, is not valid. However, as is usually the case in design, such approximate forms are sufficient. This takes into consideration that exact solutions are not required or justified, given the variation in the model and circuit parameters that naturally occur.

If better precision is, however, required, a good approximation for (C.70) is readily obtained by replacing, in (C.70), $1 + \beta_{ac}$ with $1 + \beta'_{ac} = 1 + \beta_{ac} \left[r_{op} / (r_{op} + r_{on}) \right]$. This is specifically applied to the circuit of [Fig. C.10](#), where the input is at the base of the pnp and $r_{on} \equiv R_L$ and the output resistance of the pnp is r_{op} . The approximation is based on the expectation that $V_{oce} \gg V_{oef}$ [Fig. C.15](#) such that the voltages across r_{on} and r_{op} are similar.

Equation C.71

$$a_{vb} = \frac{V_C}{V_b} = -\frac{I_C R_L}{I_b R_{ib}} = -\frac{\beta_{ac} R_L}{R_{ib}} = -\frac{\beta_{ac} R_L}{r_{\pi} + (1 + \beta_{ac}) R_E} \approx -\frac{R_L}{R_E}$$

Suppose that the gain expression is applied to the circuit of [Fig. C.1\(b\)](#) but with an emitter resistor installed. From the signal source, the gain is then

Equation C.72

$$a_v = \frac{R_b}{R_s + R_b} a_{vb}$$

where the resistance at the base, R_b , is now

Equation C.73

$$R_b = \frac{R_{ib} R_B}{R_{ib} + R_B}$$

With [\(C.71\)](#) for the base-to-output gain,

Equation C.74

$$a_v \approx - \frac{R_b}{R_s + R_b} \beta_{ac} \frac{R_L}{R_{ib}} \approx - \frac{R_b}{R_s + R_b} \frac{R_L}{R_E}$$

The result shows that the maximum gain is approximately R_L/R_E . Therefore, although the circuit can have good bias stability, the emitter resistor seriously reduces the signal gain. The circuit with a bypass capacitor would have a gain given by [\(C.19\)](#). The circuit with a bypass capacitor would still have the advantage of bias stability. Essentially all practical common-emitter circuits have the emitter resistor, with or without the bypass capacitor. A two-stage-circuit significant voltage gain without a capacitor is discussed in the next unit.

C.9.2. Emitter-Follower Amplifier Stage

When the output is taken at the emitter instead of at the collector ([Fig. C.15](#)), the circuit becomes an emitter-follower amplifier stage. (The collector resistor is unnecessary and the collector terminal is connected to the power supply.) The output voltage in this case is

Equation C.75

$$V_{oef} = (I_b + I_c) R_E = I_b (1 + \beta_{ac}) R_E$$

The emitter-follower stage gain (transfer ratio) is the ratio of [\(C.75\)](#) and [\(C.68\)](#), that is

Equation C.76

$$a_{\text{vef}} = \frac{(1 + \beta_{\text{ac}})R_E}{r_{\pi} + (1 + \beta_{\text{ac}})R_E} = \frac{(1 + \beta_{\text{ac}})R_E}{\frac{\beta_{\text{ac}}}{g_m} + (1 + \beta_{\text{ac}})R_E}$$

A magnitude assessment can be made from

Equation C.77

$$a_{\text{vef}} = \frac{(1 + \beta_{\text{ac}})R_E}{\frac{\beta_{\text{ac}}}{I_C} V_T + (1 + \beta_{\text{ac}})R_E} \approx \frac{V_{R_E}}{V_T + V_{R_E}}$$

For example, with $V_{R_E} = 1 \text{ V}$, $a_{\text{vef}} = 0.97$ at room temperature. The input resistance, from (C.70), for the same conditions in addition to $I_C = 0.1 \text{ mA}$ and $\beta_{\text{ac}} = 100$ is $R_{\text{ib}} = 1 \text{ M}\Omega$. The emitter-follower stage is seen to have a gain of approximately unity and a very high input resistance. Its primary role is therefore that of a buffer stage.

An alternative form for the emitter-follower stage gain is

Equation C.78

$$a_{\text{vef}} = \frac{R_E}{\frac{V_T}{I_C} \frac{\beta_{\text{ac}}}{1 + \beta_{\text{ac}}} + R_E} \approx \frac{R_E}{\frac{1}{g_m} + R_E}$$

This is equivalent to a unity-gain amplifier with an output resistance of $1/g_m$. The output resistance is, for example, about 26Ω at $I_C = 1 \text{ mA}$. Therefore, when an external load is attached to the emitter-follower stage output, the gain remains near unity for very small load values.

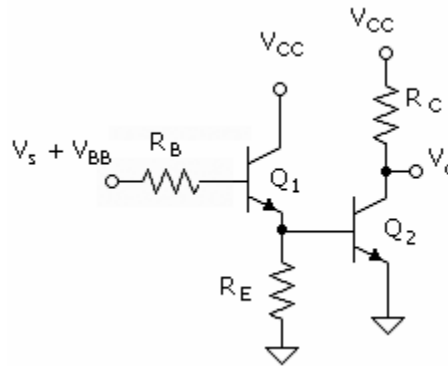
An example of an application that takes advantage of the characteristics of the emitter-follower stage is shown in Fig. C.16. This is a cascade of an emitter-follower stage and a common-emitter stage. The overall gain is the product of the input network function and the gains of the two stages. That is,

Equation C.79

$$a_v = \frac{V_{b1}}{V_s} \frac{V_{e1}}{V_{b1}} \frac{V_{c2}}{V_{b2}} = \frac{R_{ib}}{R_{ib} + R_B} a_{ve1} a_{vce} = - \frac{R_{ib}}{R_{ib} + R_B} \frac{R'_E}{1/g_m + R'_E} g_m R_C$$

where $R'_E = R_E \parallel r_{\pi ce}$. For example, suppose that $\beta_{ac} = 100$ and that the bias collector currents are $I_{C1} = 0.1$ mA and $I_{C2} = 10I_{C1} = 1$ mA. The emitter resistor is $R_E = V_{BE}/I_{C1} = 6000\Omega$ (neglecting the base current, I_{B2}) with $V_{BE} = 0.6$ V such that $R'_E \approx 4900\Omega$ and the gain of the emitter-follower stage is 0.87. With $V_{CC} = 10$ V and $V_{CE} = 5$ V, the gain of the common-emitter stage is -192 . Both gains are at room temperature. Therefore, the gain from the base of the emitter-follower stage to the output is -168 .

Figure C.16. Cascade of an emitter-follower stage and a common-emitter stage.
The overall gain is the product of the gains of the separate stages. The load on the emitter-follower stage is r_{π} of the common-emitter stage.



The input resistance at the base of the emitter follower is $R_{ib} \approx 500$ k Ω . Assume, for example, that $R_B \approx 50$ k Ω . The overall gain for this case is -136 . By comparison, the circuit, which omits the emitter-follower stage (R_B connected directly to the base of the common-emitter stage), has a gain of -9.5 .

C.10. Summary of BJT Model Parameter Relations

$g_m \approx \frac{I_C}{V_T}$	Transconductance.
$r_{\pi} = \frac{\beta_{ac}}{g_m} = \beta_{ac} \frac{V_T}{I_C}$	Emitter-base junction common-emitter stage input resistance.
$r_o = \frac{V_{AF}}{I_C}$	Collector – emitter output resistance.

$g_m \approx \frac{I_C}{V_T}$	Transconductance.
$r_i = r_b + r_\pi$	Total base – emitter input resistance.

C.11. Summary of Circuit Equations

$I_B = \frac{V_{BB} - V_{BE}}{R_B} \quad V_{BE} \approx V_T \ln \left(\frac{I_C}{I_S} \right)$	Base current equation for common-emitter amplifier.
$V_{CE} = V_{CC} - I_C R_C$	Collector current equation for common-emitter amplifier.
$\frac{V_{CC} - V_{CE}}{R_C} = \frac{V_{CC} - V_{BE}}{R_B} \beta_{DC}$	Collector power-supply voltage solution common-emitter amplifier with single power supply.
$a_{vb} = -g_m R_C$	Base-to-collector gain. Neglect internal output resistance of driver transistor.
$a_v = -\frac{R_b}{R_b + R_s} g_m R_C$	Overall gain from function generator (signal source).
$a_v = -\beta_{ac} \frac{R_C}{R_s}$	Overall gain with very large source resistor, R_s , as in amplifier gain experiments. $R_s \gg r_\pi$, $R_B \gg r_\pi$.
$a_{vb} = -g_m \frac{R_C}{1 + \frac{I_C}{V_{AF}} R_C}$	Base-to-collector gain expression that includes the output resistance of the transistor.
$a_{vb} = \frac{-I_C R_C}{V_T \ln \left(1 \pm \frac{I_c}{I_C} \right)}$	Base-to-collector large-signal "gain" that includes nonlinearity of I_C , V_{BE} relationship. Linear relation applies when I_c/I_C ratio is small enough for $\ln(1 + I_c/I_C) \approx I_c/I_C$.

$I_B = \frac{V_{BB} - V_{BE}}{R_B} \quad V_{BE} \approx V_T \ln \left(\frac{I_C}{I_S} \right)$	Base current equation for common-emitter amplifier.
$a_{vb} = -g_m \frac{r_{on} R_{op}}{r_{on} + R_{op}}$	General solution for base-to-collector gain for npn – pnp amplifier with emitter resistor, R_{Ep} .
$R_O = \left(1 + g_m \frac{R_E}{r_{\pi} + R_B + R_E} r_{\pi} \right) r_o$	Output resistance at collector with emitter resistor (general).
$a_{vb} = -g_m \frac{1}{I_C} \frac{V_{AFn} V_{AFp}}{V_{AFn} + V_{AFp}}$	Base-to-collector gain of npn – pnp amplifier, $R_{Ep} = 0$.
$a_{vb} \approx -\frac{V_{AFn}}{V_T}$	Approximate base-to-collector gain of npn – pnp amplifier with emitter resistor R_{Ep} (and $R_{Bp} = 0$).
$R_{op}(f) = R_{opmax} \frac{f_z + jf}{f_z + jf}$	Frequency-dependent output resistance at the collector of pnp with R_{Ep}
$f_z = \frac{r_{\pi p} + R_{Bp} + R_{Ep}}{r_{\pi p} + R_{Ep}} f_1$ $f_1 = \frac{1}{2\pi R_{Bp} C_b}$ $f_z = \frac{r_{\pi p} + R_{Bp} + R_{Ep}}{r_{\pi p} + R_{Ep} + \beta_{acp} R_{Ep}} f_1$	Characteristic frequencies of $R_{op}(f)$ function.
$a_v(f) = \frac{\beta_{acn}}{R_{Bn}} \times \frac{R_{opmax} r_{on}}{R_{opmax} + r_{on}} \frac{f_z + jf}{f_p + jf}$	Frequency-dependent amplifier gain of npn – pnp amplifier with emitter resistor and base-bypass capacitor.
$f_p = \frac{R_{opmax} f_z + r_{on} f_z}{R_{opmax} + r_{on}} \approx \left(1 + \frac{r_{on}}{r_{op}} \right) \frac{1}{2\pi \beta_{acp} R_{Ep} C_b}$	Characteristic frequency of $a_v(f)$ function.
$R_{opmax} = \left(1 + \frac{\beta_{acp} R_{Ep}}{R_{Ep} + r_{\pi p}} \right) r_{op}$	Maximum output resistance at the collector of the pnp with emitter resistor.
$f_{3dB} = \frac{1}{2\pi \beta_{acp} R_{Ep} C_b} \sqrt{\left(\frac{r_{on} + r_{op}}{r_{op}} \right)^2 - 2}$	Design equation for selecting C_b for npn – pnp measurement circuit.

$I_B = \frac{V_{BB} - V_{BE}}{R_B} \quad V_{BE} \approx V_T \ln \left(\frac{I_C}{I_S} \right)$	Base current equation for common-emitter amplifier.
$R_i = r_{\pi} + (1 + g_m r_{\pi}) R_E = r_{\pi} + (1 + \beta_{ac}) R_E$	Input resistance at base of common-emitter stage with emitter resistor.
$a_{vef} = \frac{(1 + \beta_{ac}) R_E}{\frac{\beta_{ac}}{I_C} V_T + (1 + \beta_{ac}) R_E} \approx \frac{V_{R_E}}{V_T + V_{R_E}}$	Gain of the emitter-follower stage.
$a_{vef} \approx \frac{R_E}{\frac{1}{g_m} + R_E}$	Alternative form of the gain of the emitter-follower stage.

C.12. Exercises and Projects

Project Mathcad Files ExerciseC1.mcd – ProjectC1.mcd – ExerciseC2 – ProjectC2.mcd

Laboratory Project C1 [NPN Common-Emitter Amplifier](#)

PC.2 [DC Circuit Setup and Parameter Determination](#)

PC.3 [Amplifier Gain at One Bias Current](#)

PC.4 [Amplifier Gain versus Bias Current](#)

PC.5 [Gain-Measurement Frequency Response](#)

Laboratory Project C2 [NPN – PNP Common-Emitter Amplifier with Current-Source Load](#)

PC.7 [Measurement of the PNP Parameters](#)

PC.8 [DC Circuit Setup](#)

PC.9 [Measurement of the Amplifier Gain](#)

C.11. Summary of Circuit Equations

$I_B = \frac{V_{BB} - V_{BE}}{R_B} \quad V_{BE} \approx V_T \ln \left(\frac{I_C}{I_S} \right)$	Base current equation for common-emitter amplifier.
$V_{CE} = V_{CC} - I_C R_C$	Collector current equation for common-emitter amplifier.

$I_B = \frac{V_{BB} - V_{BE}}{R_B} \quad V_{BE} \approx V_T \ln \left(\frac{I_C}{I_S} \right)$	Base current equation for common-emitter amplifier.
$\frac{V_{CC} - V_{CE}}{R_C} = \frac{V_{CC} - V_{BE}}{R_B} \beta_{DC}$	Collector power-supply voltage solution common-emitter amplifier with single power supply.
$a_{vb} = -g_m R_C$	Base-to-collector gain. Neglect internal output resistance of driver transistor.
$a_v = -\frac{R_b}{R_b + R_s} g_m R_C$	Overall gain from function generator (signal source).
$a_v = -\beta_{ac} \frac{R_C}{R_s}$	Overall gain with very large source resistor, R_s , as in amplifier gain experiments. $R_s \gg r_\pi$, $R_B \gg r_\pi$.
$a_{vb} = -g_m \frac{R_C}{1 + \frac{I_C}{V_{AF}} R_C}$	Base-to-collector gain expression that includes the output resistance of the transistor.
$a_{vb} = \frac{-I_C R_C}{V_T \ln \left(1 \pm \left \frac{I_C}{I_C} \right \right)}$	Base-to-collector large-signal "gain" that includes nonlinearity of I_C , V_{BE} relationship. Linear relation applies when I_C/I_C ratio is small enough for $\ln(1 + I_C/I_C) \approx I_C/I_C$.
$a_{vb} = -g_m \frac{r_{on} R_{op}}{r_{on} + R_{op}}$	General solution for base-to-collector gain for npn – pnp amplifier with emitter resistor, R_{Ep} .
$R_O = \left(1 + g_m \frac{R_E}{r_\pi + R_B + R_E} r_\pi \right) r_O$	Output resistance at collector with emitter resistor (general).
$a_{vb} = -g_m \frac{1}{I_C} \frac{V_{AFn} V_{AFp}}{V_{AFn} + V_{AFp}}$	Base-to-collector gain of npn – pnp amplifier, $R_{Ep} = 0$.
$a_{vb} \approx -\frac{V_{AFn}}{V_T}$	Approximate base-to-collector gain of npn – pnp amplifier with emitter

$I_B = \frac{V_{BB} - V_{BE}}{R_B} \quad V_{BE} \approx V_T \ln \left(\frac{I_C}{I_S} \right)$	Base current equation for common-emitter amplifier.
	resistor R_{Ep} (and $R_{Bp} = 0$).
$R_{op}(f) = R_{omax} \frac{f_z + jf}{f_z + jf}$	Frequency-dependent output resistance at the collector of pnp with R_{Ep}
$f_z = \frac{r_{\pi p} + R_{Bp} + R_{Ep}}{r_{\pi p} + R_{Ep}} f_1 \quad f_1 = \frac{1}{2\pi R_{Bp} C_b}$ $f_z = \frac{r_{\pi p} + R_{Bp} + R_{Ep}}{r_{\pi p} + R_{Ep} + \beta_{acp} R_{Ep}} f_1$	Characteristic frequencies of $R_{op}(f)$ function.
$a_v(f) = \frac{\beta_{acn}}{R_{Bn}} \times \frac{R_{opmax} r_{on}}{R_{opmax} + r_{on}} \frac{f_z + jf}{f_p + jf}$	Frequency-dependent amplifier gain of npn – pnp amplifier with emitter resistor and base-bypass capacitor.
$f_p = \frac{R_{opmax} f_z + r_{on} f_z}{R_{opmax} + r_{on}} \approx \left(1 + \frac{r_{on}}{r_{op}} \right) \frac{1}{2\pi \beta_{acp} R_{Ep} C_b}$	Characteristic frequency of $a_v(f)$ function.
$R_{opmax} = \left(1 + \frac{\beta_{acp} R_{Ep}}{R_{Ep} + r_{\pi p}} \right) r_{op}$	Maximum output resistance at the collector of the pnp with emitter resistor.
$f_{3dB} = \frac{1}{2\pi \beta_{acp} R_{Ep} C_b} \sqrt{\left(\frac{r_{on} + r_{op}}{r_{op}} \right)^2 - 2}$	Design equation for selecting C_b for npn – pnp measurement circuit.
$R_i = r_{\pi} + (1 + g_m r_{\pi}) R_E = r_{\pi} + (1 + \beta_{ac}) R_E$	Input resistance at base of common-emitter stage with emitter resistor.
$a_{vef} = \frac{(1 + \beta_{ac}) R_E}{\frac{\beta_{ac}}{I_C} V_T + (1 + \beta_{ac}) R_E} \approx \frac{V_{R_E}}{V_T + V_{R_E}}$	Gain of the emitter-follower stage.
$a_{vef} \approx \frac{R_E}{\frac{1}{g_m} + R_E}$	Alternative form of the gain of the emitter-follower stage.

C.12. Exercises and Projects

Project Mathcad Files ExerciseC1.mcd – ProjectC1.mcd – ExerciseC2 – ProjectC2.mcd

Laboratory Project [NPN Common-Emitter Amplifier](#)
C1

PC.2 [DC Circuit Setup and Parameter Determination](#)

PC.3 [Amplifier Gain at One Bias Current](#)

PC.4 [Amplifier Gain versus Bias Current](#)

PC.5 [Gain-Measurement Frequency Response](#)

Laboratory Project [NPN – PNP Common-Emitter Amplifier with Current-Source](#)
C2 [Load](#)

PC.7 [Measurement of the PNP Parameters](#)

PC.8 [DC Circuit Setup](#)

PC.9 [Measurement of the Amplifier Gain](#)

Laboratory Project 1. Basic Circuit Analysis for Electronic Circuits and Programming Exercises

[P1.1 Resistor Voltage-Divider Measurements](#)

[P1.2 Resistor Voltage Divider with Current Measurement](#)

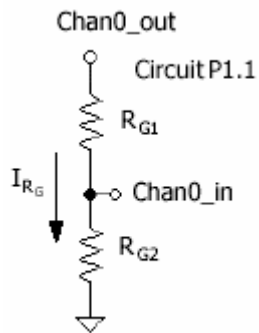
[P1.3 Resistor Voltage Divider with Resistor Measurement](#)

[P1.4 Resistor Voltage Divider with a Sine-Wave Source Voltage](#)

[P1.5 Frequency Response of a Resistor-Capacitor Circuit](#)

Exercises and Analysis Exercise01.mcd - Project01.mcd

P1.1. Resistor Voltage-Divider Measurements



Components

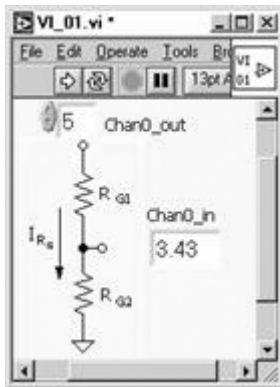
$$R_{G1} \approx R_{G2}/X$$

$$0.5 < X < 2$$

$$\text{Chan0_out}_{\text{max}} = 10 \text{ V}$$

$$I_{R_G \text{max}} = \frac{\text{Chan0_out}_{\text{max}}}{R_{G1} + R_{G2}}$$

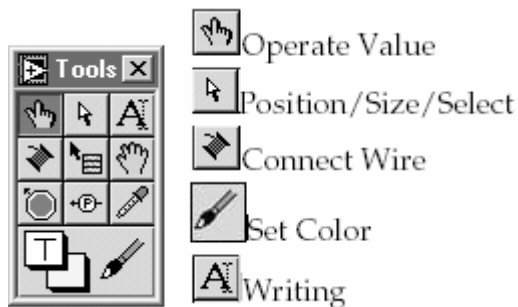
$$10 \text{ } \mu\text{A} < I_{R_G \text{max}} < 100 \text{ } \mu\text{A}$$




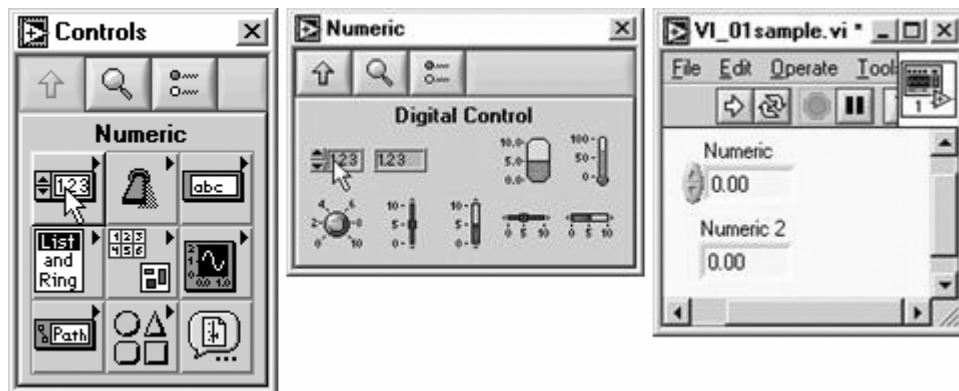
Programming Exercise 1.1

Use Tab to switch between Tools. Use Space Bar to switch between Wiring Tool and Position Tool (arrow). Get Tools Palette under Windows menu (or Shift/Right Click)

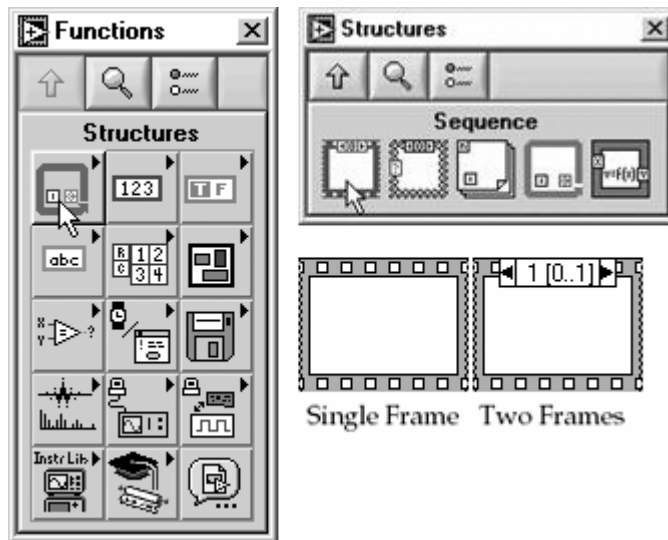
from the Diagram.



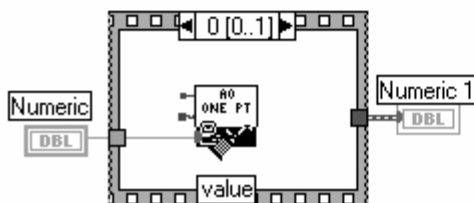
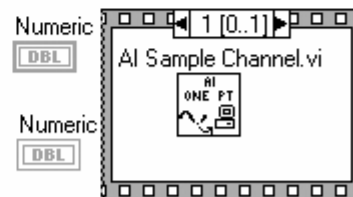
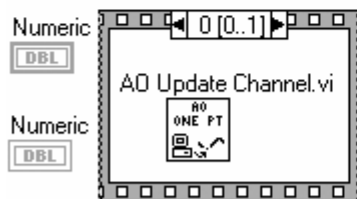
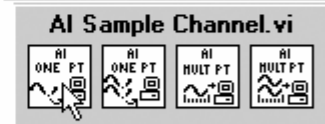
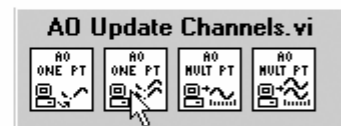
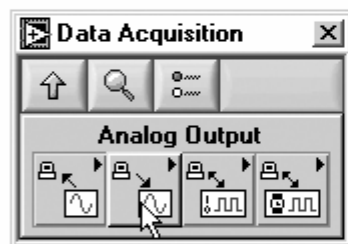
- Open a new VI to construct your **VI_01.vi**. The VI opens with a Front Panel (top) and a Diagram (under). Switch from one to the other using the Windows menu or Ctrl/E. The VI opens with the Operate Value Tool .
- Place a Digital Control and Digital Indicator on the Front Panel (example below). (Right Click on Front Panel to get the Controls Palette.) The Digital Control and Digital Indicator are in the Numeric Palette (below). Save the file with your choice of Name.



- Go to the Diagram (Ctrl/E) or Window>>Show Diagram. Place a Sequence Structure in the Diagram. (Right Click on Diagram to get Functions>>Structures>>Sequence Structure.) Enlarge the Structure by contacting the Arrow Tool (Position Tool) at the bottom-right corner of the Structure (drag).

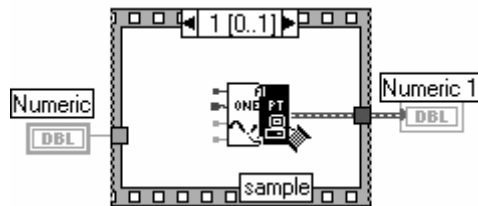


- Right Click on the edge of the Frame and Add Frame After. Click on arrows to switch between Frame 0 and Frame 1.
- Place **AO Update Channel.vi** in Frame 0.
(Functions>>Data Acquisition>>Analog Output >>AO Update Channel.vi.)
- Place **AI Sample Channel.vi** in Frame 1.
(Functions>>Data Acquisition>>Analog Input>>AI Sample Channel.vi.)

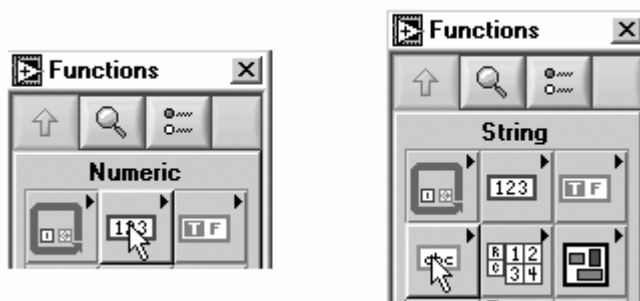


- Position the icons, etc., with the Arrow Tool (Left Click>>Drag).
- Wire the Digital Control (darker) terminal to the value terminal of the Analog Output VI, **AO Update Channel.vi**. Press the space bar to alternate between the Wiring Tool and Position (arrow) Tool.
- To wire, get the Wiring Tool (space bar), Left Click on a terminal, release, drag the Tool to the opposite terminal, and Left Click. To clean up wiring mistakes, use Ctrl/B. Note that the terminal name (value in the example) of an

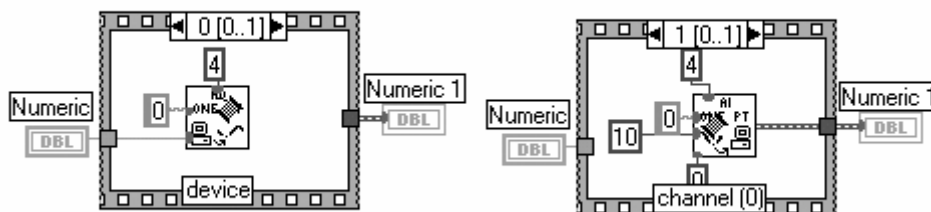
icon is displayed while the Wiring Tool is over the terminal.



- Wire the Digital Indicator terminal (lighter) to the output terminal (sample) of the Analog Input VI, **AI Sample Channel.vi**.
- Place the Constant (4) and the String (0) in the Diagram as shown in the example (Frame 0). Use the Writing Tool to type in the numbers. The location of Constants and Strings, under Functions, is shown below. Place Constants (4), (10) and (0) and String (0) in Frame 1.



- Wire the Constants and Strings as shown in the examples. To wire, get the Wiring Tool (space bar), Left Click on a terminal, release, drag the Tool to the opposite terminal, and Left Click. To clean up wiring mistakes, use Ctrl/B.

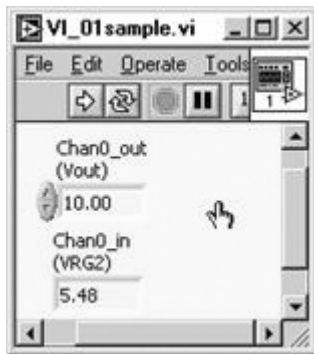


- Change the names of the Digital Control and Digital Indicator. Use the Writing Tool. This can be done from the Diagram or the Front Panel. Click

on the existing name (e.g., Numeric) and edit.

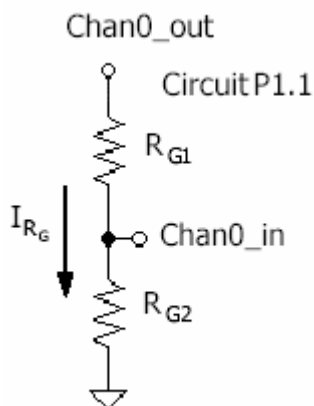
Procedure

- Calculate the resistor values. Connect the circuit.
- To Run a VI, from the Front Panel, use Operate>>Run or (better) use Ctrl/R. Run the VI with Chan0_out = 10V (Vout) and note the value of Chan0_in (VRG2). This should be consistent with your resistor ratio choice. Try a variety of Chan0_out values. (The maximum is 10 V.)



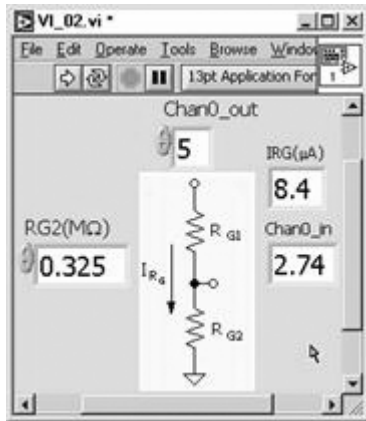
- (Optional) To install the circuit Diagram on the Front Panel, open **VI_01.vi** from **Project01.llb**. Use the Arrow Tool, Click on the circuit diagram, use Window's copy (Ctrl/C), and paste it into you VI Front Panel.

P1.2. Resistor Voltage Divider with Current Measurement



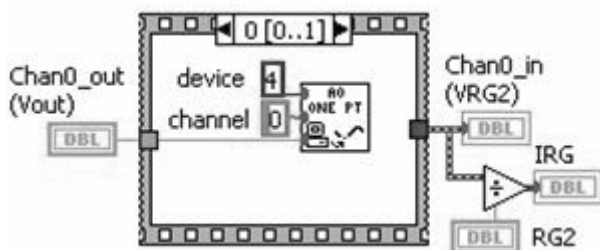
LabVIEW Computation

$$I_{R_G} = \frac{\text{Chan0_in}}{R_{G2}}$$



Programming Exercise 1.2

- Save a copy of VI_01.vi (your name) and give it a new name (such as #2).
- On the Front Panel, place a Digital Indicator for IRG (Controls>>Numeric).
- On the Front Panel, place a Digital Control for the resistor value. (Controls>>Numeric.)
- In the Diagram (below) place a Divide operation. (Functions>>Numeric>>Divide.)
- Wire the Diagram as in the example.



Procedure

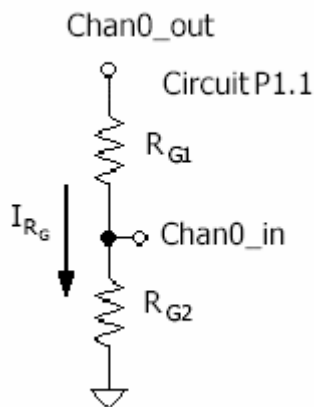
- Set your value of R_{G2} in the Digital Control ($M\Omega$). To set in three digits,

change the precision. Right Click on the Control, go to Format and Precision, and change Digits of Precision.

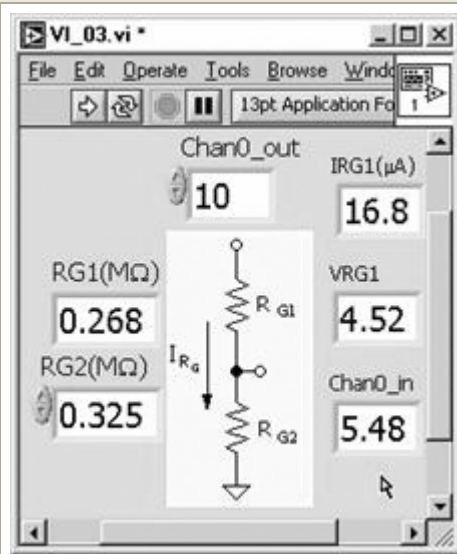


- Run VI_02.vi with Chan0_out set at 5 V and note the value of IRG. *Default and save* the Front Panel for comparison with the Mathcad evaluation file. For Default, menu Operate>>Make Current Values Default.

P1.3. Resistor Voltage Divider with Resistor Measurement



Basic sample shown below.



Programming Exercise 1.3

- Save a copy of **VI_02.vi** with a new name and add Digital Indicators for VRG1 and RG1 as in **VI_03.vi** (basic sample below). The new VI will find and indicate the value of RG1.

Functions>>



Waveform>>

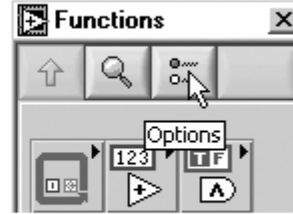
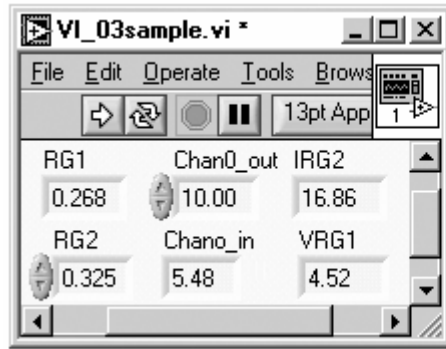


Waveform
Operations>>



Get Y Value.vi

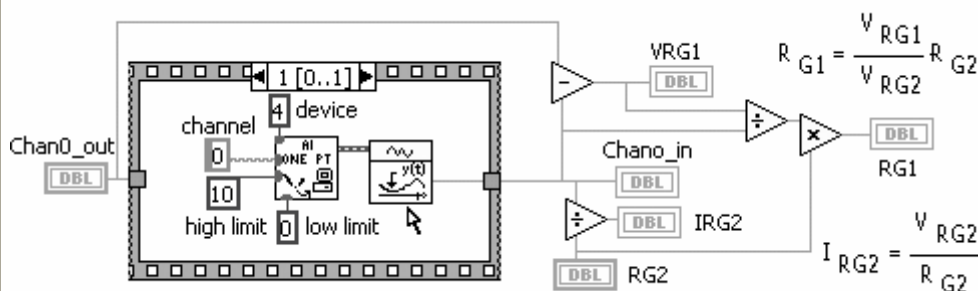
- In the Diagram, we will add **Get Y Value.vi**. This VI is found in the Functions Palette with the sequence shown on the left.
- Note that the full Palette Set is required. Your Palette may be an abbreviated form called Basic. If so, open the Palette and use the Stick Pin to keep it open. Then Click on Options as shown below. Select the default Palette Set.



- **Get Y Value.vi** is used to extract a given Y value in a waveform. AI Sample **Channel.vi** is a special case of a waveform with only one component. In the VI of part P1.2, we connected the output directly to a Digital Indicator, in which case, LabVIEW sorted out the component value automatically.
- Place **Get Y Value.vi** as shown in the Diagram (cursor, Arrow). Connect the balance of the circuit as shown. The new math formulations are indicated below and in the Diagram.

LabVIEW Computations

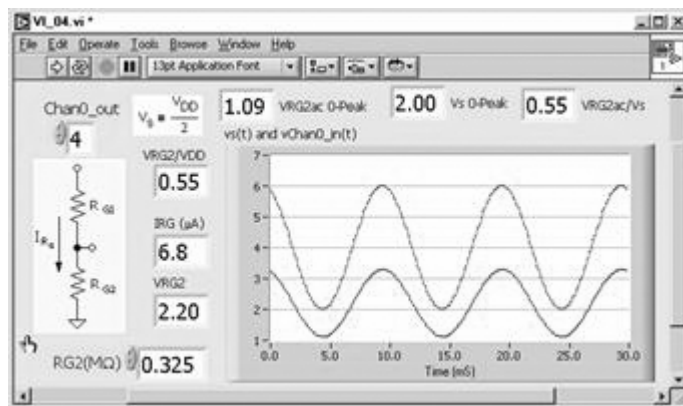
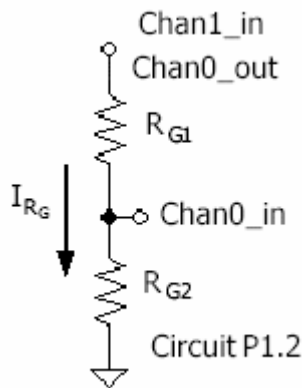
$$R_{G1} = \frac{V_{RG1}}{V_{RG2}} R_{G2} \quad V_{RG1} = \text{Chan0_out} - \text{Chan0_in}$$



Procedure

- Run the VI_03.vi for Chan0_out set at 10 V. Note the value of RG1 and VRG1. *Default and save the Front Panel for the Mathcad evaluation file.*

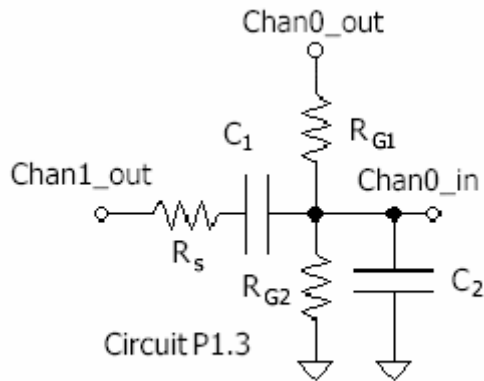
P1.4. Resistor Voltage Divider with a Sine-Wave Source Voltage



Procedure

- The VI sends out, on Chan0_out, a sine-wave superimposed on the dc value. Add the connection, Chan1_in, directly to the output channel. **Run** the VI for various values of Chan0_out. Note that the maximum allowed Chan0_out is about 6 V since the dc and ac peak must be less than 10 V. Verify that the results are consistent. For example, the peak ac values must be 1.5 of the dc values since $V_s = V_{DD}/2$. *Default and save* the Front Panel using Chan0_out = 4V.

P1.5. Frequency Response of a Resistor-Capacitor Circuit



Components

$$C_1 = C_2$$

$$f_{3dBhi} = 50f_{3dBlo}$$

$$R_s \approx R_G/50$$

$$f_{3dBlo} = 5 \text{ Hz (approximately)}$$

$$C_1 \approx \frac{1}{2\pi R_G f_{3dBlo}}$$

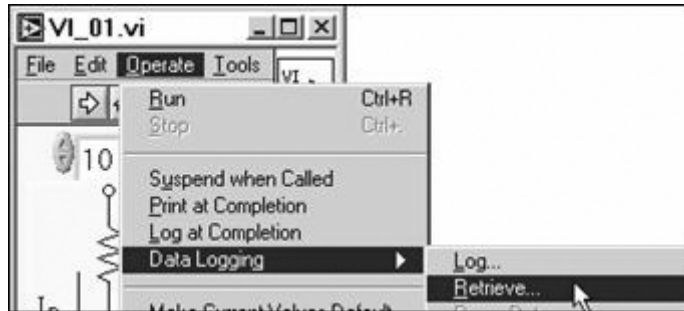
$$R_G = R_{G1} || R_{G2}$$

$$\text{Typical } C_1 \approx 0.5 \mu\text{F}$$

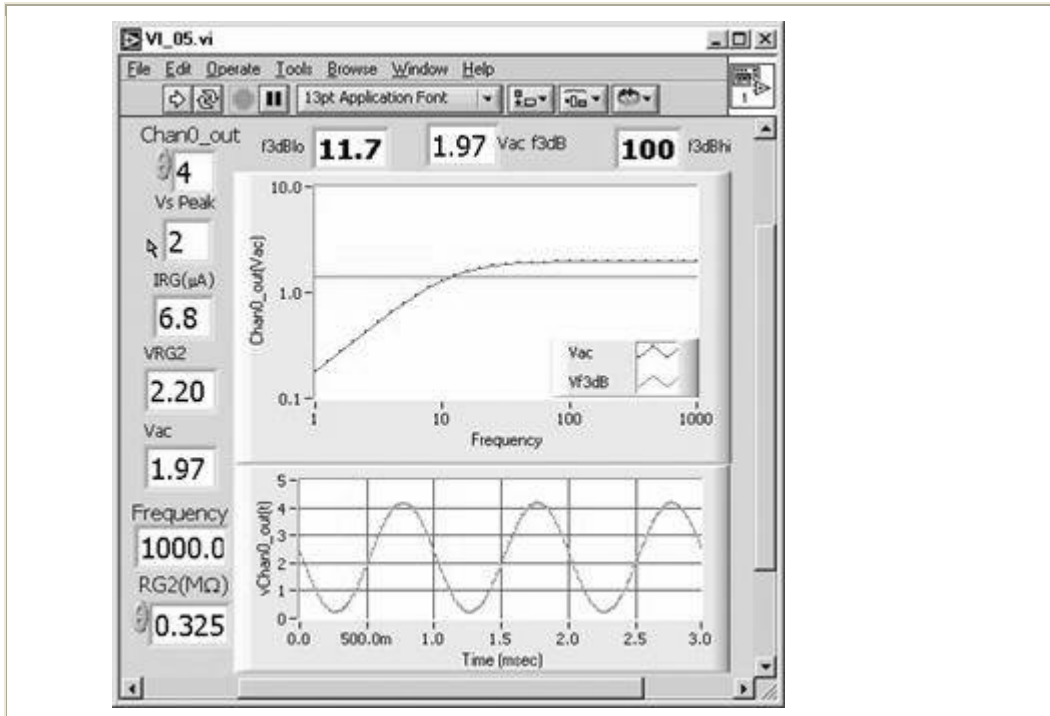
Procedure

- Configure the circuit for the low-end measurement and f_{3dBlo} . Install C_1 and do not install C_2 . **Run VI_05.vi** to obtain f_{3dBlo} . Obtain a Log of the Front Panel to save the results. To obtain a Log, go through menu sequence *Operate>>Data Logging>>Log*. To retrieve data: *Operate>>Data Logging>>Retrieve*.
- An example of *retrieving* a Log using **VI_01.vi** is shown here. Note that at the first Data Logging, you will be asked to name a Data Log file.

Select any name and Click Save, to install Log File in the Project folder.



- Now move the capacitor $C_2 = C_1$ to the C_2 location and install a large capacitor, $C_{1\text{new}}$, in place of C_1 , which satisfies $C_{1\text{new}} \gg C_2$ (e.g., $C_{1\text{new}} = 47 \mu\text{F}$). Note that the source side of the capacitor (connected to R_s) is more negative than the output side (Chan0_in). **Run VI_05.vi** to obtain $f_{3\text{dBhi}}$. *Default and save the Front Panel.* Note that when reopening the VI, the Front Panel will contain the information last defaulted from this $f_{3\text{dBhi}}$ measurement. The first measurement is in the data Log. Repeating, to retrieve the information from the data Log, go through menu sequence *Operate>>Data Logging>>Retrieve*. To then go back to the defaulted Front Panel, click OK and go through *Operate>>Reinitialize All to Default Values*.
- Note that if $C_2 = C_1$ are actually both in the circuit at the same time, the output in the plateau region is 1/2 as large (Exercise 1). This configuration is not implemented here. The use of $C_2 = C_1$ is only for convenience and $f_{3\text{dBlo}} \ll f_{3\text{dBhi}}$ is satisfied with $R_s \ll R_G$.



Laboratory Project 2. Basic NMOS Common-Source Amplifier with Programming Exercises

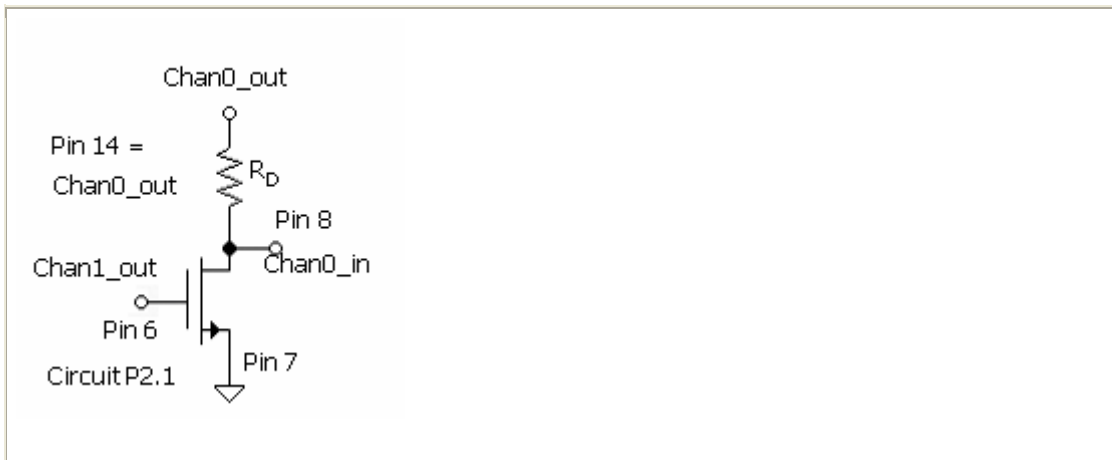
[P2.1 NMOS Common-Source Circuit with Drain Current Measurement](#)

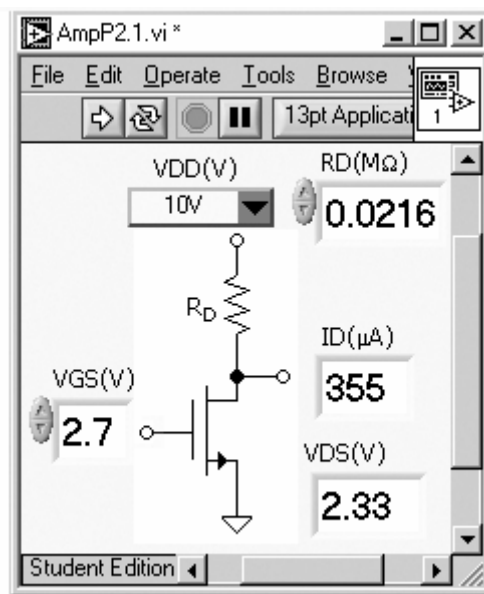
[P2.2 NMOS Common-Source Amplifier with Resistor Gate Bias Circuit](#)

[P2.3 Amplifier with Signal and Gain Measurement](#)

Exercises and Analysis Exercise02.mcd - Project02.mcd

P2.1. NMOS Common-Source Circuit with Drain Current Measurement

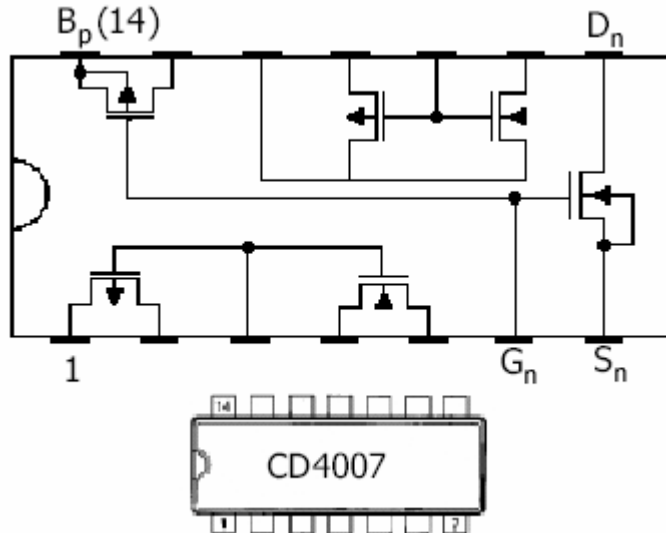




Chip Diagram

Connect pin 14 to Chan0_out.

Maintain Chan0_out > Chan1_out.



Components

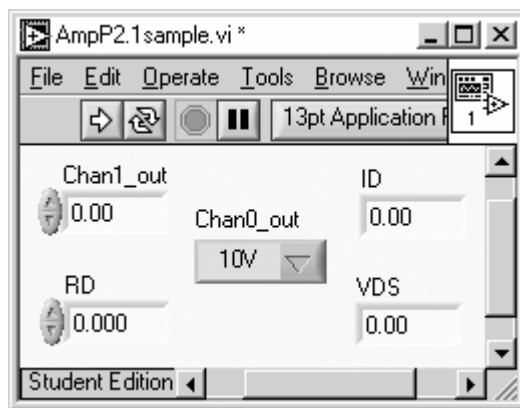
$$R_D \approx \frac{\text{Chan0_out}_{\text{max}}}{I_{D\text{max}}}$$

$$\text{Chan0_out}_{\text{max}} = 10\text{V}$$

$$I_{D\text{max}} \approx 500\mu\text{A}$$

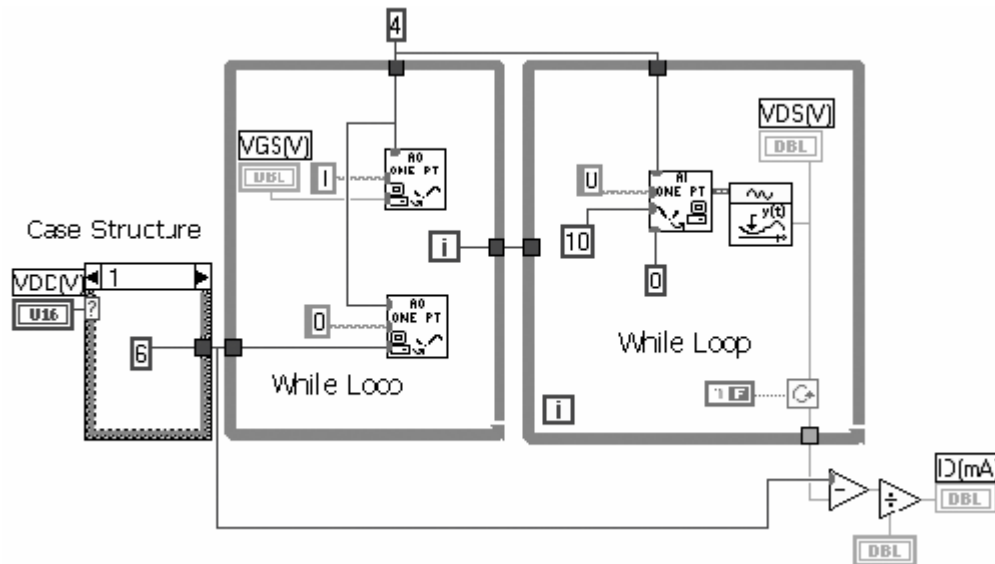
Programming Exercise 2.1

- On the Front Panel of a New VI, install two Digital Controls and two Digital Indicators (**AmpP2.1.vi**). Also install a Menu Ring (Controls>>Ring and Enum>>Menu Ring).



- In the Diagram (below), place, from left to right, one Case Structure: Functions>>Structures>>Case, and two While Loops:

Functions>>Structures>>While Loop.



- On the Front Panel, using the Text Tool (*Shift/Right Click* for Tool Palette) in the Menu Ring, type 10 V. *Right Click* on the Menu Ring and execute Add Item After. In the new listing, type 6 V. Go to the Diagram and wire the output of the Menu Ring to the "?" on the edge of the Case Structure. Note that the Items correspond to integers 0 and 1 at the terminal.
- In the Diagram, place two copies of **AO Update Channel.vi** in the first (left) While Loop and place **AI Sample Channel.vi** in the right-hand Loop. In the latter, also place **Get Y Value.vi** (Functions>>Waveform>>Waveform Operations>> **Get Y Value.vi**). Connect the constants and Front Panel terminals as in the example.
- Install constant values 10 and 6 in the 0 and 1 case states, respectively, and wire to the value terminal of the Chan0_out **AO Update Channel.vi** icon.
- Configure the current computation function as in the example (lower-right side in Diagram) and wire to the current Digital Indicator and resistor Digital Control. Optionally, relabel the Digital Controls and Digital Indicators according to their functions (as in **AmpP2.1.vi**).

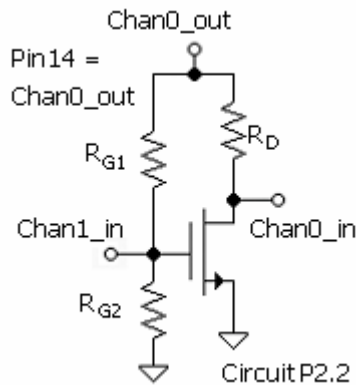
Procedure

- Connect the circuit using the R_D selected. Install the value of $R_D(M\Omega)$ in the Digital Control. To obtain three digits of precision in the resistor Digital Control, *Right Click* on the Control and go to Format and Precision... Set Digits of Precision.
- **Run AmpP2.1.vi** with VDD (Chan0_out) set at 10 V. Adjust VGS

(Chan1_out) for a drain voltage, VDS, of roughly 8 to 9 V. *Obtain a log* of the Front Panel to preserve the current (ID) and VGS information for the Mathcad file. Reminder: Data Logging is under the *Operate* menu.

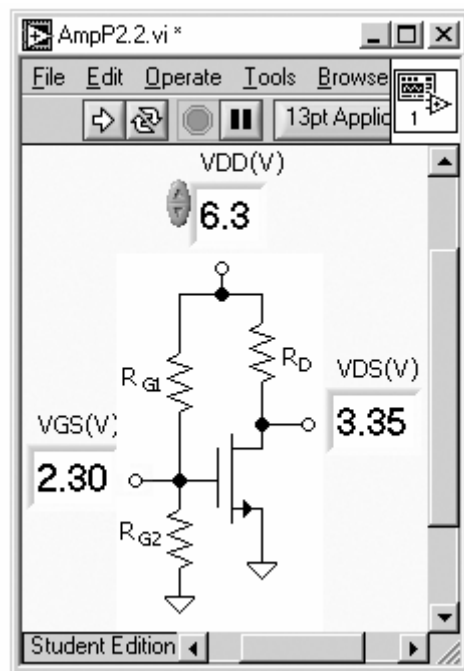
- Now reset VDD to 6 V and re-**run**. Note that VDS drops by roughly 4 V, indicating that the transistor drain terminal is a current source. Note that the change in the ID indicated is only slight.
- Reset VDD = 10 V. Increase VGS by an amount that makes VDS about 2 to 3 V. Note the increase in drain current. *Log* the Front Panel to preserve the information for the Mathcad file.

P2.2. NMOS Common-Source Amplifier with Resistor Gate Bias Circuit



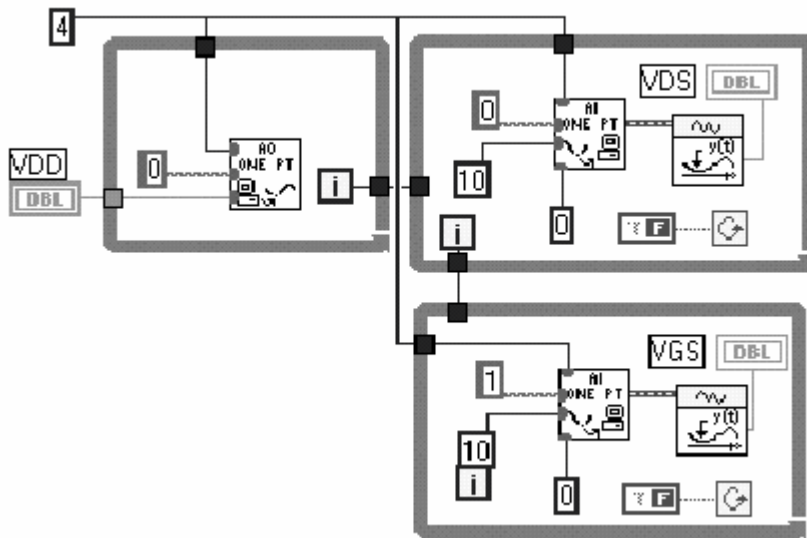
Components

$$R_{G1} \approx 2 R_{G2}$$



Programming Exercise 2.2

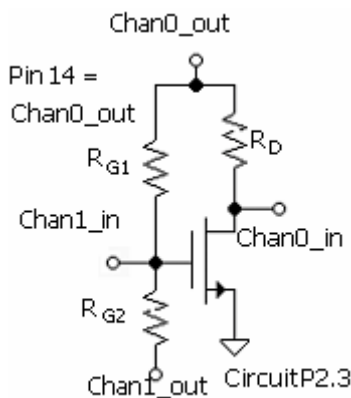
- Save a copy of **AmpP2.1.vi** (your name) and give it a new name.
- As in the example (below), use three While Loops to send Chan0_out and receive on Chan0_in and Chan1_in. Note that the index "i" of a loop on the left is connected to the border of the following Loop to establish the proper order of execution of the program.
- Provide Digital Indicators for VGS and VDS.



Procedure

- Run **AmpP2.2.vi** with VDD set at 6 V to obtain VGS of about 2 V (due to $R_{G1} \approx 2R_{G2}$). Rerun and adjust VDD to obtain VDS equal to about one-half of VDD. Note that decreasing VDD raises VDS toward VDD. Default and save the Front Panel for the Mathcad evaluation.

P2.3. Amplifier with Signal and Gain Measurement



Programming Exercise 2.3 follows below.

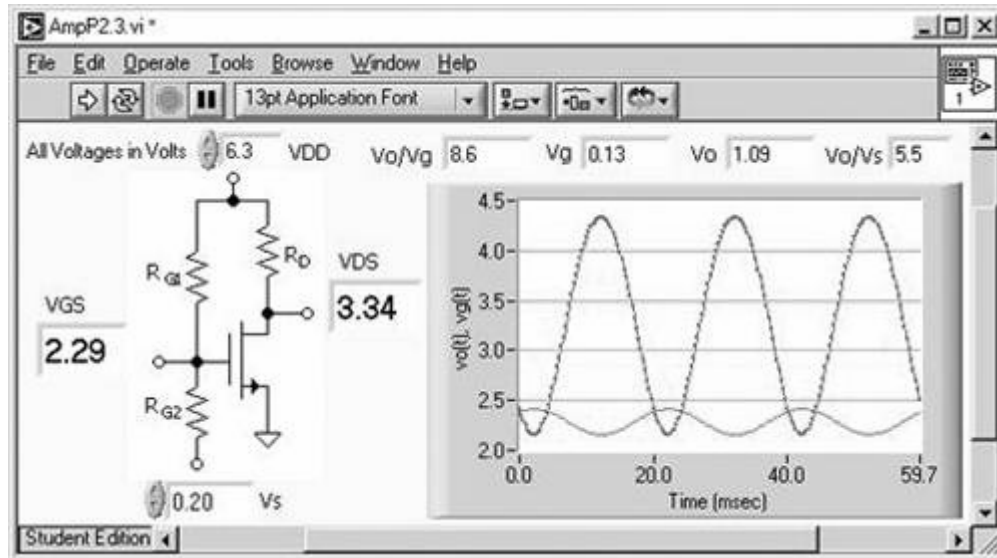
(Advanced-Optional)

Procedure

- Move the bottom of RG2 from ground to Chan1_out. The VI will send out a

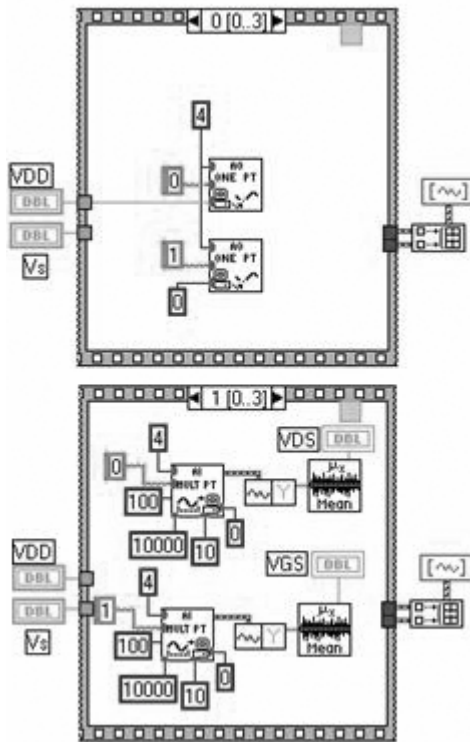
sine-wave signal via this channel with peak V_s . Set Chan0_out (VDD) as in the circuit for P2.2 above (for $V_{DS} \approx V_{DD}/2$).

- **Run AmpP2.3.vi (Project02.11b)** for various values of V_s . Re-set for a V_o peak of about 1V ($V_{DS}/3$). Note the gains, which are V_o/V_s and V_o/V_g . Verify that the gain from the source is about two-thirds of the gain the gate, based on the selection (ratio) of the gate bias resistors. *Default and save* the Front Panel for the Mathcad evaluation.



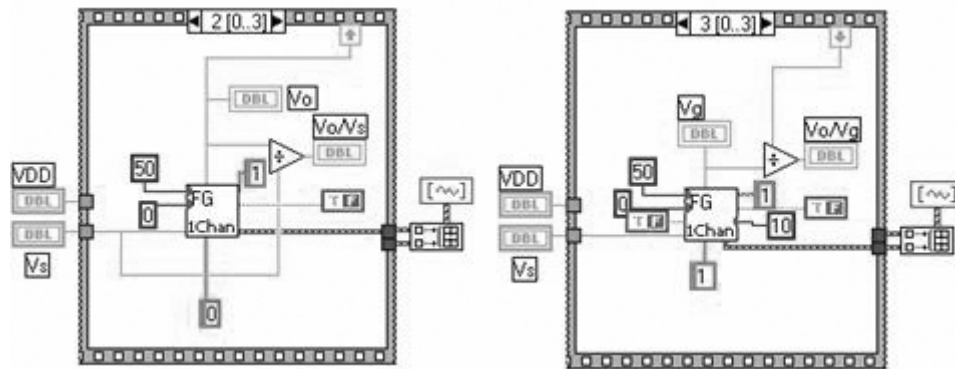
Programming Exercise 2.3

- Make a copy of **AmpP2.2.vi** with a new name.
- Go to the Diagram and delete all except Control and Indicator terminals.
- Install a Sequence Structure in the Diagram (below). In the first Frame, install **AO Update Channel.vi**. (Functions>>Data Acquisition>>Analog Output.) Press Ctrl and drag (with Arrow Tool) an additional copy, also in Frame 0.
- Connect a voltage-out (VDD, Chan0_out) value as shown in the example. Connect a numeric 0 (value) to set Chan1_out initially to 0 V for the dc measurements. Connect channel numbers (String) and device number (Numeric, 4 in the example).



- Add a Frame After. In this Frame (1), install two copies of **AI Acquire Waveform.vi**. (in menu Functions>>Data Acquisition>>Analog Input.) These will be used for the dc measurements. A given node voltage will be sampled 100 times and the samples will be averaged for the result. Connect constants to the icons as shown in the example. These include device (Numeric, 4, default), channel (String), number of samples (Numeric, 100), sample rate (Numeric, 10000), **high limit** (10 V) and **low limit** (0 V).
- Place two copies of **Get Waveform Components** in Frame 1 (Palette Functions>>Waveform>>Get Waveform Components). Then *Left Click* on the output (right side) of these and select "Y". Connect the inputs to the waveform outputs of **AI Acquire Waveform**.
- Connect the outputs of **Get Waveform Components** to the inputs of **Mean.vi** functions. These are located in Functions>>Mathematics>>Probability and Statistics or in Functions>>Analyze>>Mathematics>>Probability and Statistics. Connect the outputs of **the Mean.vi** icons to the terminals of the Digital Indicators of VDS (Chan0_in) and VGS (Chan1_in).
- Add a Frame After (2). In this Frame we place a general-purpose function generator and oscilloscope function. This is **FG1Chan.vi** and is in Functions>>User Libraries>>FuncGen. A Digital Control and a Digital Indicator are required for this Frame. Install Digital Control, Vs, and Digital

Indicator, Vo.



- Connect to **FG1Chan.vi**, the various terminals, constants, and strings. The connections include Frequency (Numeric, 50), Vacin (Vo), Chan_out (String, 1), Sinewave - SqWave (False), Graph Out (leave disconnected for now), Chan_in (String, 0), Vs (Digital Control), and VDCout (Numeric, 0).
- Now add another Frame after (3). In this Frame, install the same function, **FG1Chan.vi**. This can be copied and pasted from Frame 2. Add, in the Front Panel, an additional Digital Indicator, Vg. Make the connections the same as in Frame 2 except Chan_in (String, 1).
- In Frame 2, configure, using a divide function, Vo/Vs, as shown in the example. Add a Digital Indicator in the Front Panel with the label Vo/Vs and connect the output from the divide function to the terminal of this Digital Indicator.
- Click on the edge of Frame 2 and install Add Sequence Local and, to this, Connect Vacin (Vo). Move to Frame 3, install a Divide function, and configure Vo/Vg as in the example. Note that Vo comes from the Add Sequence Local from Frame 2. In the Front Panel, add an additional Digital Indicator to read Vo/Vg.
- Now add the graph in the Front Panel. (A detailed description of using a graph is given in [Section A.1.5.](#)) The graph for this case is a Waveform Graph. Get this in the Front Panel under Controls>>Graph>>Waveform Graph. Use the Coloring Tool to adjust the color of the graph background by *Right Clicking* on the graph with the Coloring Tool. *Right Click* on the sample trace in the Plot Legend to adjust the color of the trace. Using the Operating Tool, *Right Click* on the X Scale or Y Scale>>Formatting to set the Grid Options.
- Configure the Diagram for connecting to the graph. Outside the Sequence Structure and on the right, install a Build Array function. (Functions>>Array>>Build Array.) Connect the Plot output of **FG1Chan.vi**

in the two Frames, 2 and 3, to the two inputs of Build Array. Connect the output of Frame 2 on the top input.

Laboratory Project 3. Characterization of the PMOS Transistor for Circuit Simulation

[P3.1 SPICE Parameters and Pin Diagram](#)

[P3.2 SPICE Equations](#)

[P3.3 PMOS Transistor](#)

[P3.4 Low-Voltage Linear Region of the Output Characteristic](#)

[P3.5 PMOS Parameters from the Transfer Characteristic](#)

[P3.6 PMOS Lambda from the Transfer Characteristic](#)

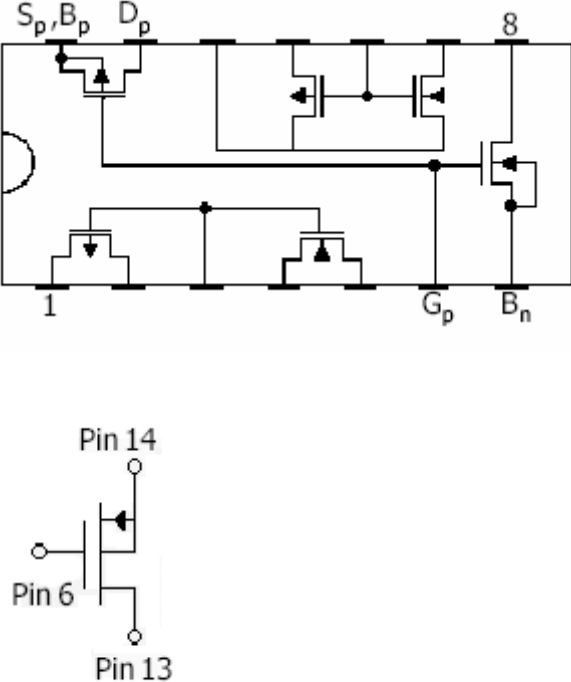
[P3.7 PMOS Output Characteristic](#)

[P3.8 PMOS Lambda](#)

Exercises and Analysis Exercise03.mcd - Project03.mcd

P3.1. SPICE Parameters and Pin Diagram

SPICE PARAMETERS		
SPICE Name	Math Symbol	Description
VTO	$V_{t_{no}}, V_{t_{po}}$	Zero VSB threshold voltage.
KP	$k_{n,p} = \frac{K_{P_{n,p}}}{2} \frac{W}{L}$	Transconductance parameter.
GAMMA	γ_n, γ_p	Threshold voltage parameter.
LAMBDA	λ_n, λ_p	Active-region slope parameter.
CD4007 Pin Diagram		

SPICE PARAMETERS		
SPICE Name	Math Symbol	Description
 <p>The top diagram shows a cross-section of a PMOS transistor. Labels include S_p, B_p for the source and bulk regions, D_p for the drain, G_p for the gate, and B_n for the n-well. The bottom diagram shows the pin configuration: Pin 14 is the gate, Pin 6 is the source, and Pin 13 is the drain.</p>		

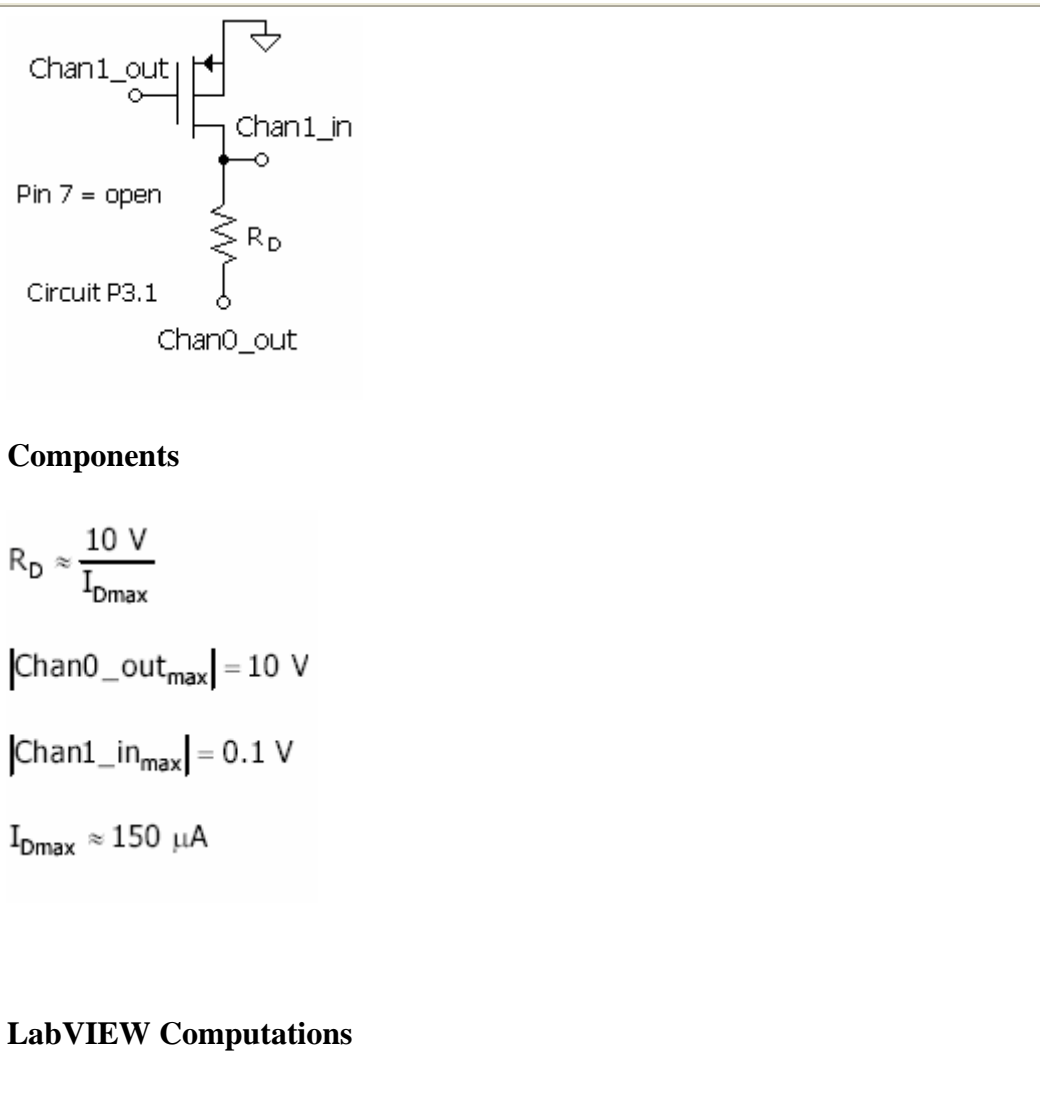
P3.2. SPICE Equations

SPICE Equation (PMOS)	NMOS Eq.	Description
$G_{\text{chan}} = \mu_p C_{\text{ox}} \frac{W}{L} (V_{\text{SG}} - V_{\text{tpo}}) = 2k_p V_{\text{effp}}$ $V_{\text{effp}} = (V_{\text{SG}} - V_{\text{tpo}})$	3.2	Output characteristic low-voltage conductance.
$I_D = 2k_p \left(V_{\text{effp}} V_{\text{SD}} - \frac{V_{\text{SD}}^2}{2} \right) (1 + \lambda_p V_{\text{SD}})$	3.10	Output characteristic for full linear range ($0 < V_{\text{SD}} < V_{\text{effp}}$).
$I_D = k_p V_{\text{effp}}^2 (1 + \lambda_p V_{\text{SD}})$	3.8	Active region transfer characteristic and output characteristic ($V_{\text{SD}} > V_{\text{effp}}$).
$V_{\text{SG}} = \sqrt{\frac{I_D}{k'_p}} + V_{\text{tpo}}$ $k'_p = k_p (1 + \lambda_p V_{\text{SD}})$	3.12	Active region relation solved for V_{SG} .

P3.3. PMOS Transistor

Use pins 6 (gate), 14 (source), and 13 (drain). Connect the NMOS body (pin 7) to the most negative node, Chan1_out, for PMOSparam.vi. Leave pin 7 disconnected when running PMOSoutput.vi. Note that the source and body for this transistor are internally connected. Note that Chan0_in is used for positive voltages only. Thus, it is always set automatically for the unipolar mode, which slightly improves the resolution. Chan1_in must measure positive and negative voltages.

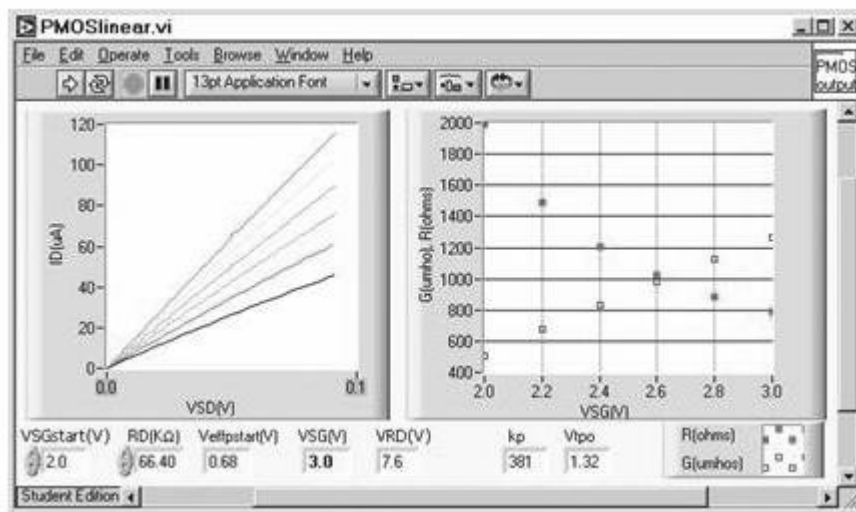
P3.4. Low-Voltage Linear Region of the Output Characteristic



$$k_p = \frac{\text{Slope}(G_{\text{chan}}, V_{\text{SG}})}{2}$$

$$V_{\text{effp}}(V_{\text{SG}}) = \frac{\text{Slope}(I_D, V_{\text{SD}})}{2k_p}$$

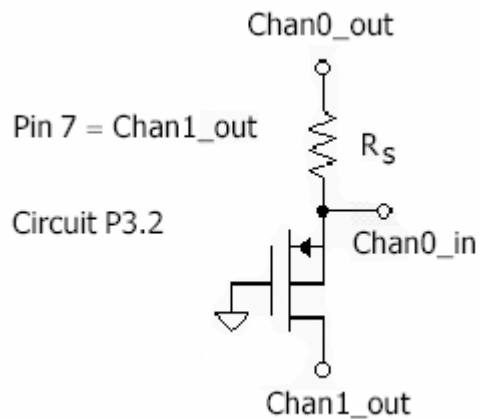
$$V_{\text{tpo}} = V_{\text{SG}} - V_{\text{effp}}(V_{\text{SG}})$$



Procedure

- **Run PMOSlinear.vi** and adjust $V_{\text{SGstart}}(\text{V})$ such that $V_{\text{effpstart}}(\text{V}) > 0.5 \text{ V}$. The maximum I_D is set for 0.1 V. Run the VI to obtain parameters as indicated in the Front Panel. *Default and save* the Front Panel to save information for the Mathcad analysis.

P3.5. PMOS Parameters from the Transfer Characteristic



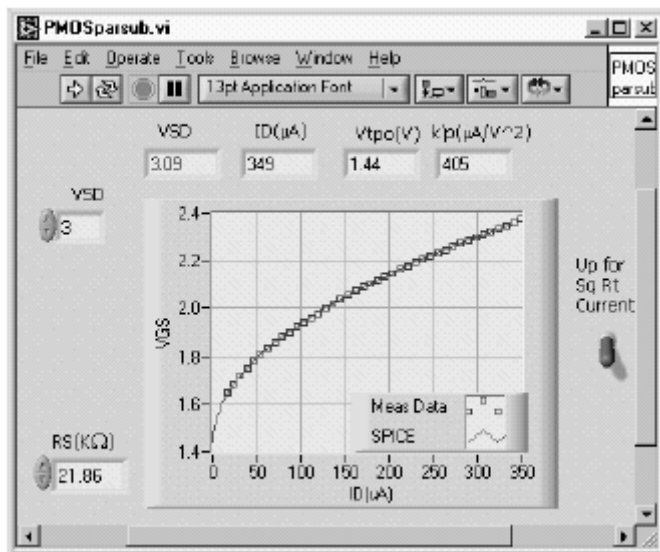
Components

$$R_S \approx \frac{|Chan0_out_{max}| - |Chan0_in_{min}|}{I_{Dmax}}$$

$$|Chan0_out_{max}| = 10 \text{ V}$$

$$|Chan0_in_{min}| = 3 \text{ V}$$

$$I_{Dmax} \approx 500 \text{ } \mu\text{A}$$



LabVIEW Computations

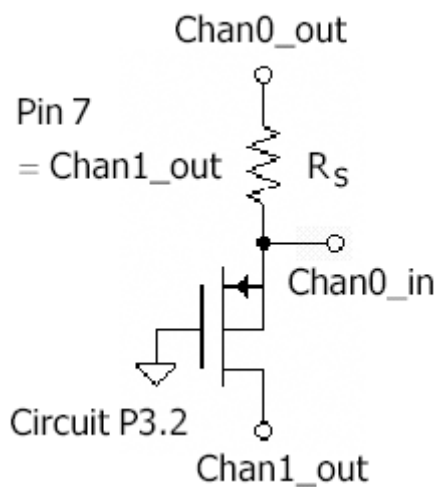
$$k_p' = \frac{1}{(\text{slope}(V_{SG}, \sqrt{I_D}))^2}$$

$$V_{tp0} = f(V_{SG}, \sqrt{I_D}) \Big|_{I_D=0}$$

Procedure

- **Run PMOSparsub.vi** with $V_{SD} = 3 \text{ V}$ and $V_{SD} = 9 \text{ V}$. V_{DD} (Chan0_out) starts at 2 V such that the minimum current is $I_{Dmin} = (2 \text{ V} - V_{SGmin})/R_S$. I_D stops automatically at $I_D = 500 \mu\text{A}$ or Chan0_out = 10 V . Verify that the VI functions properly. **PMOSparsub.vi** will run as a subVI of the next VI.

P3.6. PMOS Lambda from the Transfer Characteristic



Components

$$R_S \approx \frac{7 \text{ V}}{I_{D\max}}$$

$$|\text{Chan0_out}_{\max}| = 10 \text{ V}$$

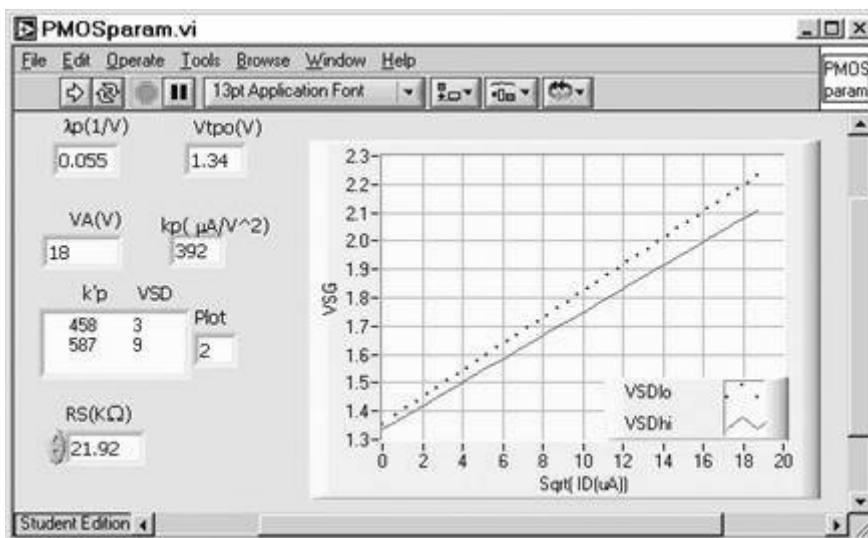
$$|\text{Chan0_in}_{\min}| = 3 \text{ V}$$

$$I_{D\max} \approx 500 \text{ } \mu\text{A}$$

LabVIEW Computations

$$\lambda_p = \frac{\frac{k'_p(9 \text{ V})}{k'_p(3 \text{ V})} - 1}{9 - \frac{k'_p(9 \text{ V})}{k'_p(3 \text{ V})} 3}$$

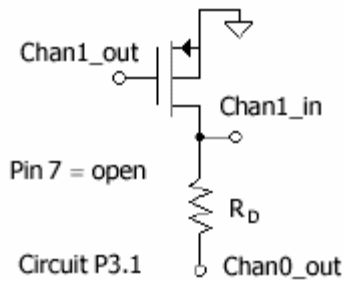
$$k_p = \frac{k'_p(9)}{1 + \lambda_p 9}$$



Procedure

- **Run PMOSparam.vi to run subVI PMOSparsub.vi** at $V_{SD} = 3\text{ V}$ and $V_{SD} = 9\text{ V}$ to resolve λ and final values of k_p and V_{tp0} . *Default and save* the Front Panel to save the parameter values for the Mathcad results analysis file.

P3.7. PMOS Output Characteristic

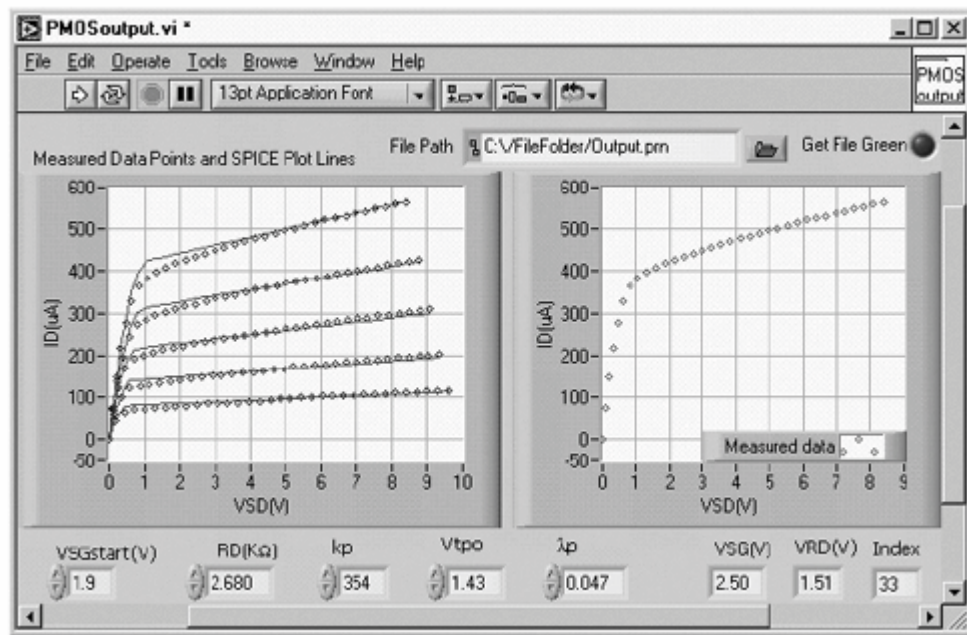


Components

$$R_D \approx \frac{|Chan0_out_{max}| - |Chan1_in|}{I_{Dmax}}$$

$$|Chan0_out_{max}| = 10\text{ V}$$

$$|Chan1_in| \approx 5\text{ V at } I_{Dmax} \approx 500\text{ }\mu\text{A}$$



LabVIEW Computations

Linear Region

$$I_D = 2k_p \left(V_{\text{effp}} V_{SD} - \frac{V_{SD}^2}{2} \right) (1 + \lambda_p V_{SD})$$

Active Region

$$I_D = k_p V_{\text{effp}}^2 (1 + \lambda_p V_{SD})$$

Procedure

- Install PMOS parameters from P3.6 into the Front Panel of **PMOSoutput.vi**. A given VSD sweep will halt at Chan0_out = 10 V. Set VSGstart(V) as necessary to obtain $I_D \approx 100 \mu\text{A}$ for VSD in the active region. Verify that the measured data and SPICE calculation agree reasonably well. Write in you file path and *save a data file* for Mathcad. The file will contain the data points for the highest current output characteristic. *Default Front Panel and save*. These data will be used in the Mathcad analysis file.
- With the data defaulted and saved in the graph of the VI, a data file can be obtained at a later time. From the Diagram of **PMOSoutput.vi**, click on **XYtoDataFile.vi**, copy the data from **PMOSoutput.vi**, and paste it

into the Control Graph of **XYtoDataFile.vi**. Name the path and data file and run the VI. **XYtoDataFile.vi** is located in the User.lib in the LabVIEW folder. Therefore, it can also be opened from the Diagram of a VI under Functions>>User Libraries>>Dat_File.

P3.8. PMOS Lambda

LabVIEW Computations

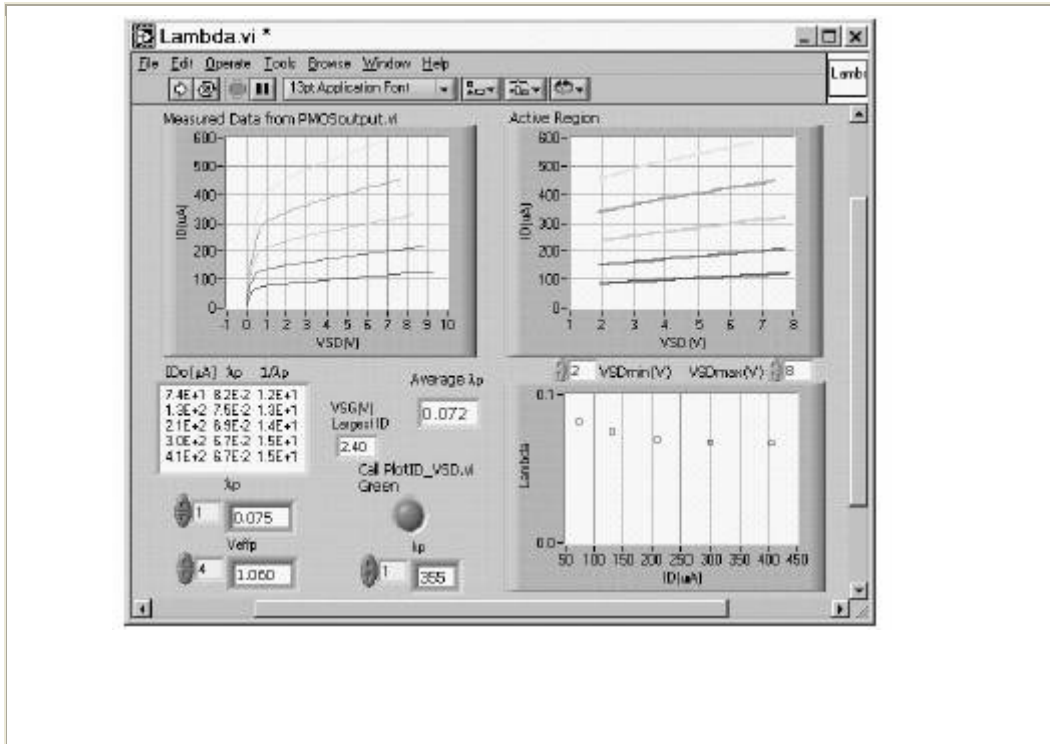
Straight-line curve fit from active region data:

$$I_D = \text{Slope} \cdot V_{SD} + I_{D0}$$

$$\lambda_p = \text{Slope}/I_{D0} \quad k_p = I_{D0}/V_{\text{effp}}^2$$

Procedure

- **Lambda.vi** is configured for Enable Data Base Access to read the data from **PMOSOutput.vi**. **Lambda.vi** can read all of the Front Panel information in **PMOSOutput.vi** when a log of the Front Panel has been obtained.
- From the Diagram of **Lambda.vi**, open **PMOSOutput.vi**. Obtain a data log of the Front Panel of the **PMOSOutput.vi** (under Operate Menu).
- **Run Lambda.vi** to read the graph of measured data (from **PMOSOutput.vi**) and to plot the measured data from the active region. The VI obtains K_p and λ_p from these data, for each of the voltage sweeps. Note that λ varies. Compare the average λ value with that from **PMOSparam.vi**.
- Click on **Call PlotID_VSD.vi** Green to bring up a plot of the active-region equation plot for $-1/\lambda_p < V_{SD} < V_{SD\text{max}}$, compared with the measured data. **Lambda.vi** will be used in the Mathcad analysis file.



Laboratory Project 4. Characterization of the NMOS Transistor for Circuit Simulation

[P4.1 SPICE Parameters and Chip Diagram](#)

[P4.2 NMOS Transistor](#)

[P4.3 SPICE Equations](#)

[P4.4 NMOS Parameters from the Transfer Characteristic](#)

[P4.5 NMOS Lambda from the Transfer Characteristic](#)

[P4.6 NMOS Gamma SubVI](#)

[P4.7 NMOS Gamma](#)

[P4.8 NMOS Circuit with Body Effect](#)

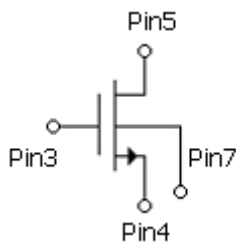
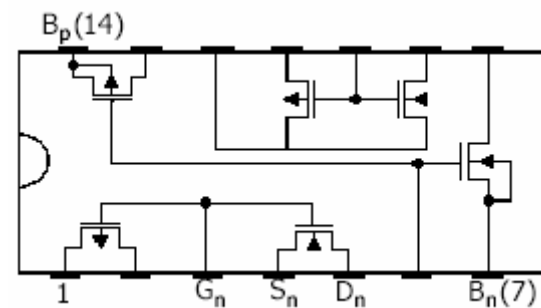
Exercises and Analysis Exercise04.mcd - Project04.mcd

P4.1. SPICE Parameters and Chip Diagram

SPICE PARAMETERS

SPICE Name	Math Symbol	Description
VTO	V_{tno}, V_{tpo}	Zero VSB threshold voltage.
KP	$k_{n,p} = \frac{KP_{n,p}}{2} \frac{W}{L}$	Transconductance parameter.
GAMMA	γ_n, γ_p	Threshold voltage parameter.
LAMBDA	λ_n, λ_p	Active-region slope parameter.

CD4007 Pin Diagram



P4.2. NMOS Transistor

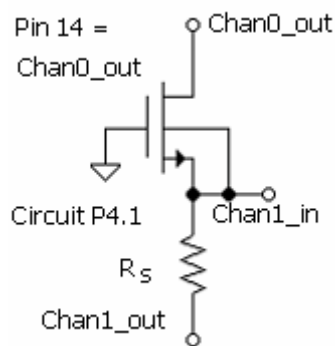
Use CD4007 pins 3 (gate), 4 (source) and 5 (drain). Connect pin 4 (source) to pin 7 (body). Connect PMOS body (pin 14) to the most positive node in the NMOS measurements, Chan0_out or ground, depending on the circuit. In the γ_n measurement (NMOSgamma.vi) and the transfer characteristic measurement (NMOS_Cir.vi), pin 14 is connected to ground. Note that the NMOS of pins 6, 7, and 8 has internally connected body and source. This transistor is used in a following project.

P4.3. SPICE Equations

SPICE Equation		Description
$I_D = k_n V_{effn}^2 (1 + \lambda_n V_{DS})$ $V_{effn} = (V_{GS} - V_{tno})$	3.8	Active region transfer characteristic and output characteristic ($V_{DS} > V_{effn}$, $V_{SB} = 0$).

SPICE Equation		Description
$V_{GS} = \sqrt{\frac{I_D}{k'_n}} + V_{tno}$ $k'_n = k_n (1 + \lambda_n V_{DS})$	3.12	Active-region equation solved for V_{SG} .
$V_{tn} = V_{tno} + \gamma_n (\sqrt{V_{SB} + 2\phi_F} - \sqrt{2\phi_F})$	3.14	Threshold voltage dependence on V_{SB} .
$X = \sqrt{V_{SB} + 2\phi_F} - \sqrt{2\phi_F}$		X variable for V_{tn} versus X plot in γ_n determination.

P4.4. NMOS Parameters from the Transfer Characteristic



LabVIEW Computations

$$k'_n = \frac{1}{[\text{slope}(V_{GS}, \sqrt{I_D})]^2}$$

$$V_{tno} = f(V_{GS}, \sqrt{I_D}) \Big|_{I_D=0}$$

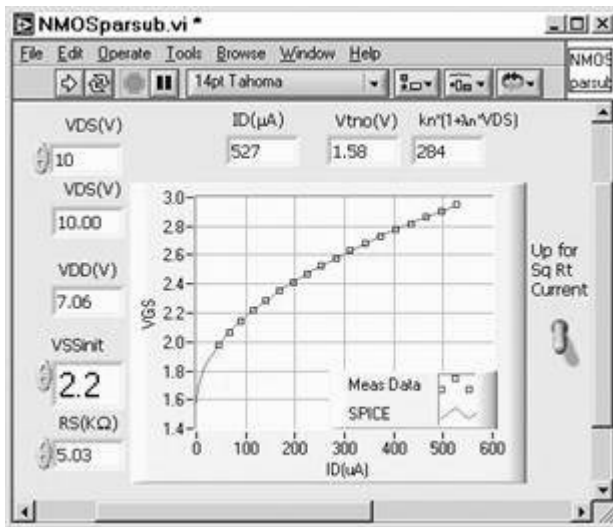
Components

$$R_S \approx \frac{7 \text{ V}}{I_{D\max}}$$

$$|\text{Chan1_out}_{\max}| = 10 \text{ V}$$

$$|\text{Chan1_in}_{\max}| = V_{GS\max}$$

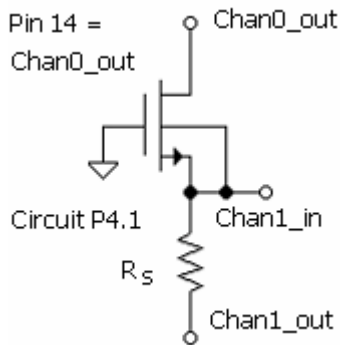
$$I_{D\max} \approx 500 \text{ } \mu\text{A}$$



Procedure

- Set RS value. **Run NMOSparsub.vi** and adjust VSSinit for a minimum ID $\approx 50 \text{ } \mu\text{A}$. Test run **NMOSparsub.vi** with VDS = 3 V and 10 V. Execution halts at ID $\approx 500 \text{ } \mu\text{A}$. The VI runs as a subVI in the next part. *Default and save.* Saved VSSinit is used in the Top VI in the next part.

P4.5. NMOS Lambda from the Transfer Characteristic



Components

$$R_S \approx \frac{7 \text{ V}}{I_{D\max}}$$

$$|\text{Chan1_out}_{\max}| = 10 \text{ V}$$

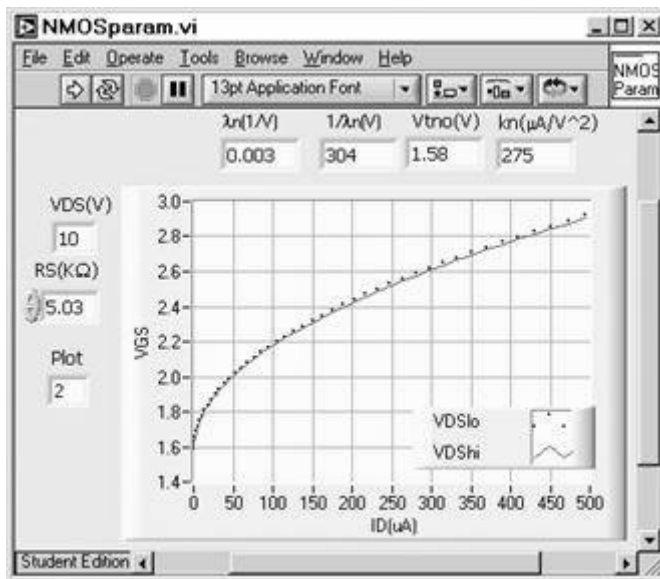
$$|\text{Chan1_in}_{\max}| = V_{GS\max}$$

$$I_{D\max} \approx 500 \text{ } \mu\text{A}$$

LabVIEW Computations

$$\lambda_n = \frac{\frac{k'_n(10 \text{ V})}{k'_n(3 \text{ V})} - 1}{10 - \frac{k'_n(10 \text{ V})}{k'_n(3 \text{ V})} 3}$$

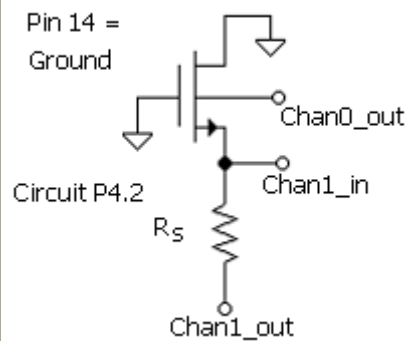
$$k_n = \frac{k'_n(10)}{1 + \lambda_p 10}$$



Procedure

- Set R_S . Run **NMOSparam.vi** to obtain a set of NMOS parameters. The VI runs **NMOSparsub.vi** automatically with $V_{DS} = 3\text{ V}$ and $V_{DS} = 10\text{ V}$. The sweep stops at $I_D \approx 500\mu A$ or $|Chan0_out| = 10\text{ V}$. *Default and save* the VI.

P4.6. NMOS Gamma SubVI



Components

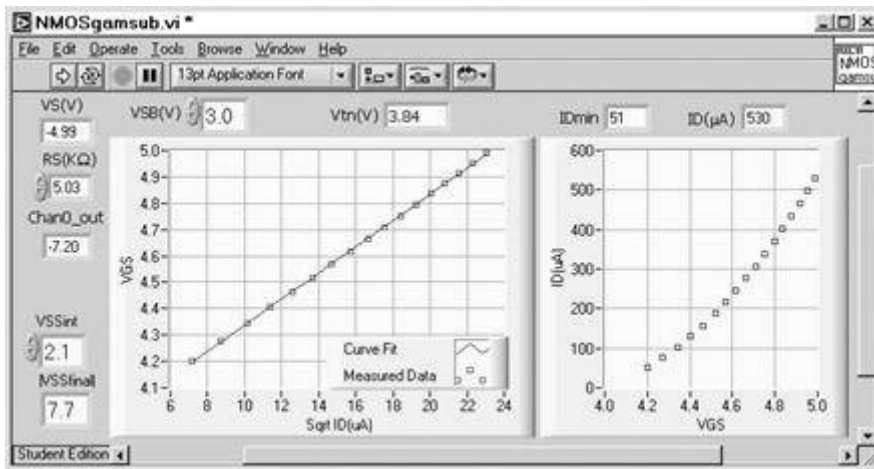
$$R_S \approx \frac{|Chan1_out_{max}| - |Chan1_in_{max}|}{I_{Dmax}}$$

$$|Chan1_out_{max}| = 10 \text{ V}$$

$$|Chan1_in_{min}| \approx 5 \text{ V}$$

$$I_{Dmax} \approx 500 \text{ } \mu\text{A}$$

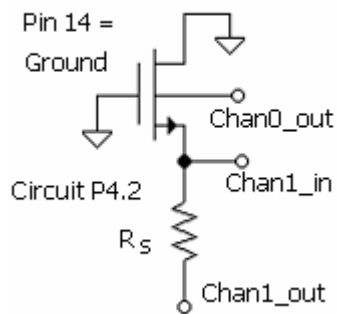
Note: V_{GS} will be larger with the body effect.



Procedure

- Note that PMOS body pin 14 and the NMOS drain terminal are at ground. Set R_S and V_{SSinit} from **NMOSparsub.vi**. Run **NMOSgamsub.vi** with $V_{SB} = 0 \text{ V}$ and 3 V to verify that circuit is functioning properly. Note the change in the V_{GS} range for the larger V_{SB} . V_{SSinit} is adjusted automatically for larger values of V_{SB} , to obtain, approximately, a starting I_D of about $50 \text{ } \mu\text{A}$. *Default and save* the Front Panel to save V_{SSinit} .

P4.7. NMOS Gamma



Components

$$R_S \approx \frac{5 \text{ V}}{I_{D\max}}$$

$$|\text{Chan1_out}_{\max}| = 10 \text{ V}$$

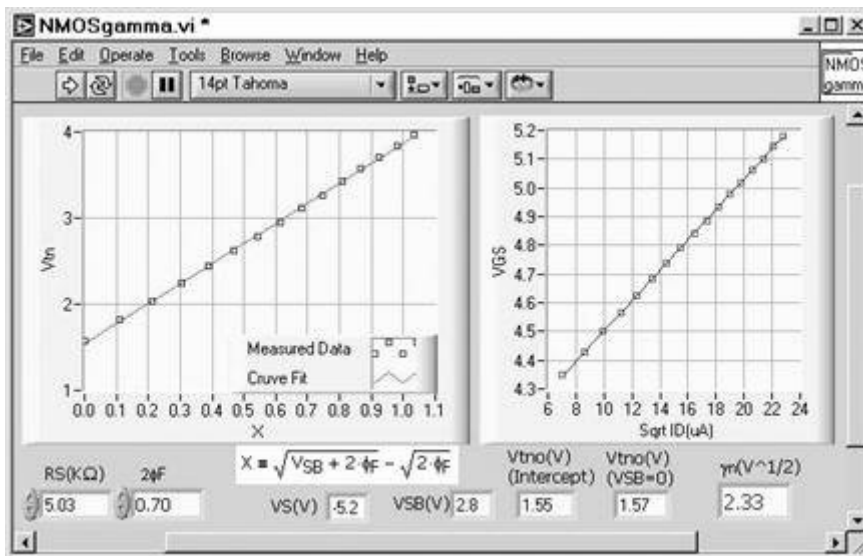
$$|\text{Chan1_in}_{\max}| \approx 5 \text{ V}$$

$$I_{D\max} \approx 500 \text{ } \mu\text{A}$$

LabVIEW Computations

$$X = \sqrt{V_{SB} + 2\phi_F} - \sqrt{2\phi_F}$$

$$\gamma_n = \text{Slope}(f(V_{tn}, X))$$



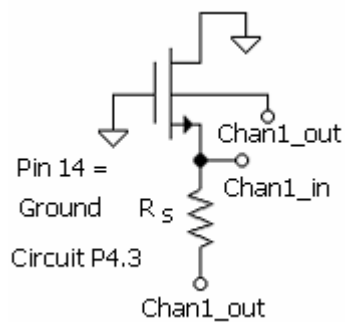
Procedure

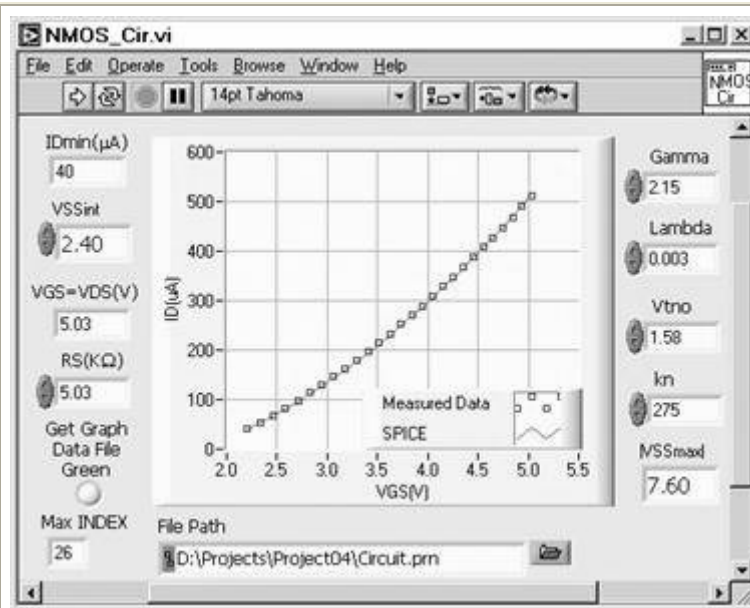
- Set R_S . With **NMOSgamsub.vi** open, run **NMOSgamma.vi** to obtain Gamma. Adjust $2\phi_F$ to try to match V_{th0} (intercept, curve fit) with V_{th0} ($V_{SB} = 0$, transfer characteristic, graph on right). Save the results.

P4.8. NMOS Circuit with Body Effect

Components

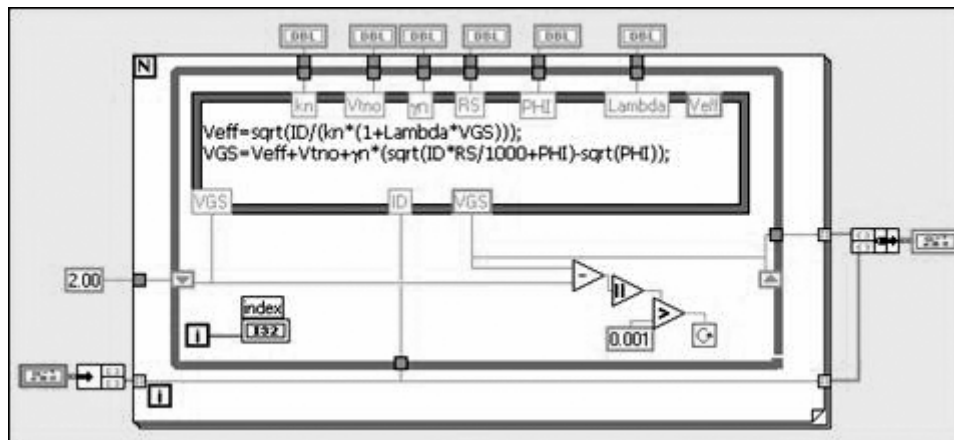
Use R_S from **NMOSgamma.vi**.





Procedure

- Set RS. Install SPICE parameters from **NMOSparam.vi** and **NMOSgamma.vi**. Run **NMOS_Cir.vi** to obtain ID versus VGS with a sweep of VSS. The iteration solution obtained in **SPICEgamma.vi** (Diagram below) is compared in the graph of **NMOS_Cir.vi**. Adjust VSSinit to obtain IDmin of about 50 μ A. Re-adjust Gamma for best fit. Then *save a data file* with your data file name. The data file must be in the folder with the Mathcad file. The file name must have a *.prn extension.



- A file can be obtained from the graph of the defaulted and saved VI using XY1toDataFile.vi. The **VI XY1toDataFile.vi** is located in \\LabVIEW 6\User.lib\Dat_Files\Dat.llb.
- Note that **XY1toDataFile.vi** is different from **XYtoDataFile2.vi**, as used to obtain a data file in the Diagram of **NMOS_Cir.vi**. The Control Graph

of **XYtoDataFile2.vi** accepts single plots while **XY1toDataFile.vi** is for graphs with two Y functions (with only one Y function in the data file). For convenience, a copy of **XY1toDataFile.vi**, **GetDataFile.vi**, is included in the Project04.llb. For clarification, **XY1Y2toDataFile.vi** accepts graph data with two Y functions and includes both in the data file.

Laboratory Project 5. PMOS Common-Source Amplifier

[P5.1 SPICE Equations and Pin Diagram](#)

[P5.2 PMOS Common-Source Amplifier DC Setup](#)

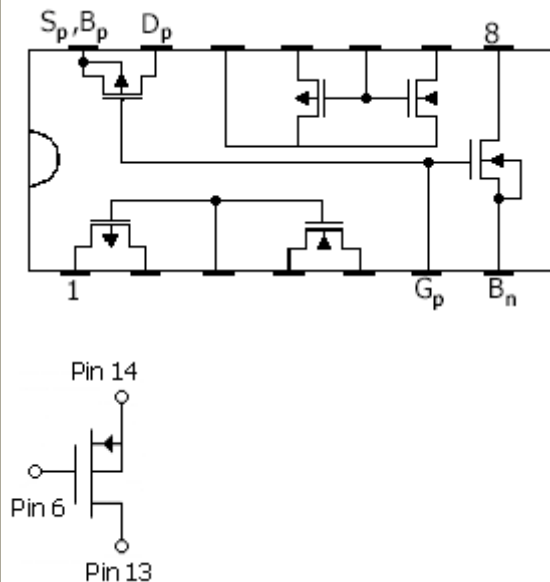
[P5.3 Amplifier Gain at One Bias Current](#)

[P5.4 Amplifier Gain versus Bias Current](#)

Exercises and Analysis Exercise05.mcd - Project05.mcd

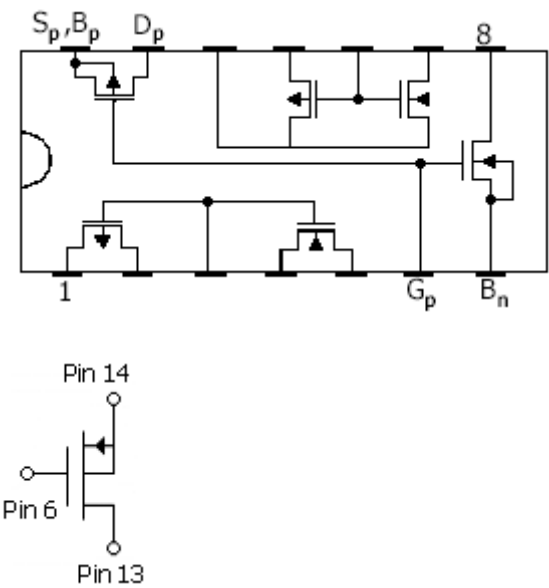
P5.1. SPICE Equations and Pin Diagram

CD4007 Pin Diagram



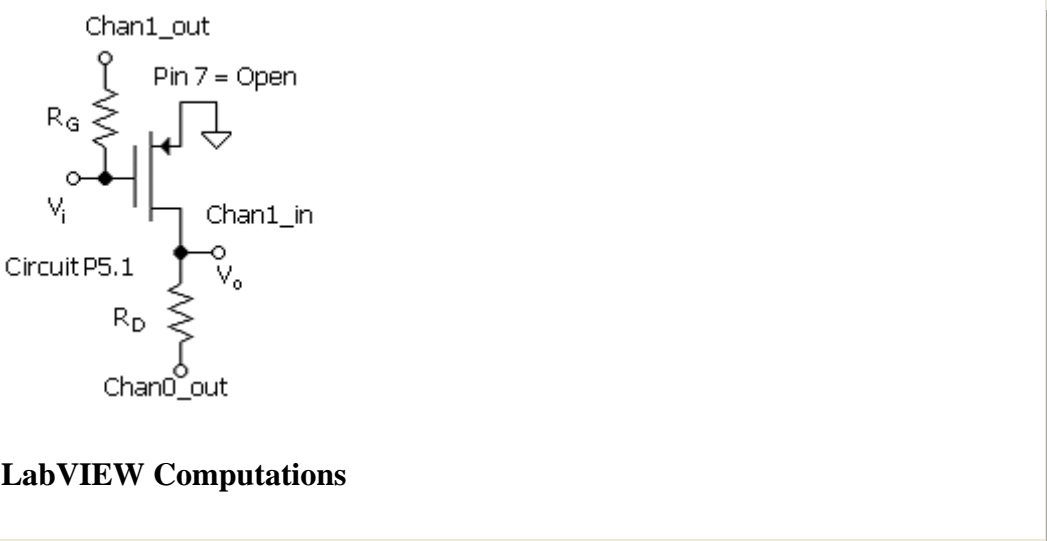
SPICE Equation		Description
$I_D = k_p V_{effp}^2 (1 + \lambda_p V_{SD})$	3.8 (NMOS)	DC drain current voltage relation (PMOS).

CD4007 Pin Diagram



SPICE Equation		Description
$a_{VO} = -2\sqrt{k_p I_D} \frac{R_D}{1 + \lambda_p I_D R_D}$	5.6	Gain versus I_D .
$a_v = -2 \frac{I_D}{V_{effp}} \frac{R_D}{1 + \lambda_p I_D R_D}$	5.7	Gain versus I_D , V_{effp} .

P5.2. PMOS Common-Source Amplifier DC Setup



$$I_D = \frac{\text{Chan1_in} - \text{Chan0_out}}{R_D}$$

$$V_D = V_{SS} - I_D R_D$$

$$V_{SS} = \text{Chan0_out}$$

Components

$$R_D \approx \frac{6 \text{ V}}{I_{D\max}}$$

$$|\text{Chan0_out}_{\max}| = 10 \text{ V}$$

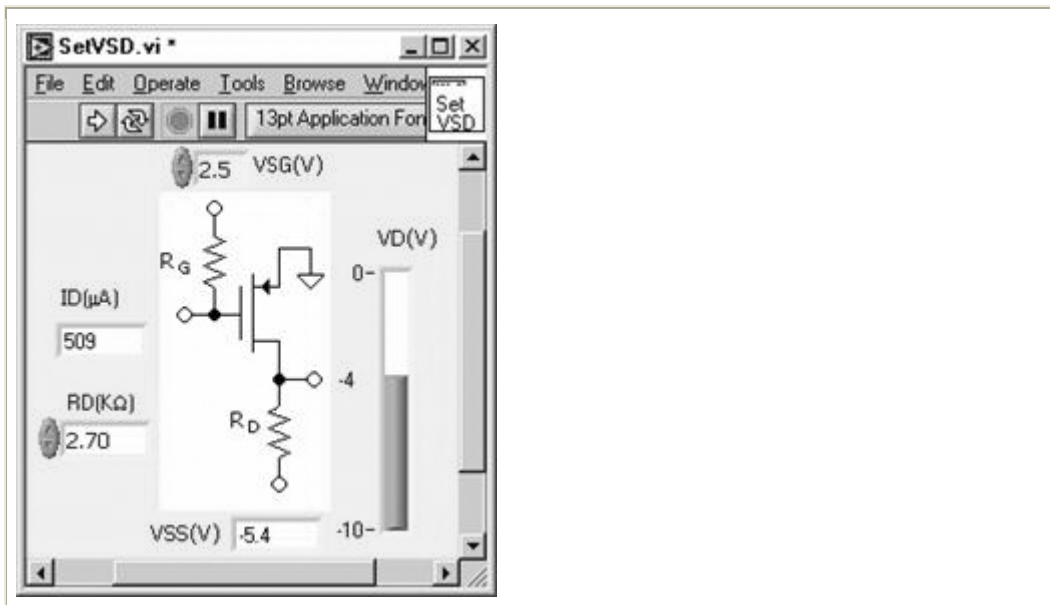
$$|\text{Chan1_in}| = 4 \text{ V}$$

$$I_{D\max} \approx 500 \text{ } \mu\text{A}$$

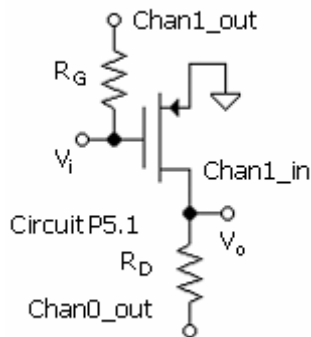
$$R_G > 500 \text{ k}\Omega$$

Procedure

- Install RD value. Adjust VSG in **SetVSD.vi** and **run** the VI to find the value for $I_{D\max} = 500 \text{ } \mu\text{A} \pm 50 \text{ } \mu\text{A}$. VSS is ramped up from 10 V (default value) to find the value for $V_D = -4 \text{ V}$. Note that if RD is too large, VD will be $> -4 \text{ V}$ initially (for $V_{SS} = -10 \text{ V}$) at $I_{D\max}$. *Default and save* the Front Panel.



P5.3. Amplifier Gain at One Bias Current



LabVIEW Computations

$$a_{vi} = \frac{\text{Chan1_in}_{acPeak}}{VSG/20}$$

Components

Same as **SetVSD.vi**

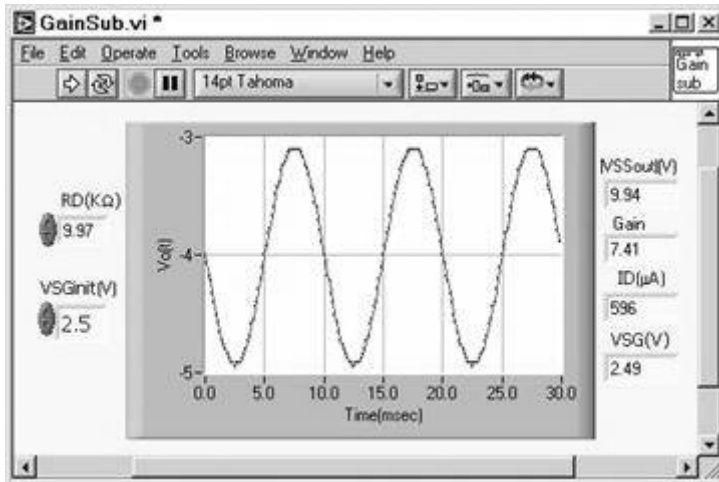
SubVI's

SetVSD.vi (Set Bias)

FG1Chan.vi (Function Generator)

SR.vi (Send-Receive Function)

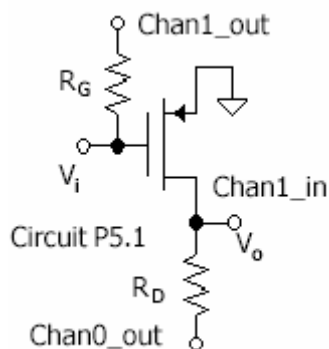
Oscilloscope.vi (Chan1_in Read Waveform)



Procedure

- Sub VI **FG1Chan.vi** sends out the bias VSG and, in series, a sine-wave signal equal to $V_{SG}/20$. Set VSGinit as determined with SetVSD.vi as the VSG required to obtain $I_D \approx 500 \mu A$. This is the initial VSG in the sweep of the next part. **Run** the VI to obtain the gain for this bias current. Obtain a *data log* of the Front Panel. *Default and save*. **Re-run** with a new $VSG_{new} = VSG - 0.5 V$ to obtain the gain at a lower bias current. Obtain a *data log*.

P5.4. Amplifier Gain versus Bias Current



LabVIEW Computations

$$a_{vi} = \frac{\text{Chan1_in}_{acPeak}}{VSG/20}$$

Components

Same as **SetVSD.vi**

Sub VI's

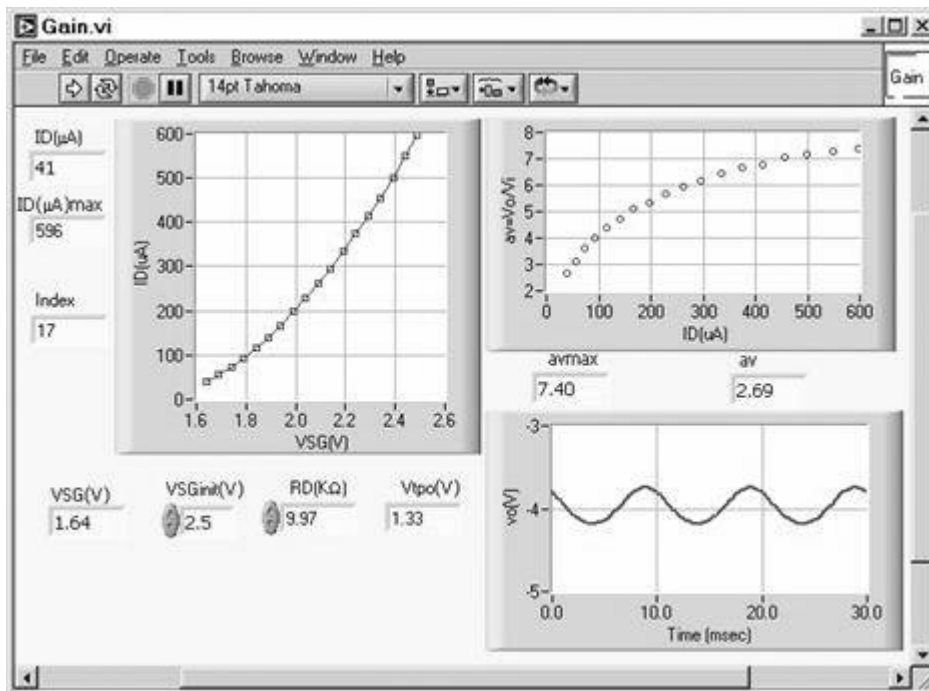
GainSub.vi

SetVSD.vi (Set Bias)

FGsine.vi (Function Generator)

SR.vi (Send-Receive Function)

Oscilloscope.vi (Chan1_in Read Waveform)



Procedure

- **Run Gain.vi to run GainSub.vi** over a range of currents. Set VSGinit(V) to the value obtained above for $I_D \approx 500 \mu\text{A}$ (defaulted in **SetVSD.vi**). The VI automatically halts (decreasing) at $I_D \approx 50 \mu\text{A}$. *Default and save the Front Panel. Now obtain a data file as follows.*
- Open **GraphToDataFile.vi** (copy of **XYtoDataFile.vi**) in **Project05.llb**. Copy the data from the graph ID versus VSG of **Gain.vi**. (*Right Click* on the graph, then Data Operations>>Copy Data.) Then paste the data into the Control Graph of **GraphToDataFile.vi**. (*Right Click* on the graph and then Data Operations>>Paste Data.) Type the data file path and name the data file. **Run** the VI to obtain a data file to be read by the Mathcad evaluation file. Repeat for the gain plot, a_v versus ID. Recall: The data files must be in the folder with the Mathcad file. The file name must have a *.prn extension.

Laboratory Project 6. PMOS Common-Source Amplifier Stage with Current-Source Biasing

[P6.1 PMOS Schematic and Pin Diagram](#)

[P6.2 SPICE PMOS and Circuit Equations](#)

[P6.3 PMOS Current-Source Amplifier DC Setup](#)

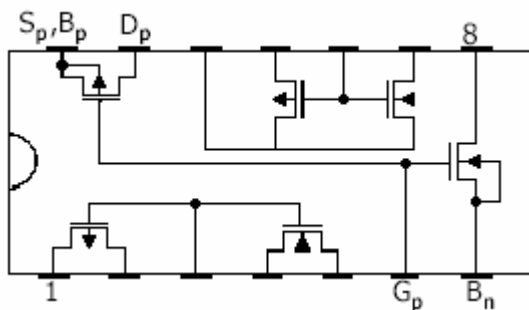
[P6.4 Amplifier Gain](#)

[P6.5 Amplifier Frequency Response](#)

Exercises and Analysis Exercise06.mcd - Project06.mcd

P6.1. PMOS Schematic and Pin Diagram

CD4007 Pin Diagram

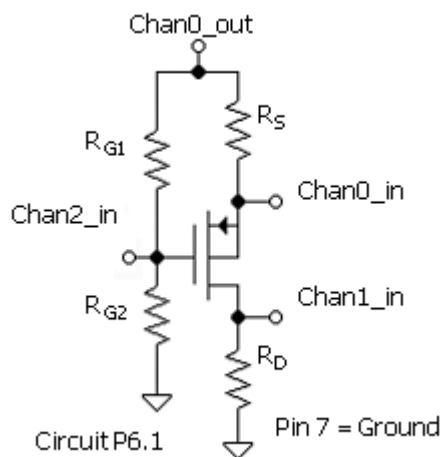




P6.2. SPICE PMOS and Circuit Equations

SPICE Equation		Description
$I_D = k_p V_{effp}^2 (1 + \lambda_p V_{SD})$	3.8 (NMOS)	DC drain current – voltage relation (PMOS).
$a_v = -\frac{g_m R_D}{1 + g_m R_S}$ $g_m \approx 2\sqrt{k_p I_D}$	5.15	Midfrequency gain versus I_D .
$a_{vO} = -2\sqrt{k_p I_D} \frac{R_D}{1 + \lambda_p I_D R_D}$	5.6	Midfrequency gain versus I_D with bypass capacitor C_s .
$V_g = \frac{R_G}{R_G + R_S} \frac{1}{1 - j\frac{f}{f_g}} V_s$ $f_g = \frac{1}{2\pi(R_G + R_S)C_g}$	6.1	Gate signal voltage, V_g , frequency-dependent relation to signal source, V_s .
$a_v(f) = -\frac{g_m R_D}{1 + g_m R_S} \frac{1 + j\frac{f}{f_z}}{1 + j\frac{f}{f_s}}$ $g_m = 2\frac{I_D}{V_{effp}}$ $f_s = (1 + g_m R_S) f_z$ $f_z = \frac{1}{2\pi R_S C_S}$	6.4	Gain frequency response with C_s only (C_g infinite).
$f_{3dB} \approx \frac{g_m}{2\pi C_s}$	6.9	Approximate f_{3dB} with C_g infinite.

P6.3. PMOS Current-Source Amplifier DC Setup



Components

$$V_{R_{G1}} = V_{SG} + I_D R_S = \text{Chan0_out}/2$$

$$I_D \approx 300 \mu\text{A}$$

$$\text{Chan0_out} = 10 \text{ V}$$

$$R_D = R_S \approx \frac{\text{Chan0_out}/2 - V_{SG}}{I_D}; \quad V_{SG} \approx 2 \text{ V}$$

$$R_{G1} = R_{G2} > 500 \text{ k}\Omega$$

V_{SG} versus I_D from Lab Projects 4,5.

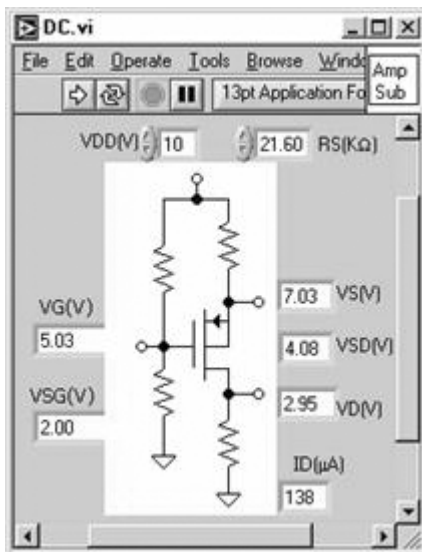
LabVIEW Computations

$$I_D = \frac{\text{Chan0_out} - \text{Chan0_in}}{R_S}$$

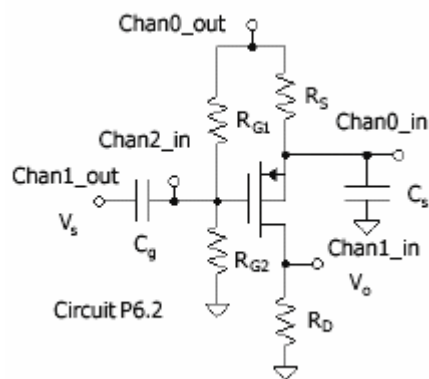
Procedure

- Install your value of R_S in the Front Panel. Verify that $V_{DD} = 10\text{V}$. Run DC.vi to verify your selection of $R_S \approx R_D$. Goal is $I_D \approx 300 \mu\text{A}$. Verify

that V_G is about $V_{DD}/2$. *Default and save* the Front Panel. Note that V_{SD} is about $2V_{SG}$.



P6.4. Amplifier Gain



LabVIEW Computations

$$C_g = \frac{1}{2\pi R_G f_g} \quad f_g = 1 \text{ Hz}$$

$$R_G = R_{G1} || R_{G2}$$

$$C_s \approx \frac{g_m}{2\pi f_{3dB}} \quad f_{3dB} \approx 20 \text{ Hz}$$

$$g_m = 2 \frac{I_D}{V_{SG} - V_{t_{po}}} = 2\sqrt{k_p I_D}$$

Use **Project06.mcd** for calculations.

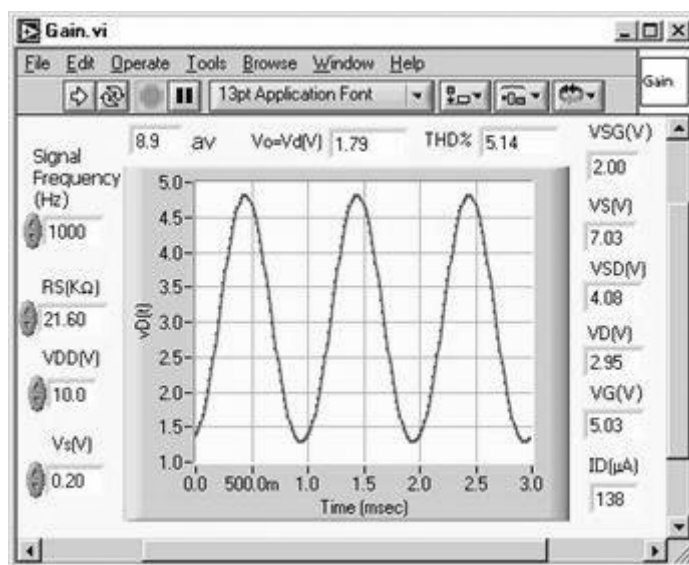
SubVI's

FG1Chan.vi

SpecAnaly.vi

LabVIEW Computation

$$a_v = \frac{\text{Chan1_in}_{\text{acpeak}}}{V_s(\text{V})}$$



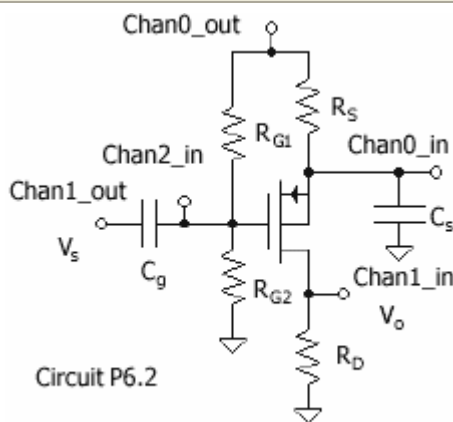
Procedure

- The SubVI of **Gain.vi**, **FG1Chan.vi**, sends out a signal with sine-wave peak V_s (Chan1_out). The first measurement is with only capacitor C_g installed. Be sure to conform to the capacitor polarity requirement. Set the Signal Frequency to 1000 Hz. Set your value of RS and check VDD = 10 V. **Run Gain.vi** to measure the gain. The gain should be less than 1 without the source bypass capacitor. A signal of $V_s \approx 0.5$ V should thus suffice for good linearity and measurement precision.
- Now install the capacitor C_s . Note that the source is at a positive voltage. Run the VI while setting V_s at various values. Adjust V_s until the

THD% (total harmonic distortion, %) is reduced to below about 5%.

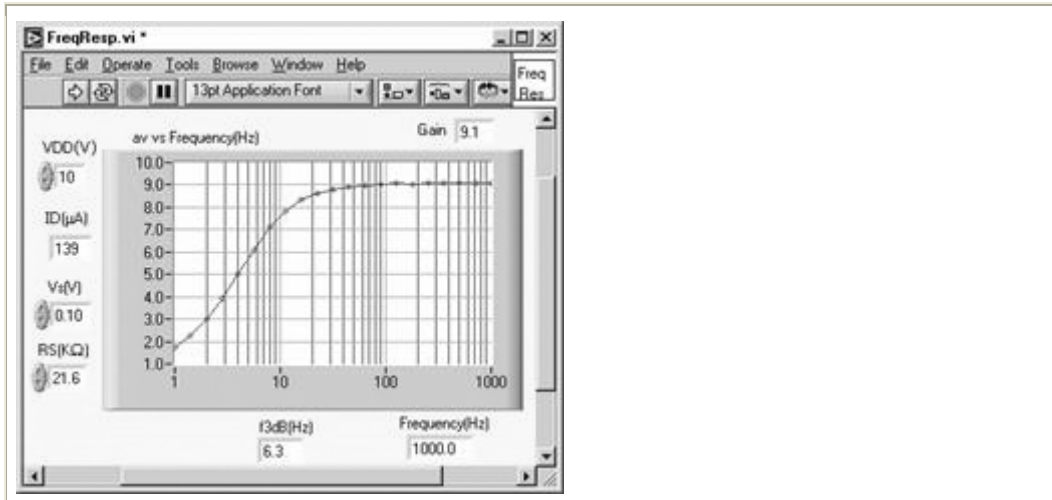
- Continue to lower V_s and **run Gain.vi**. Verify that the measured gain, a_v , is invariant for smaller values of V_s except as eventually limited by DAQ resolution and noise. Find the largest V_s (up to 5% THD), which is consistent with a constant gain measurement with increasing V_s .
- **Run** the VI again with a lower frequency (e.g., 800 Hz) to verify that the gain is not frequency dependent in this frequency range. Now *default and save* the Front Panel.

P6.5. Amplifier Frequency Response



Procedure

- The frequency of the input source voltage is swept from 1 to 1000 Hz to 1000 Hz. Set V_s in **FreqResp.vi** as determined above for a valid voltage gain measurement. Open **FG1Chan.vi** to observe the waveform at the output.
- Set your value of R_S . **Run FreqResp.vi** without C_s . Recall that the gain will be less than 1. The frequency response is dictated by C_g only. Verify that f_{3dB} is about equal to or less than 1 Hz. Note that $f_{3dB} < 1$ H if the digital indicator f3dB indicates 1 Hz.
- Install C_s and run the VI. Obtain a value for f_{3dB} . *Default and save* the Front Panel. Then use **XYtoDataFile.vi** to obtain a *data file* for the frequency response. A copy of the VI is in **Project06.llb**.



Laboratory Project 7. NMOS Common-Source Amplifier Stage with Source-Resistor Bias

[P7.1 SPICE Equations and Pin Diagram](#)

[P7.2 NMOS Common-Source Amplifier DC Evaluation](#)

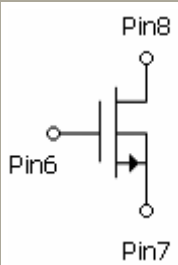
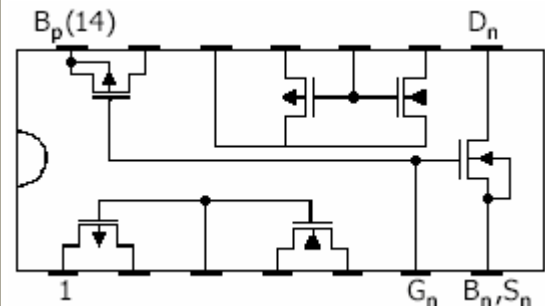
[P7.3 Amplifier Gain at Optimum Bias for Linear Output](#)

[P7.4 Optimum Bias Stability Test](#)

[P7.5 Amplifier Frequency Response](#)

Exercises and Analysis Exercise07.mcd - Project07 1.mcd - Project07 2.mcd

P7.1. Chip Diagram and SPICE Equation

 <p>CD4007 Pin Diagram</p> 		
SPICE Equation		Description
$I_D = k_n V_{effn}^2 (1 + \lambda_n V_{DS})$	3.8 (NMOS)	Dc current – voltage relation.
$a_v = -\frac{g_m R_D}{1 + g_m R_S}$ $g_m \approx 2\sqrt{k_n I_D}$	5.15	Voltage gain versus I_D .
$a_{vO} = -2\sqrt{k_p I_D} \frac{R_D}{1 + \lambda_p I_D R_D}$	5.6	Voltage gain versus I_D with bypass capacitor C_s .
$f_{3dB} = \sqrt{f_s^2 - 2f_z^2} = \sqrt{(1 + g_m R_S)^2 - 2f_z^2} \approx f_s$	6.8	Corner frequency (f_{3dB}) with capacitor C_s .
$f_s = \left[1 + \frac{(g_m + g_{ds})R_S}{1 + g_{ds}R_D} \right] f_z$	6.12	Precision f_s .

P7.2. NMOS Common-Source Amplifier DC Evaluation

Procedure (Design)

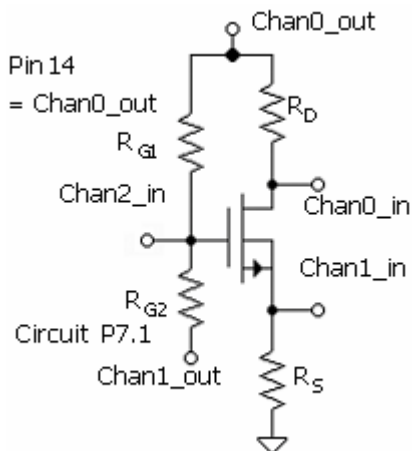
- Start in the Mathcad file Project07.mcd to compute circuit component values as follows:
- We will design for a selected I_{Dmax} at $Chan0_out_{max} = 10$ V with V_{DSmin}

= 1V. Select your I_{Dmax} in the range $100 \mu A < I_{Dmax} < 500 \mu A$.

- Make $R_{G1} = R_{G2} \approx 100 \text{ k}\Omega$. Thus, the gate voltage is

$$V_G = \text{Chan0_out}_{max}/2.$$

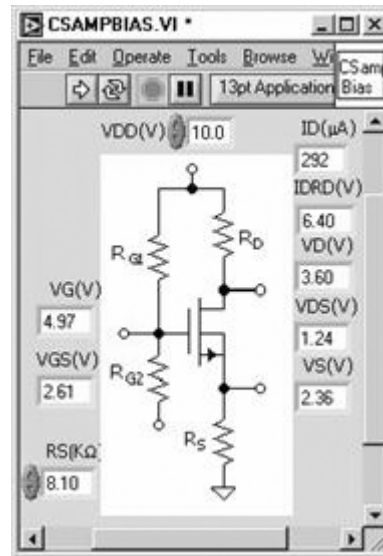
- Select the circuit source resistor, R_S , to satisfy $V_G = V_{GS}(I_{Dmax}) + I_{Dmax}R_S$. $V_{GS}(I_{Dmax})$ can be calculated using previously measured parameters or obtained graphically from, for example, **NMOSparam.vi** from [Project 4](#).
- After acquiring a *real* R_S (which is close to the calculated value), calculate an *actual, new* I_{Dmax} using the same equation in reverse with the real R_S . Neglect the change in V_{GS} .
- We will select R_D to set the condition of $V_{DSmin} = 1 \text{ V}$ (at Chan0_out_{max}) from $R_D = (V_{DD} - V_{DSmin})/I_{Dmax} - R_S$. (Use actual, new I_{Dmax} .)



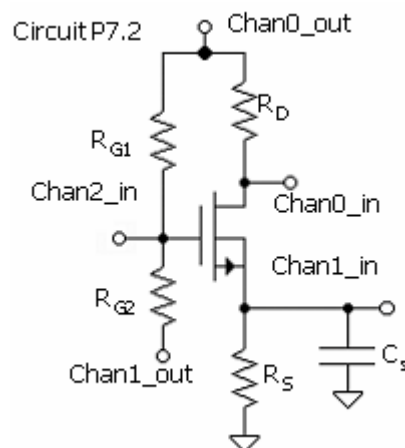
Procedure (Measurement)

- Install your value of R_S in CSAMPBIAS.vi (for current sensing). With $V_{DD} = 10 \text{ V}$ (V_{DDmax}), run the VI to verify the design. V_{DSmin} can be significantly different from design 1 V due to parameter variation. For example, $1 < V_{DSmin} < 1.5 \text{ V}$ is satisfactory, where the low limitation is based on $V_{DSmin} > V_{effn}$. This is the requirement to be in the active region. In the example, the design drain current is $I_{Dmax} = 300 \mu A$.
- Verify that as V_{DD} is reduced, I_D decreases and V_{DS} increases. Check $V_{DD} = 4 \text{ V}$, the minimum in the sweep for parameter measurement of the next part. Verify that at the minimum V_{DD} , $V_{DS} > I_D \cdot R_D$. Obtain a log for $\text{Chan0_out} = 10V$.
- Reduce V_{DD} from the maximum $V_{DD} = 10 \text{ V}$ and determine,

approximately, the value between $4V < V_{DD} < 10V$ that corresponds to $V_{DS} \approx I_D R_D$. This is the operating point (bias) condition for the amplifier gain measurement. This will be found automatically in the gain measurement VI.



P7.3. Amplifier Gain at Optimum Bias for Linear Output



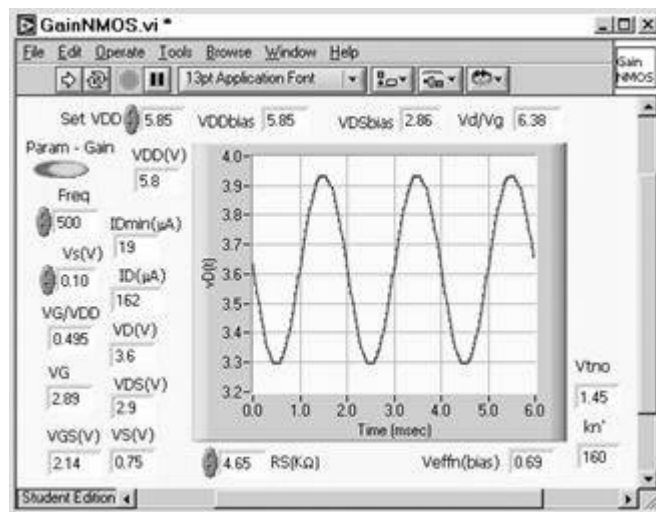
Components*

$$C_s \approx \frac{g_m 10^{-6}}{2\pi f_{3dB}}$$

$$f_{3dB} \approx 10 \text{ Hz} \quad g_m = 2 \frac{I_D}{V_{effn}}$$

Note that g_m is in $\mu\text{A/V}$.

***Run** the first part of Procedure with **GainNMOS.vi**, below, without the capacitor, to obtain V_{effn} and bias I_D , for computing g_m .



LabVIEW Computations

Parameters: V_{tno} , k_n

$$a_{vg} = \frac{\text{Chan0_in}_{acpeak} / V_s}{1 - \frac{\text{Chan2_in}_{DC}}{\text{Chan0_out}}}$$

$$\frac{R_{G2}}{(R_{G1} + R_{G2})} = \frac{\text{Chan2_in}_{DC}}{\text{Chan0_out}}$$

Procedure (Obtain Parameters and Bias Variables)

- Open **GainNMOS.vi**. Install your R_S value. *Without* the capacitor installed, set the Mode Switch to Param (red, switch in logic state 0). **Run** the VI for a VDD sweep to get the parameters. (Note: The sweep rate is configured to be slow in case the capacitor is in place during the sweep.) The VDD sweep is from 4 V to 10 V. Obtain a *log* of the results.

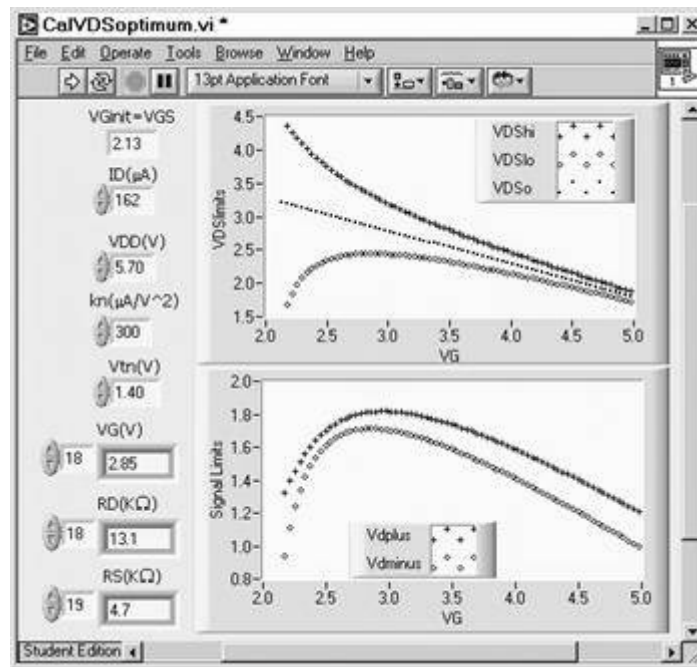
Procedure (Gain Measurement)

- In the Param mode, the VI finds and indicates VDDbias corresponding to the optimum $V_{DS} = (V_{DD} - V_{R_S} + V_{effn})/2$. This permits maximum signal swing without distortion. Set the indicated value of VDDbias into the Set VDD Digital Control. **Run** the VI with the Mode Switch set to Gain (green, switch position logic 1) without the capacitor. Obtain a *log* of the Front Panel. *Default and save* the Front Panel.
- At the end of the *parameter* sweep, Veffn is indicated for VDDbias. The value is retained in the Digital Indicator in the gain measurement. The value of ID indicated in the *gain* measurement is the amplifier gain bias value. Use Veffn along with bias ID, to compute g_m and thus C_s . Use Project07.mcd for the calculation.
- *Install* the capacitor. Make certain that the polarity of the capacitor is correct. Check that the Mode Switch is set to Gain (switch position logic 1). Check that the value of VDDbias matches the value in Set VDD Digital Control. The VI will measure the gain at this one VDD setting (optimum for distortion-free output). The signal frequency is initially 500 Hz.
- **Run** the VI and vary the value of V_s to determine that the gain result, V_d/V_g , is not affected by changes, that is, by distortion. *Default and save* the Front Panel. Obtain a *log* of the results. Also, run with slight variations in Freq (e.g., 300 Hz). Note that the parameter values are preserved when running the gain mode. Parameter, bias variable, and gain data are required for the project Mathcad file.

P7.4. Optimum Bias Stability Test

- As will be explored in a project Mathcad file, the *basis stability* computation is performed here with **LabVIEW**. Recall ([Section 5.5](#)) that this takes into consideration a possible range of V_{tn0} and k_n values. Here, an analysis is made regarding the extent to which your design is optimum in terms of bias stability. Note that this is separate from the

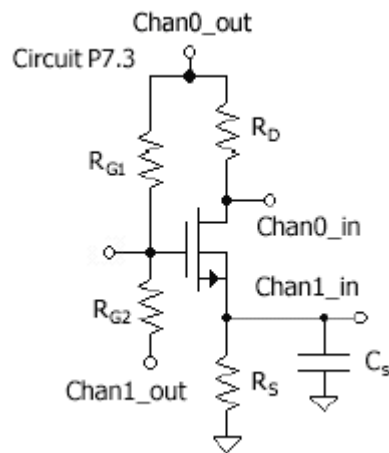
maximum signal condition that has been included in the design.



- Open **CalVDSOptimum.vi**. In the Digital Controls, set your values for VDD, k_n , V_{tn0} , and ID. ID is the nominal value of the design for the nominal values of the parameters. Your value of ID, along with your parameter values, will be assumed to be, for this evaluation, the nominal values. The computation holds the bias current constant at the design value (as entered into the Front Panel). As VG is increased (X axis), the added increment of VG is the added drop across the new RS, for a given new VG. **Run** the VI.
- Verify that the value of VGS in the top Digital Indicator matches reasonably well your measured value (Front Panel of **GainNMOS.vi**). This SPICE computation uses your measured parameter values and I_D . The values should be consistent.
- The top graph is its VDSLimits versus VG. The center (dashed) curve is for your parameters. The downward slope reflects the increasing drop across VG. The top and bottom plots are for the two extremes of V_{DS} that occur for the worst case of the combination of the limits of k_n and V_{tn0} . The computation is for $V_{tn0\pm} = V_{tn0} \pm 100 \text{ mV}$ and $k_{n\mp} = k_n \mp 100 \mu\text{A/V}^2$. The same optimum signal level condition as used in your design is maintained throughout.

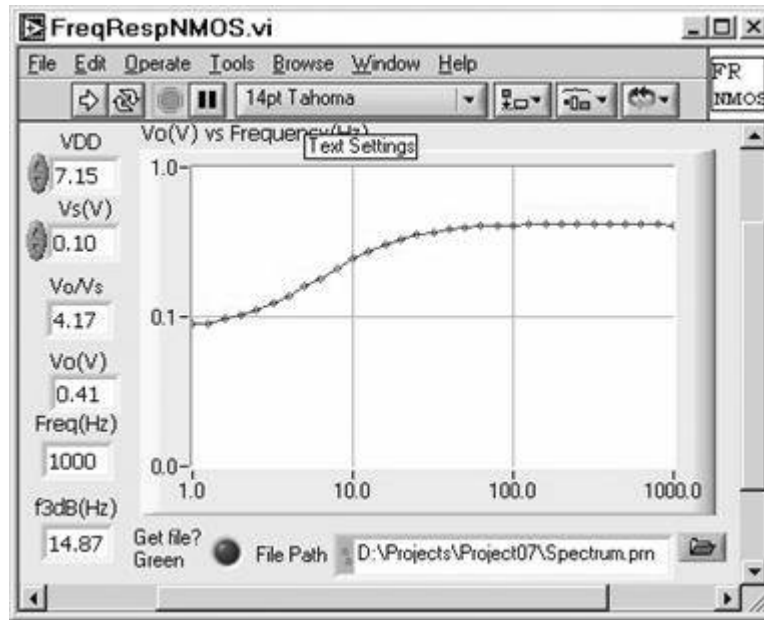
- The lower graph contains plots of the output signal limiting values. The best combination of bias stability and signal level is at the peak of these curves.
- Step through the RS Array Digital Indicator to locate your RS and the associated index. Then determine the corresponding VG and RD (same index). The value of RD should match your design value. The value of VG should match the value from the Front Panel of **GainNMOS.vi**.
- Locate your value of VG on the X-axis of the two graphs. If it falls in the range of values surrounding the peaks, the circuit is optimized both in terms of signal limits and bias stability. Note that bias stability was not taken into consideration in the design. In the design, though, often a given criterion serves as the basis and the design may then be evaluated for other criteria.

P7.5. Amplifier Frequency Response



Procedure

- In **FreqRespNMOS.vi**, the frequency of the with source voltage (sine-wave Chan1_out) is swept from 1 to 1000 Hz. Chan2_in should be disconnected to reduce the stray capacitance at the gate. In the Front Panel of the VI, set V_{DDbias} and V_s at the values determined in the gain measurement with **GainNMOS.vi**.



- Along with **FreqRespNMOS.vi**, open **FG1Chan.vi** to observe the output waveform. Verify that C_s is installed and run the VI to obtain a value for f_{3dB} .
- Open, from the Diagram of **FreqRespNMOS.vi**, Frame 2, **XYtoDataFile2.vi**. Run **FreqRespNMOS.vi** again, with the data mode switch set to Green (logic stage 1), to obtain a *data file* of the frequency-response plot in **XYtoDataFile2.vi**. (Note that with the data VI open, the data are transferred to the VI and can then be saved in the VI.) The data file is used in the Mathcad project file. *Default and save* the Front Panels of both **XYtoDataFile2.vi** and **FreqRespNMOS.vi**. Note the maximum index for the Mathcad file. The f_{3dB} result will differ from the design, as the design was based on the simple form. This will be explored in the Mathcad file.
- A data file can be obtained later from the saved data in the graph of **FreqRespNMOS.vi**, with **XYtoDataFile2.vi**. As noted above, the data file VI can be obtained from the Diagram of **FreqRespNMOS.vi**. The data file VI is located in **Dat.llb** in the User.lib folder (Program Files>>National Instruments>>LabVIEW 6). The VI can also be accessed from the menu sequence in **FreqRespNMOS.vi**, Browse>>Show VI Hierarchy, and open the data file VI from the Hierarchy Window.

Return to Project07.mcd.

Laboratory Project 8. NMOS Source-Follower Stage

[P8.1 SPICE Equations and Pin Diagram](#)

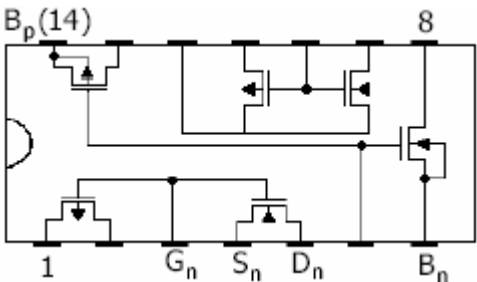
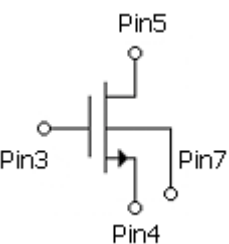
[P8.2 Source-Follower DC Evaluation](#)

[P8.3 Source-Follower Voltage Transfer Relation](#)

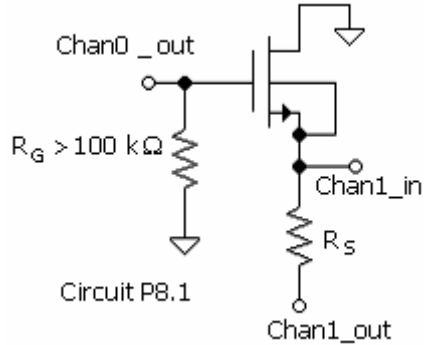
[P8.4 Source-Follower Voltage Transfer Relation with Body Effect](#)

Exercises and Analysis Exercise08.mcd - Project08.mcd

P8.1. SPICE Equations and Pin Diagram

SPICE Equation		Description
$V_{tn} = V_{tn0} + \gamma_n (\sqrt{ V_{SS} + 2\phi} - \sqrt{2\phi})$	7.1	Threshold voltage for $V_B = V_{SS}$.
$a_v = \frac{g_m}{g_m + \frac{1}{R_S} + g_{ds}}$	7.7	Source-follower voltage-transfer relation with $V_{sb} = 0$.
$a_v = \frac{g_m}{g_m(1 + \eta) + \frac{1}{R_S} + g_{ds}}$	7.12	Source-follower voltage transfer relation with body effect.
$g_{mb} = \eta g_m \quad \eta = \frac{\gamma_n}{2\sqrt{V_{SB} + 2\phi_F}}$	4.11	Body-effect transconductance.
CD4007 		Do not connect pin 14. 

P8.2. Source-Follower DC Evaluation



Pin 14 must *not* be connected. Chan0_out will be plus and minus and the drain is at ground.

Components

For **Gain.vi**

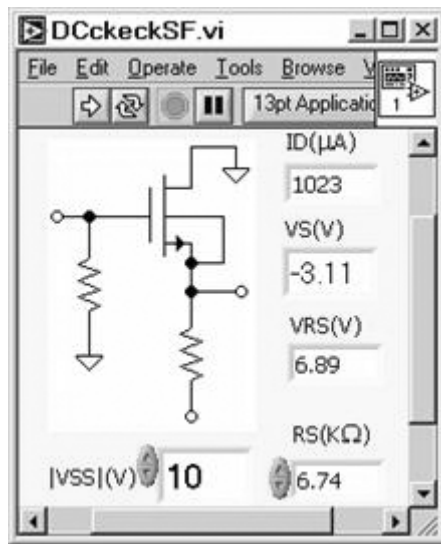
$$I_{Dmax} \approx 500 \mu A \quad | \text{Chan1_out}_{max} | = 10 \text{ V}$$
$$V_{GSmax} \approx 5 \text{ V}$$

$$R_S \approx \frac{| \text{Chan1_out}_{max} | - | V_{GSmax} |}{I_{Dmax}}$$

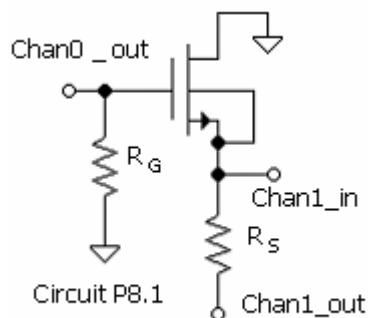
Note that with R_S is selected for the condition of maximum body-effect ($V_{GSmax} \approx 5 \text{ V}$), $I_{Dmax} > 500 \mu A$ for the case without body effect. The current sweep VI's halt at $I_D = 500 \mu A$.

Procedure

- In the Front Panel of **DCcheck.vi**, set the value of R_S in the Digital Control. With V_{SS} (magnitude) set at 10 V, **run** the VI to verify the dc design. Note that $|V_S|$ is equal to the V_{GS} magnitude. Verify that $I_D > 500 \mu A$ for this case of no body effect.



P8.3. Source-Follower Voltage Transfer Relation



Pin 14 must not be connected to ground. Chan0_out will be greater than 0 for the positive signal.

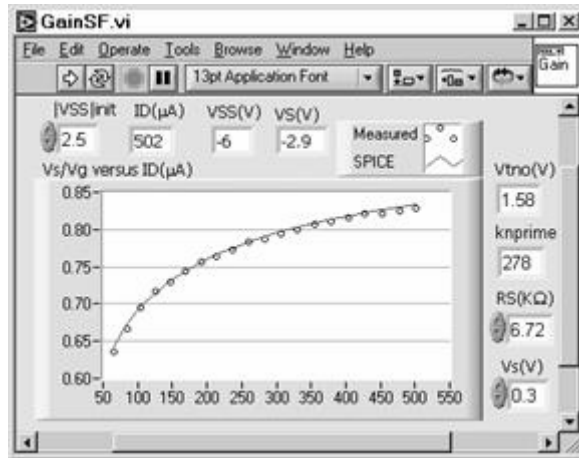
LabVIEW Computation



Procedure

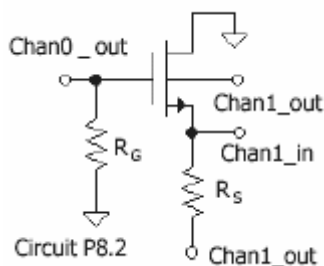
- **GainSF.vi** measures the gain (transfer relation) of the circuit over a

range of drain current. Parameters V_{tn0} and k_n are obtained from the I_D sweep. V_{tn0} is used in the voltage transfer-relation calculation as plotted along with the measured data.



- Open **GainSF.vi** and install your value of R_S . Open **FG1Chan.vi** to observe the waveform at the output. **Run** the VI and verify that the curves match reasonably well as in the example. Adjust the magnitude of V_{SS} to obtain a minimum I_D of about $50 \mu A$
- Adjust the value of V_S while **rerunning** the VI. Use the largest V_S without distortion. Large-signal distortion is manifested by a poor curve fit, particularly at the low end of the current range.
- *Default and save* the Front Panel. The value obtained for k_n must be available for the next VI. It is used in the calculation of
$$g_m = 2\sqrt{k_n I_D}$$
. The alternative form, $g_m = 2I_D/V_{effn}$, is used in the calculations for this VI.
- Obtain a *data file* of the plot using **XY1ToDataFile.vi**.

P8.4. Source-Follower Voltage Transfer Relation with Body Effect



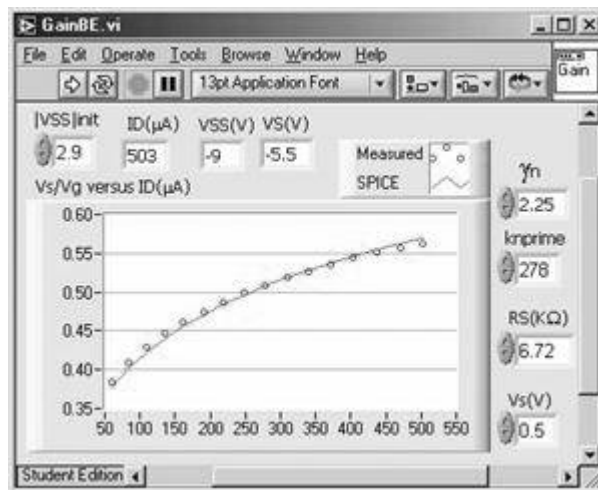
Pin 14 must not be connected to ground. Chan0_out will be greater than 0 for the positive signal.

LabVIEW Computation



Procedure

- Move the body pin connection as in Circuit P8.2. In **GainBE.vi**, install a first-guess value for γ_n ([Project 4](#)). Install the value of knprime, obtained from the Front Panel of **GainSF.vi**. **Run** the VI and readjust γ_n for the best fit. Adjust |VSS| init for a minimum ID of about 50 μA . Note the reduced value of the voltage transfer relation (gain).



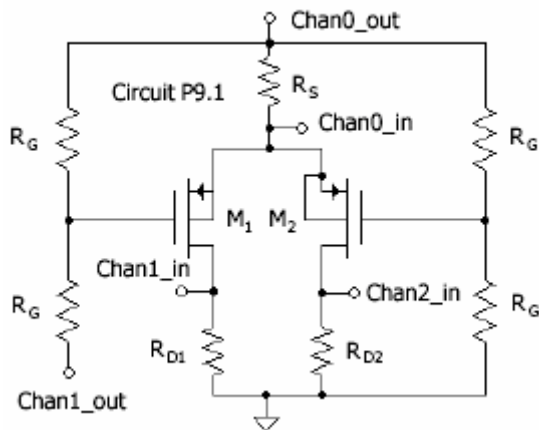
- Open **Compare.vi** from **Project08.llb**. Paste the results from the graphs of the plots for with and without body effect into the control graphs. **Run** the VI to compare the results.
- Use **XY1ToDataFile.vi** to obtain a *data file* of the plot in **GainBE.vi**.

Laboratory Project 9. MOSFET Differential Amplifier Stage

[P9.1 SPICE Equations and Pin Diagram](#)

SPICE Equation		Description
$a_{v1} = \frac{-g_{m1}R_{D1}}{(R_{D1} + R_s)g_{ds1} + (1 + g_{m1}R_s)}$	8.34	Inverting gain.
$a_{v2} = a_{vsf} a_{vcg} = \frac{g_{m1}R_s}{(R_{D1} + R_s)g_{ds1} + (1 + g_{m1}R_s)} \frac{(g_{m2} + g_{ds2})R_{D2}}{1 + g_{ds2}R_{D2}}$	8.40	Noninverting gain.
$R_s = R_{bias} / R_{is2} \quad R_{is2} = \frac{1 + g_{ds2}R_{D2}}{g_{m2} + g_{ds2}}$	8.38	Signal resistance at source node.
CD4007 	PMOS 1 	PMOS 2

P9.2. DC Evaluation of the Single-Power-Supply Differential Amplifier



Components

$$R_{D1} = R_{D2} = R_D$$

$$R_s = R_D/2$$

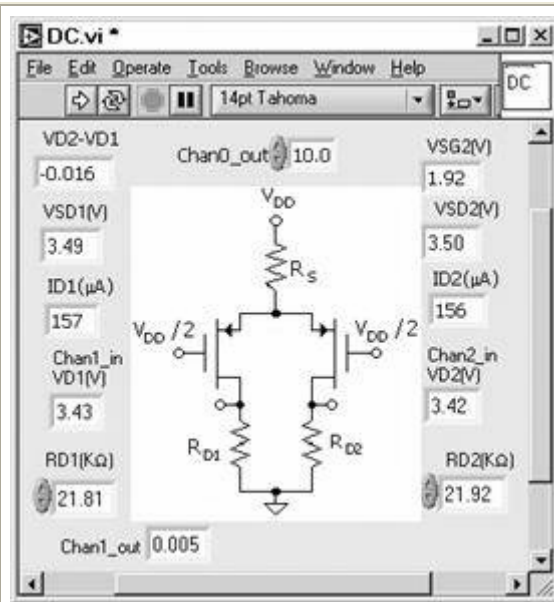
$$V_D \approx V_{DD}/2 - V_{SG}$$

For Chan0_out = 10 V:

$$100 \mu A < I_D < 400 \mu A$$

(Choose One)

$$R_G < 100 \text{ k}\Omega$$

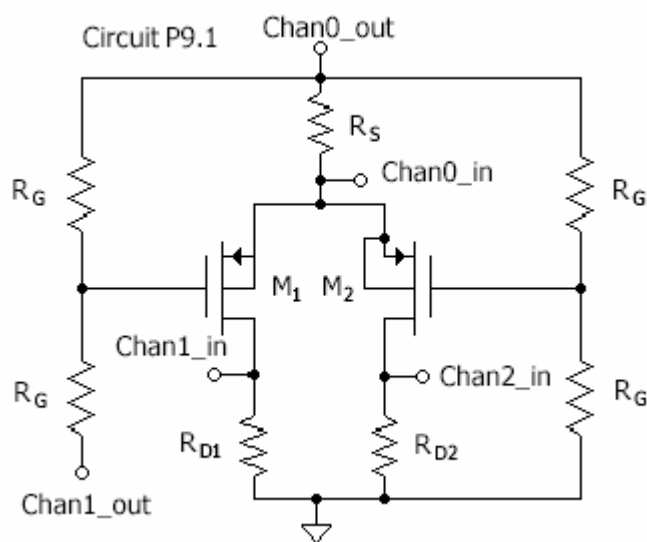


Match the gate resistors on each side to obtain $V_G = V_{DD}/2$. On a given side, the resistors must be equal, but can be different on opposite sides.

Procedure

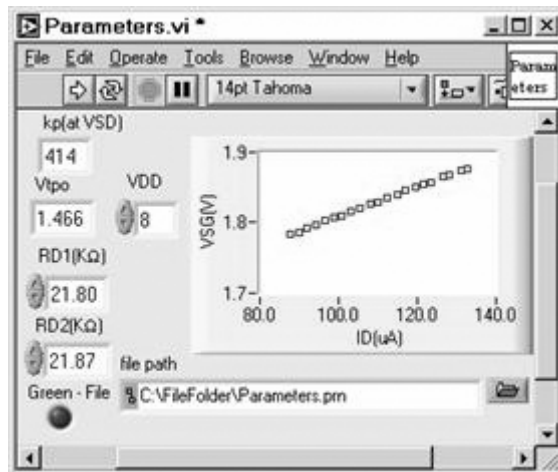
- After completing the circuit connections, open **DC.vi**. Set Chan0_out = 10 V and run the VI. **DC.vi** scans Chan1_out to balance the circuit (i.e., to set $V_{D1} = V_{D2}$ within 20 mV). Try Chan0_out in the range 5 to 10 V. Verify that $100 \mu A < ID < 400 \mu A$ (your design) with maximum Chan0_out = 10 V.
- Note the value sent out by Chan1_out. This should be only mV if the gate resistors were selected properly.
- Verify that VSD is roughly 2VSG. *Default and save* the Front Panel with Chan0_out = 10 V. The VI serves as a subVI in the next part to bias and balance the circuit.

P9.3. Determination of the PMOS Parameters



Procedure

- **Parameters.vi** sweeps output channel Chan0_out from $V_{DD} - 1$ to $V_{DD} + 1$ (where the design bias V_{DD} is set in the Front Panel). The gain measurement will be made at V_{DD} , intermediate to the sweep range. Since voltage V_{SD} is approximately equal to $2V_{SG}$, V_{SD} will be relatively constant during the sweep, as is required for a reliable measurement of K_p and V_{tp0} .

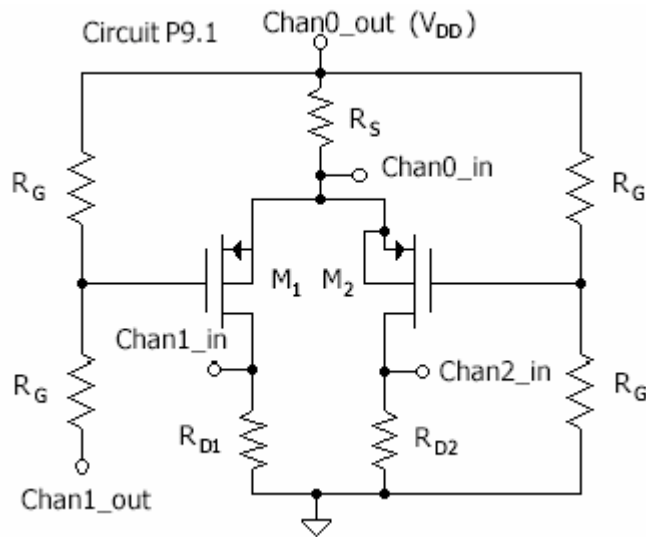


- **Run** the VI for V_{DD} set at 6 V and 8V and verify that V_{tp0} is close to the same for both V_{DD} values. For the larger V_{DD} (and V_{SD}), k_p (at V_{SD}) should be slightly larger.
- Now set $V_{DD} = 6, 7$, or 8 V and the File Mode switch to Green (on) and **re-run** to obtain a parameter *data file*. The file has R_{D1} , R_{D2} , k_p

(technically, k'_p) and V_{tp0} . The Parameters are actually for M_2 . These will be read by the evaluation Mathcad file.

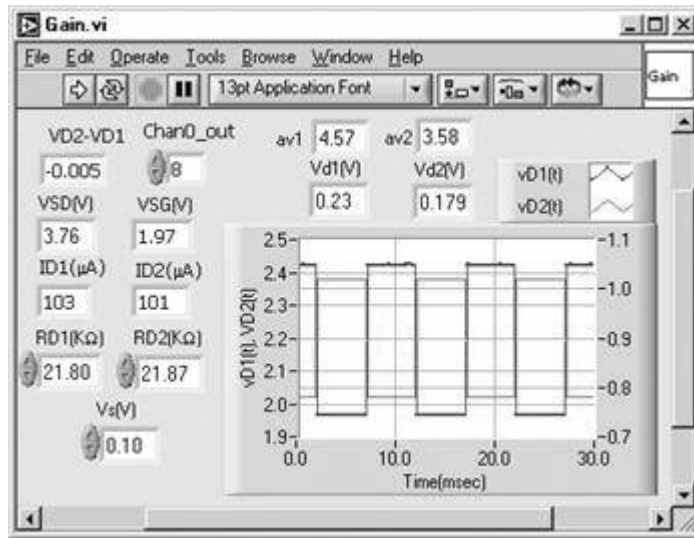
- The parameters can be obtained with **SimParam.vi** from the data saved in **Parameters.vi**. Paste the data from the Indicator Graph (sample, $V_{DD} = 7\text{ V}$) of **Parameters.vi** into the Control Graph of **SimParam.vi** and run the VI with the Data Mode switch set to on (Green).

P9.4. Amplifier Gain Measurement

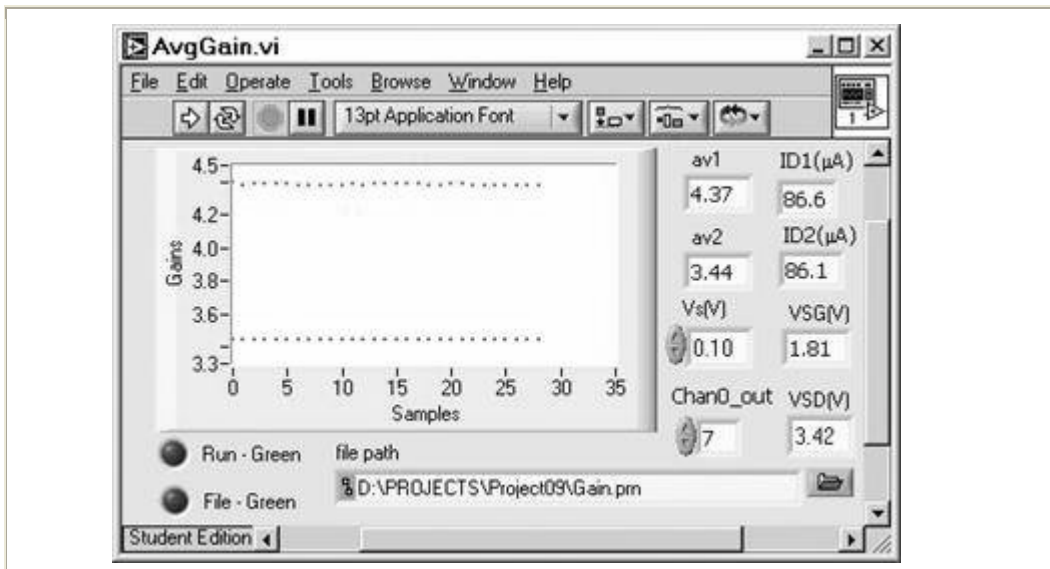


Procedure

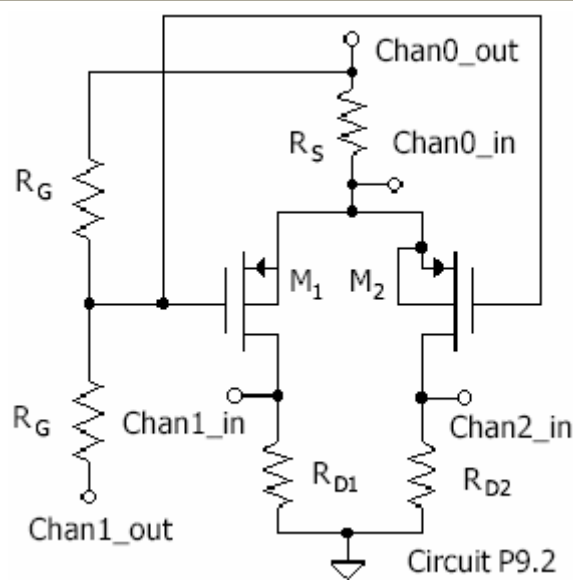
- **Gain.vi** runs **DC.vi** to set up the bias and balance the circuit. It then sends out a signal squarewave and reads the response at the two outputs. Gain is calculated as output voltage divided by $V_s/2$. The $1/2$ accounts for the voltage divider of the two gate resistors.



- Set in Chan0_out (V_{DD}), which you selected above in **Parameters.vi**. **Run Gain.vi** to obtain the gain at the two outputs. Adjust V_s for an output voltage magnitude similar to that in the example.
- Now run **AvgGain.vi** with your V_{DD} (Chan0_out) and V_s from **Gain.vi** as a subVI repeatedly and averages the gains continuously during the measurements. With the Run Mode on (Green), allow the VI to continue to **run** until the average gain no longer appears to be changing. Reset Run Mode to halt execution.
- Repeat with the File Mode button set to on (Green). The data file contains I_{D1} , I_{D2} , V_{SG} , V_{SD} , a_{v1} , and a_{v2} . The Mathcad evaluation file will read these data.
- Use **SimAvgGain.vi** to obtain the data file from the data saved in **AvgGain.vi**. Paste graph data and copy and paste the Digital Control values. Gains will be obtained from the graph data and need not be copied from the Digital Indicators.

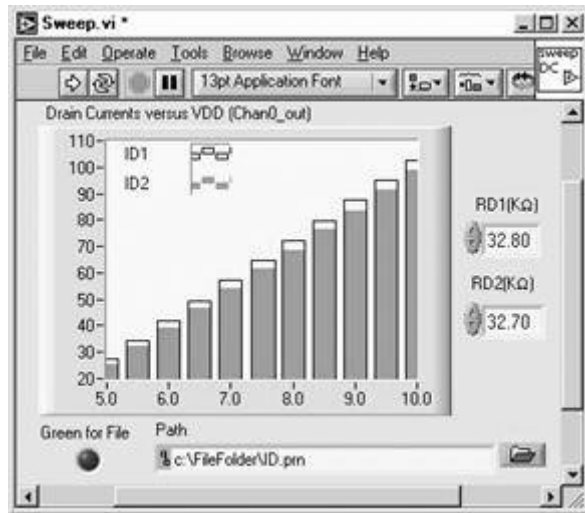


P9.5. Transistor Parameters and DC Imbalance

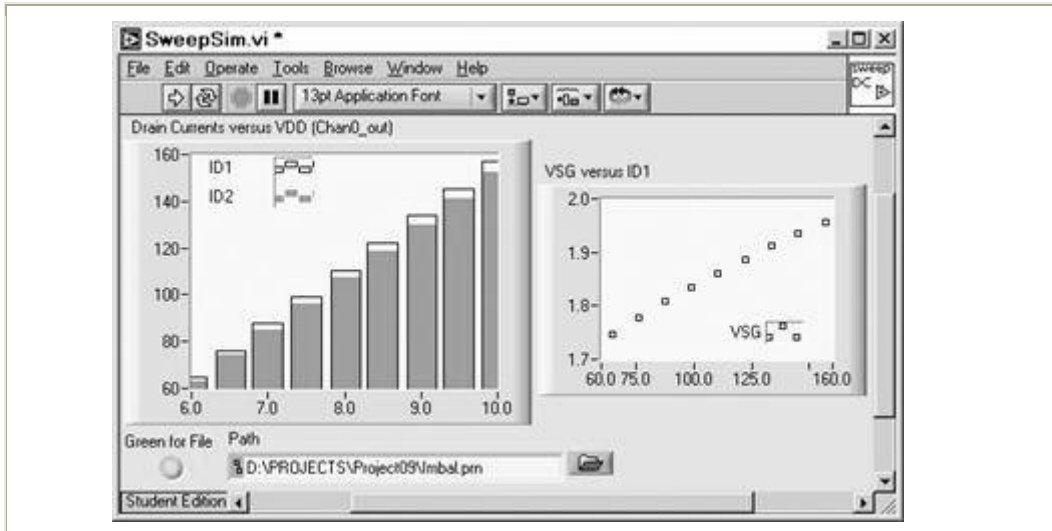


Procedure

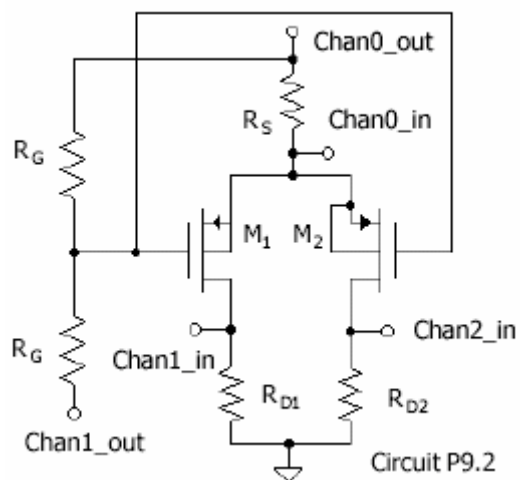
- **Sweep.vi** runs a special version of **DC.vi**, **DCimbal.vi**, as a subVI of **Sweep.vi**. **Sweep.vi** sends out a range of bias voltage, $5\text{ V} < V_{DD} < 10\text{ V}$ (Chan0_out), in 0.5 V steps. V_{SG} , I_{D1} , and I_{D2} are measured at each step. A three-column data file is saved for the Mathcad project evaluation.



- Note that Circuit P9.2 has the gates connected to make certain that the gate voltages are equal. There is no balancing in this circuit, as the goal is to study circuit imbalances. **DCimbal.vi** calls for Chan1_out = 0 V.
- Open **Sweep.vi** and set in the resistor values. Also open **DCimbal.vi**. **Run DCimbal.vi** to verify that the circuit is functioning properly.
- **Run Sweep.vi.** Set the File-Mode switch to on (Green), write in the file path (or click on the Browse button) and data file name and **run** the VI to *obtain the data file*. The data dc variables will be used in the project Mathcad file to obtain k_p and V_{tp0} . The values are used in a computation for a graphical comparison of measured and computed current ratios versus drain current.
- **Run SweepSim.vi** to obtain the data file from the graph data saved. [Note that the VSG graph is not shown in the sample **Sweep.vi** (above). The actual **Sweep.vi** has a Front Panel as the sample **SweepSim.vi** (below).] Copy and paste data from **Sweep.vi**.

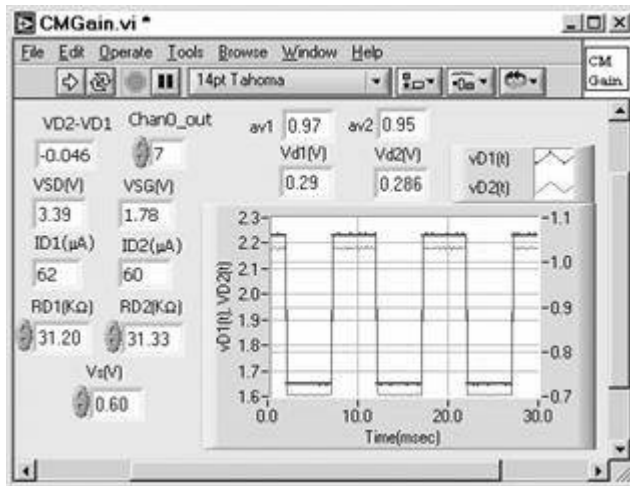


P9.6. Common-Mode Gain Measurement



Procedure

- The common-mode gain can be obtained directly with Circuit P9.2. This is accomplished with the VI, CBGain.vi. The squarewave input signal is applied with Chan1_out.



- Open **CMGain.vi** and set in the resistor values. Set V_S to a larger value than that used in **Gain.vi**, to compensate for lower gain. **Run** the VI to verify that the circuit is functioning properly. Expect the gain to be about unity, based on the design $R_S = R_D/2$. (Ideal common-mode gain is zero.)
- Open and **run AvgCMGain.vi**. Obtain a data file consisting of drain currents, V_{SG} , gains, and V_{SD} .
- Use **SimAvgGain.vi** to obtain a data file from data saved in **AvgCMGain.vi**. Copy and paste the graph data and the drain currents, V_{SG} and V_{SD} . Gains will be averaged from the graph data.

Laboratory Project 10. Current Mirror and Common-Source Amplifier with Current-Source Load

[P10.1 SPICE Equations and Pin Diagram](#)

[P10.2 Evaluation of the Current-Source Circuit](#)

[P10.3 Evaluation of the Mirror-Current Circuit](#)

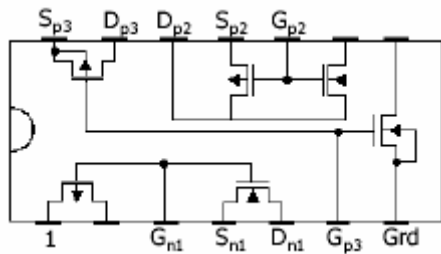
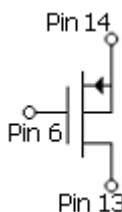
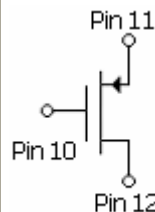
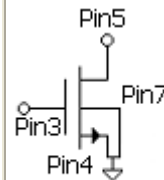
[P10.4 Evaluation of the Bias Setup](#)

[P10.5 Measurements of the Amplifier Gain versus Bias Current](#)

Exercises and Analysis Exercise10.mcd - Project10.mcd

P10.1. SPICE Equations and Pin Diagram

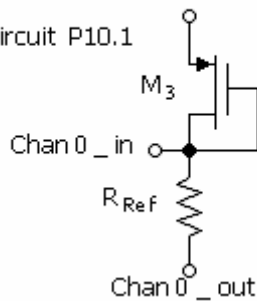
SPICE Equation	Description
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SPICE Equation		Description	
$a_v \approx -\frac{2}{(\lambda_n + \lambda_p) V_{\text{effn1}}}$ $V_{\text{effn1}} = V_{\text{GS1}} - V_{\text{tn0}}$	10.3	Amplifier gain, V_{GS} form.	
$a_v \approx -\frac{2}{\lambda_n + \lambda_p} \sqrt{\frac{k_{n1}}{I_{\text{D1}}}}$	10.4	Amplifier gain, I_{D} form.	
	PMOS3 	PMOS2 	NMOS1 

P10.2. Evaluation of the Current-Source Circuit

Power Supply = 5 V
Pin 14 on Conn. Block

Circuit P10.1



Components

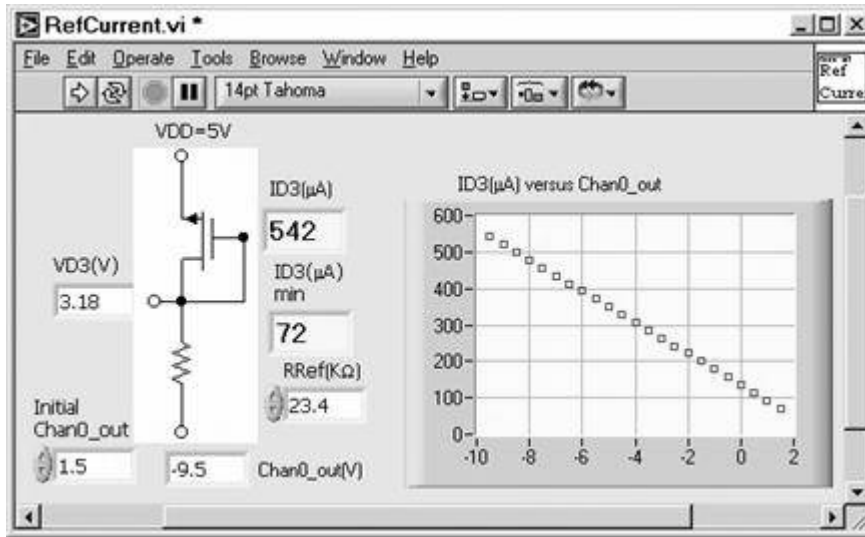
$$I_{Dmax} \approx 500 \mu A \quad \text{Chan0_out}_{min} = -10 \text{ V}$$

$$R_{Ref} \approx \frac{5 \text{ V} - V_{SG3} + 10 \text{ V}}{I_{Dmax}}$$

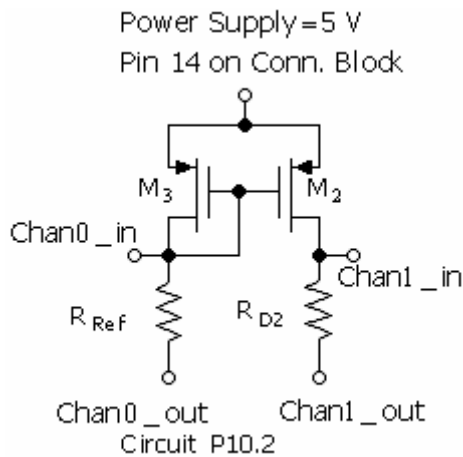
Procedure

- Set your value of RRef in the Digital Control. **Run RefCurrent.vi** to determine the current range. Chan1_out is swept from the Initial value to $\approx -10 \text{ V}$. Readjust Initial Chan1_out to obtain a minimum drain current

of about $50\ \mu\text{A}$. Verify that the maximum current is about $500\ \mu\text{A}$.



P10.3. Evaluation of the Mirror-Current Circuit

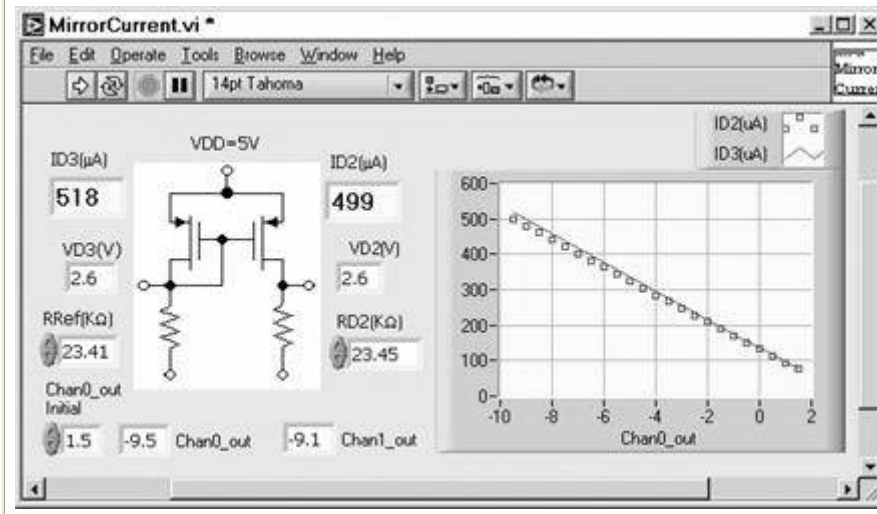


Procedure

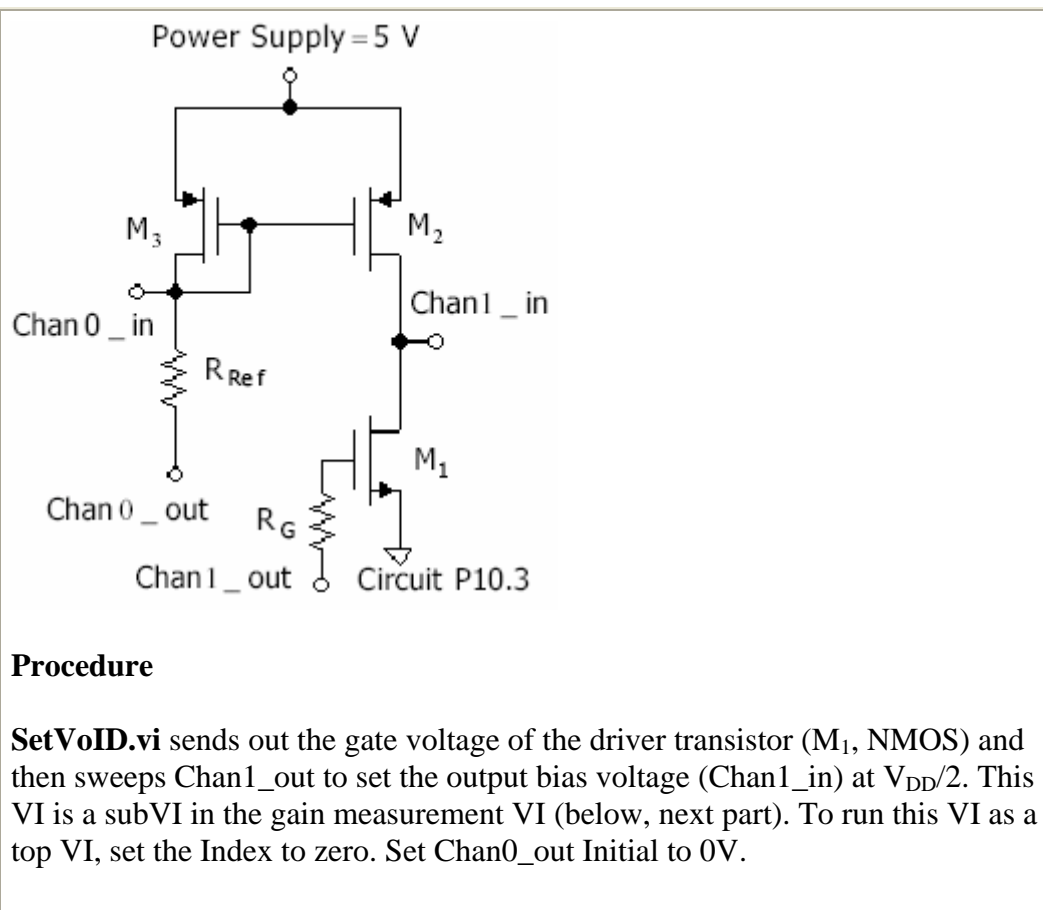
- Install the additional circuit components, which form the current source. Set your resistor values in the Digital Controls. Use $Initial\ Chan0_out$ from **RefCurrent.vi**. Run **MirrorCurrent.vi** to assess the mirror current (current-source current). $Chan0_out$ is swept from the Initial value to $\approx -10\text{ V}$. $Chan1_out$ is varied downward to maintain $V_{SD2} = V_{SD3}$. Any difference between the currents indicates a difference in transistor parameters of M_2 and M_3 .

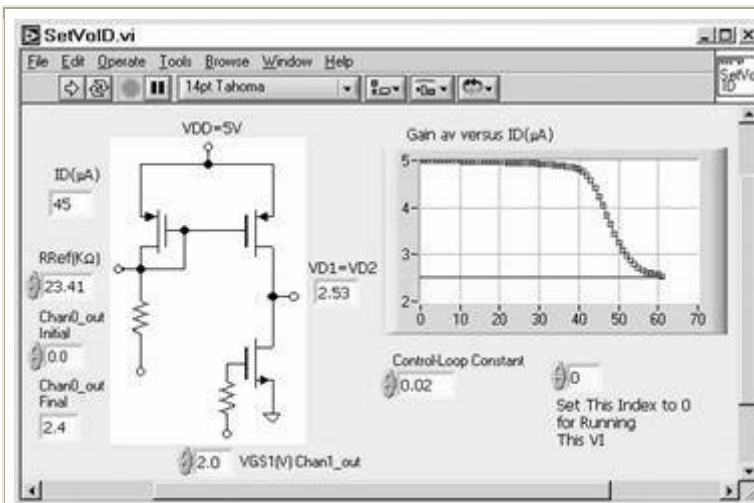
Components

$$R_{D2} \approx R_{Ref}$$



P10.4. Evaluation of the Bias Setup



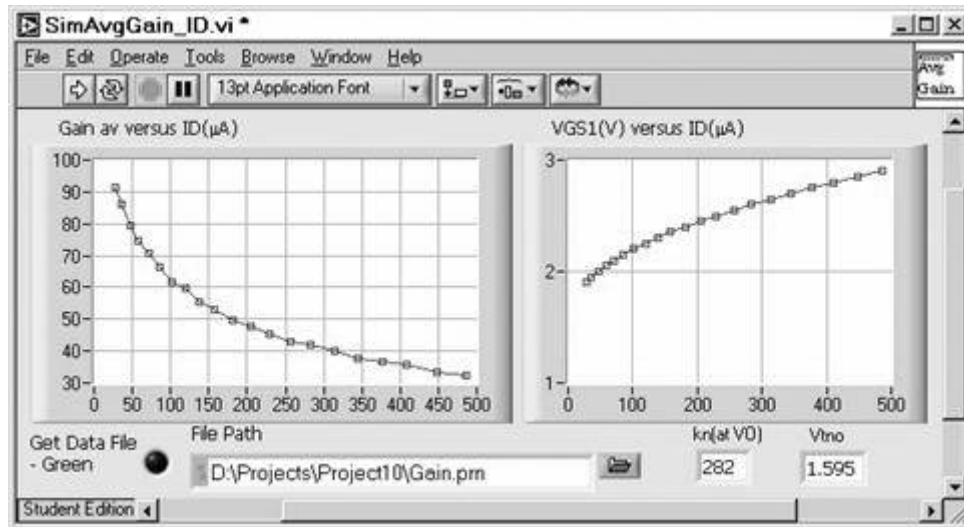


- **Run SetVoID.vi** while adjusting V_{GS} for the lowest value of the I_D range of about 50 μA . Verify that the bias-setting circuit functions properly. Adjust the Control-Loop Constant to obtain a final dc value convergence rate similar to that in the example. *Default and save* the Front Panel.

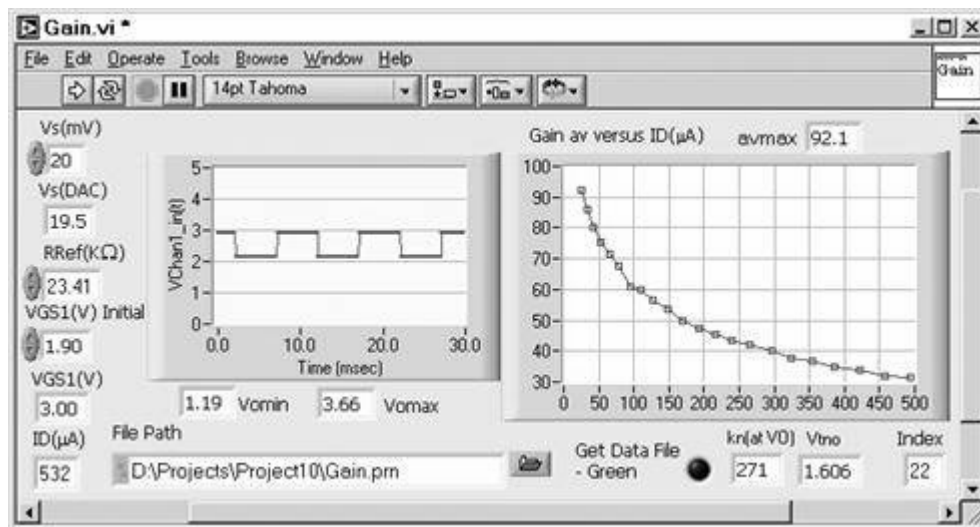
P10.5. Measurement of the Amplifier Gain versus Drain Current

Procedure

- In **Gain.vi**, set V_{GS1} Initial to the value from the evaluation of **SetVoID.vi** above. The VI will sweep V_{GS1} over a range to include a maximum response of about 500 μA , according to the design above. Due to the high gain, the input signal voltage will be small in terms of the DAC. A subVI computes the actual output signal voltage from the value requested. The value computed is used in the gain computation.



- Open **SetVoID.vi** along with **Gain.vi**. Set all numbers in the Digital Controls of **Gain.vi**. **Run** the VI and adjust V_s such that V_o min and V_o max are confined between 1 and 4 V, as in the example. These are indicated for the initial bias current (highest gain). Also, adjust (down) the Control-Loop Constant of **SetVoID.vi** if the bias setting function oscillates out of control. Save the Front Panel when the VI is functioning properly.
- Open **AvgGain_ID.vi**. This VI runs **Gain.vi** as a subVI repeatedly and averages the plots of each execution. Set the Run Mode switch to Continuous and the File Mode switch to on (Green) and **run** the VI for data smoothing. Switch to One Plot to halt the execution. The data file includes gain, V_{GS1} , and I_D . This is used in the project Mathcad analysis file.
- Obtain a data file from saved data in **AvgGain_ID.vi** with **SimAvgGain_ID.vi**.



Laboratory Project 11. Operational Amplifier with Resistor Feedback

[P11.1 SPICE Equations](#)

[P11.2 Bias Circuit Setup](#)

[P11.3 Opamp Offset Voltage](#)

[P11.4 Evaluation of the Bias Balancing Circuit](#)

[P11.5 Evaluation of the Gain and Signal Limits with Swept Input](#)

[P11.6 Evaluation of the Gain with Sine-Wave and Square-Wave Signals](#)

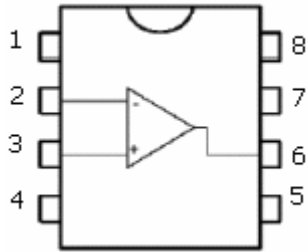
[P11.7 Determination of the Opamp Frequency Response](#)

Exercises and Analysis Exercise11.mcd - Project11.mcd

P11.1. SPICE Equations

SPICE Equation		Description
$V_O = \frac{-A_{vNI}}{1 + \frac{A_{vNI}}{a_{vO}}} V_{off} \approx -A_{vNI} V_{off}$	11.16	Output voltage of resistance-feedback opamp with offset voltage, V_{off} .
$A_v = \frac{A_{vNI}}{1 + \frac{A_{vNI}}{a_{vO}}} \quad A_{vNI} = 1 + \frac{R_f}{R_y}$	11.3, 11.4	Midfrequency gain dependence on opamp gain a_{vO} .
$A_v(f) = \frac{A_{vNI}}{1 + \frac{A_{vNI}}{a_{vO}}} \frac{1}{1 + j \frac{f}{f_{BW}}} = A_{vO} \frac{1}{1 + j \frac{f}{f_{BW}}}$	11.20	Frequency-response function of resistance-feedback opamp amplifier.
$f_{BW} = f_{3dB} \left(1 + \frac{a_{vO}}{A_{vNI}} \right) = f_{3dB} \left(1 + a_{vO} \frac{R_y}{R_y + R_f} \right)$	11.21	Bandwidth (corner frequency) of opamp amplifier with opamp open-loop corner frequency, f_{3dB} .

P11.2. Bias Circuit Setup



1 Offset Null 1

2 Invert. Input

3 Non-Invert. Input

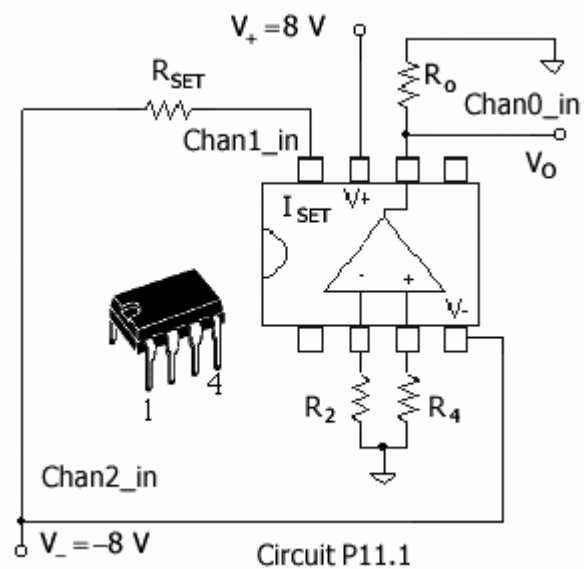
4 V-

8 ISET

7 V+

6 Output

5 Offset Null 2



Components

$$R_3 = R_4 = N_2 N_1 0 \Omega$$

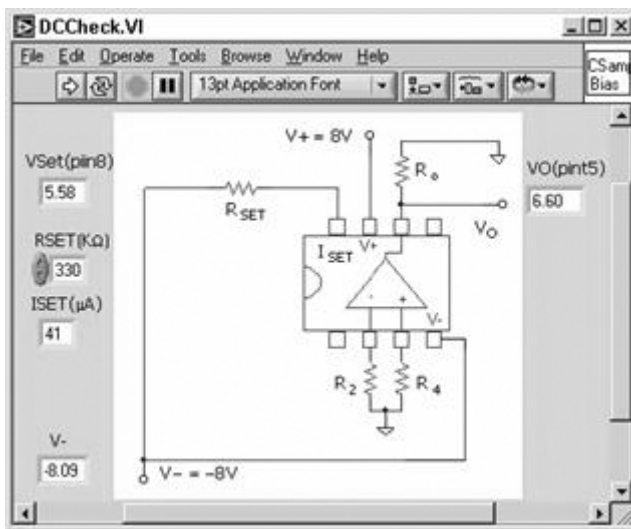
$$|V_-| = V_+ = 8 \text{ V}$$

$$R_{\text{SET}} \approx \frac{V_{\text{pin8}} - V_-}{I_{\text{SET}}}$$

$$I_{\text{SET}} \approx 25 \mu\text{A}$$

$$V_{\text{pin8}} \approx 5.7 \text{ V}$$

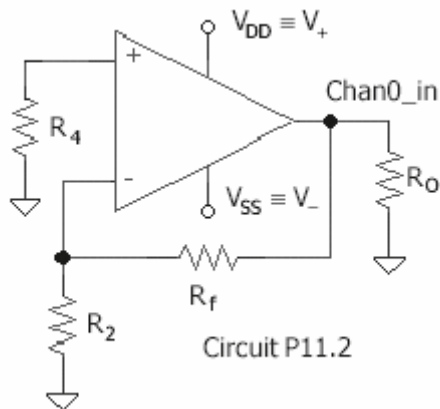
$$R_O \approx 100 \text{ k}\Omega$$



Procedure

- Do not initially turn on the power supply. Connect the circuit after having made precision measurements of the resistors. Turn on the power supply only with the circuit completely connected, Install the resistor value RSET in the Digital Control of **DCCheck.vi**. **Run** the VI. Check the dc values. Verify that ISET is roughly $25\mu\text{A}$ V_O is normally close to V_+ or V_- . *Default and save the Front Panel.*

P11.3. Opamp Offset Voltage



Components

$$R_f \approx 4 \times 10^3 R_2$$

(Very Approximate)

$$A_{vNI} \approx 4000$$

$$V_{\text{off}} = - \frac{1 + \frac{A_{vNI}}{a_{vO}}}{A_{vNI}} V_O \quad (11.16)$$

$$A_{vNI} \approx 1 + \frac{R_f}{R_2} \approx \frac{R_f}{R_2}$$

Procedure

- Install R_f and re-run **DCCheck.vi**. Verify that $|V_O|$ is less than the value without the feedback resistor. If not, use a smaller R_f . *Default and save* the information in the Front Panel.

Datasheet Offset Voltage

		Min.	Typ.	Max.	Unit
Vio	Input Offset Voltage				mV
	$V_O = 1.4 \text{ V}$, $V_{ic} = 0$ TS271C/I/M		1.1	10	

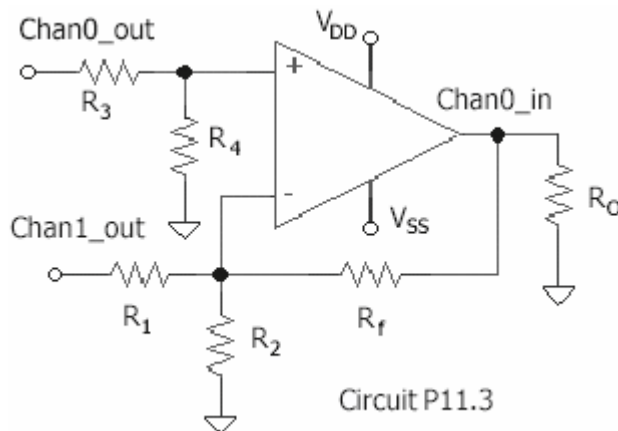
P11.4. Evaluation of the Bias Balancing Circuit

Components

$$R_1 = R_3$$

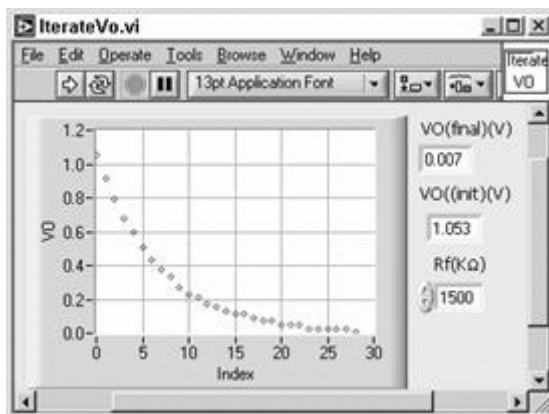
$$R_1/R_2 = R_3/R_4 = 1000$$

$$R_f \approx 4000R_2$$



Procedure

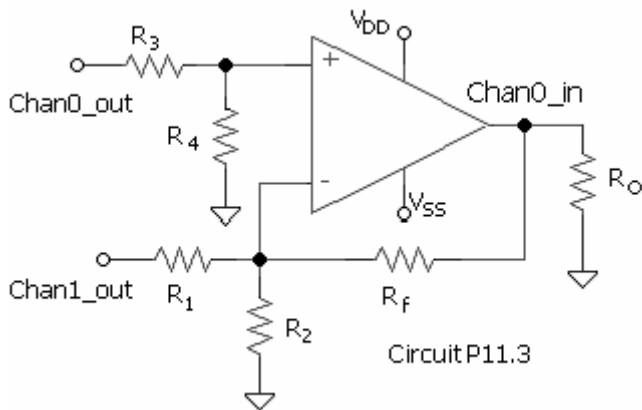
- Connect the additional circuit components, R_1 and R_3 . (Gain measurement results depend on the precision of the values entered.) The VI will sweep Chan1_out to drive the output to near 0 V. Set the value of R_f in the Digital Control of **IterateVo.vi**.



- Run** the VI to verify that the balancing circuit is functioning properly. The VI, **IterateVo.vi**, is a subVI in the amplifier gain evaluations, for balancing the circuit (i.e., for setting the dc output to near 0 V). The indicated $\text{VO}(\text{init})$ should match the value from the result of P11.3. *Save*

the Front Panel.

P11.5. Evaluation of the Gain and Signal Limits with Swept Input

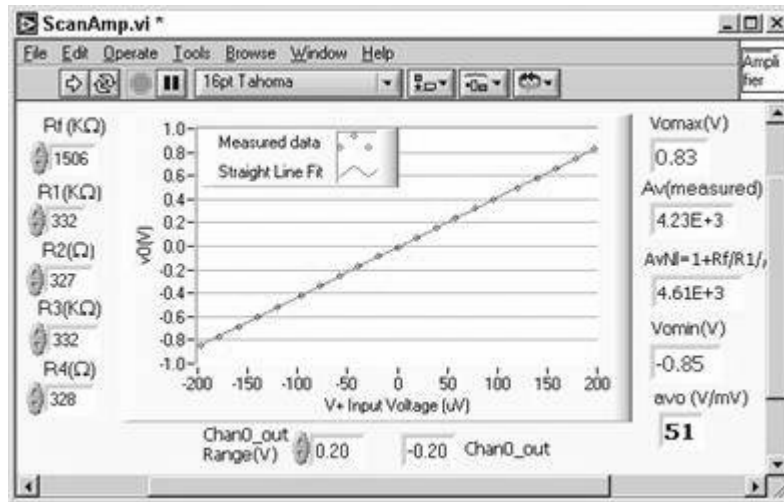


Procedure

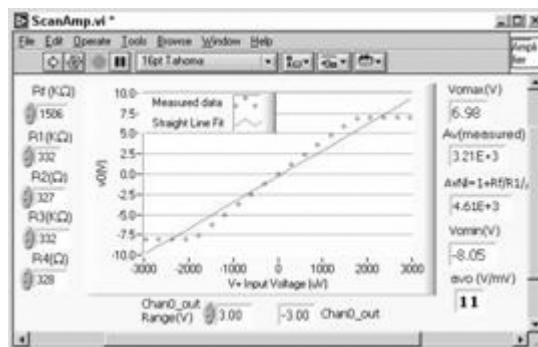
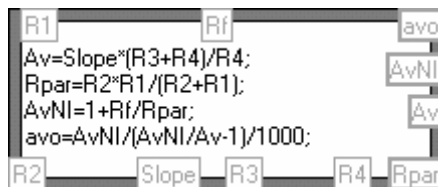
- Enter the resistor values in the Digital Controls of **ScanAmp.vi**. The gain of interest is amplifier gain, $A_v = \text{Chan0_in}/V_+$. This is obtained from the measured Slope of n Chan0_in versus Chan0_out and then

$$A_v = \frac{R_4 + R_3}{R_4} \cdot \text{Slope}$$

- Opamp gain, a_{vo} , is calculated in a Formula Node using A_v and the ideal gain $A_{vNI} = 1 + R_f/R_2 \parallel R_1$. Thus, the gain-measurement results depend critically on the precise value of the resistors.
- Run the VI and reset Chan0_out Range as necessary to obtain an output range of about plus and minus 1 V [graph Y axis and Indicators, Vomax(V) and Vomin(V)]. Save the Front Panel.



Formula Node Computation of avo



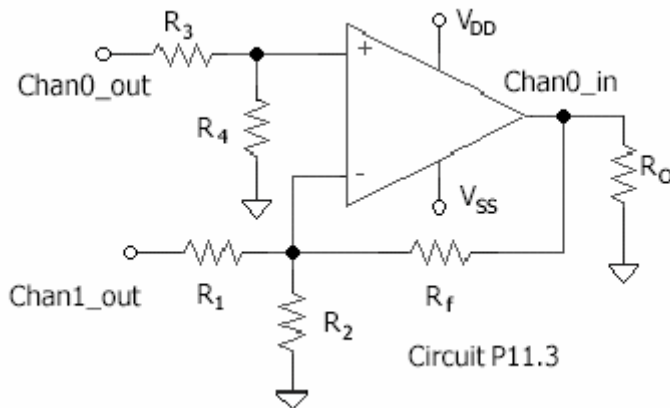
- Reset the Chan0_out Range, such as in the example (left), to drive the output to the limits. Run the VI and note the limits Vomax(V) (V_{OH}) and Vomin(V) (V_{OL}). Compare these with the datasheet values.
- Note that the datasheet numbers (below) were obtained for a single power supply and $V_+ = 10V$ and $V_- = 0V$. Thus, Vomax approximately 1 V below the power supply and Vomin is approximately 50 m above the negative supply (ground in the datasheet case and $-8V$ for our opamp amplifier).

Datasheet Limit Voltages (Power Supply = 0 to +10V)

		Min.	Typ.	Max.	Unit
--	--	------	------	------	------

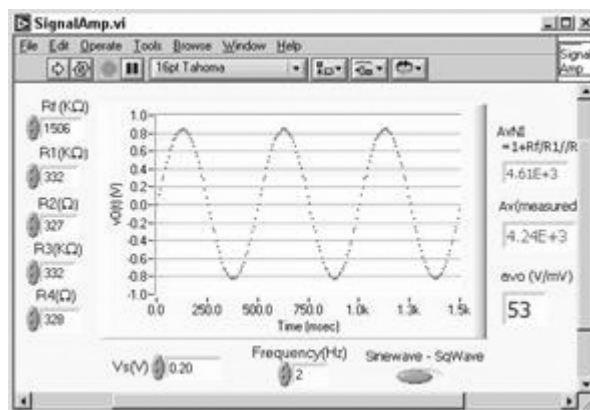
V_{OH}	High-level output voltage				V
	$V_{id} = 100 \text{ mV}$, $R_L = 100 \text{ k}\Omega$	8.7	8.9		
V_{OL}	Low-level output voltage ($V_{id} = -100 \text{ mV}$)			50	mV

P11.6. Evaluation of the Gain with Sine-Wave and Square-Wave Signals



Procedure

- **SignalAmp.vi** sends out a sine- or square-wave input signal with a frequency as selected from the Front Panel. Use a frequency of 1 or 2 Hz. It must be low as the frequency response of the amplifier is very low with the high gain of this configuration.



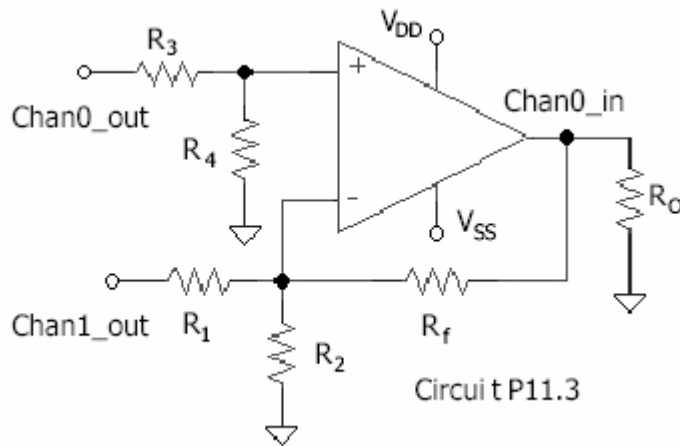
- Install resistor values in the Front Panel of **SignalAmp.vi**. Run the VI and compare the gain for sine- and square-wave signals. Compare for two different frequencies. Adjust Vs for a maximum output signal of

about 1 V. *Save* the Front Panel.

Datasheet Large-Signal Voltage Gain

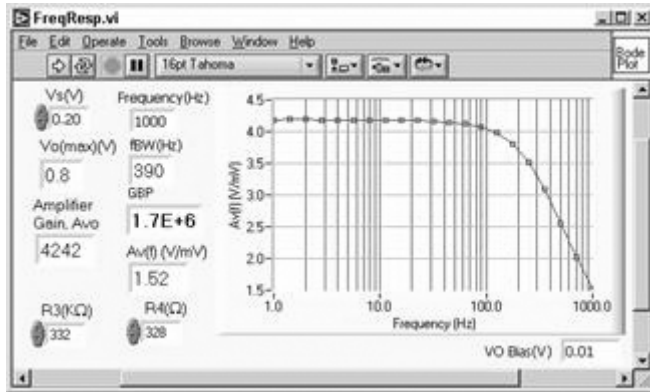
Symbol	Parameter	Min.	Typ.	Max.	Unit
A _{vd}	Large-signal voltage gain				V/mV
	V _o = 1 to 6 V, R _L = 100 k Ω , V _{ic} = 5V	30	50		
	T _{min.} < T _{amb.} < T _{max.}	20			

P11.7. Determination of the Opamp Frequency Response



Procedure

- The VI sweeps the input sine-wave signal over a range of frequencies, $1\text{Hz} < f < 1\text{kHz}$. The gain is computed and plotted and the bandwidth of the amplifier is computed and indicated. The GBP is then computed and indicated.



- Install precision resistor values for R3 and R4 in the Front Panel of **FreqResp.vi**. **Run** the VI to obtain the frequency response. Verify that Vo(max) is less than about 1 V. This must be less than the oscilloscope limit of 2 V. *Save* the Front Panel.

Datasheet Gain – Bandwidth Product

Symbol	Parameter	Min.	Typ.	Max.	Unit
GBP	Gain – Bandwidth product ($A_v = 40$ dB, $R_L = 100$ k Ω , $C_L = 100$ pF, $f_{in} = 100$ kHz)		0.7		MHz

Laboratory Project 12. Operational Amplifier Integrator and Oscillator

[P12.1 SPICE Equations](#)

[P12.2 Opamp Integrator](#)

[P12.3 Opamp Oscillator](#)

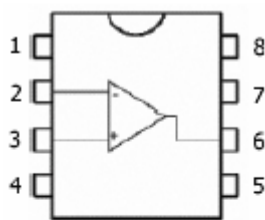
Exercises and Analysis Exercise12.mcd - Project12.mcd

P12.1. SPICE Equations

SPICE Equation		Description
$2V_{\text{opeak}} \approx \frac{V_s}{R_y C_f} \frac{T}{2}$	12.4	Opamp-integrator peak voltage amplitude of the triangular-wave for applied square-wave with peak V_s and period T and with $R_y = R_1 R_2$ and C_f .

SPICE Equation		Description
$v_o(t) = V_s \sin\left(2\pi \frac{t}{T}\right) + \frac{T}{2\pi R_y C_f} V_s \cos\left(2\pi \frac{t}{T}\right)$	12.6	Cosine-wave output for sinewave input with peak V_s , period T , $R_y = R_1 \parallel R_2$ and C_f .
$T \approx 2.2 R_f C_o$	12.26	Period of oscillator for R_f with C_o and $R_1 = R_3$.
$T_{\text{plus}} = R_f C_o \ln \left(\frac{V_{op} + \frac{R_3}{R_3 + R_1} V_{om}}{\frac{R_1}{R_3 + R_1} V_{op}} \right)$ $T = T_{\text{plus}} + T_{\text{minus}}$	12.18	General form for T_{plus} in terms of R_1 , R_3 , V_{om} , and V_{op} .
$T_{\text{minus}} = R_f C_o \ln \left(\frac{V_{om} + \frac{R_3}{R_3 + R_1} V_{op}}{\frac{R_1}{R_3 + R_1} V_{om}} \right)$	12.22	General form for T_{minus} in terms of R_1 , R_3 , V_{om} , and V_{op} .

P12.2. Opamp Integrator



- 1 Offset Null 1
- 2 Invert. Input
- 3 Non-Invert. Input
- 4 V-
- 5 Offset Null 2
- 6 Output
- 7 V+
- 8 ISET

Components

Refer to [Project 11](#) for resistor values. Use Project12.mcd to calculate the capacitor value.

Obtain C_f from the square-wave equation

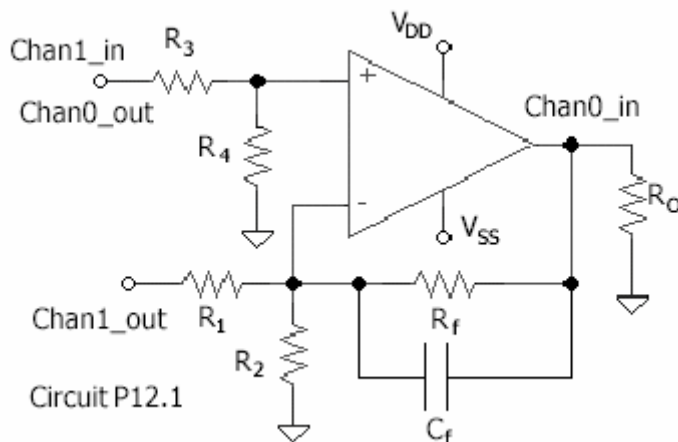
$$V_{\text{opeak}} \approx \frac{V_{s+}}{R_2 C_f} \frac{T}{4}$$

(11.4)

$$C_f \approx \frac{T}{4000 R_2} \quad V_{s+} = V_s / 1000 \quad V_s \approx V_{\text{opeak}}$$

V_{s+} is the voltage applied at the noninverting terminal (pin 3) of the opamp, and V_s is the voltage from Chan0_out.

Use $T_{\text{max}} = 0.1$ S for the initial frequency of $f = 10$ Hz.

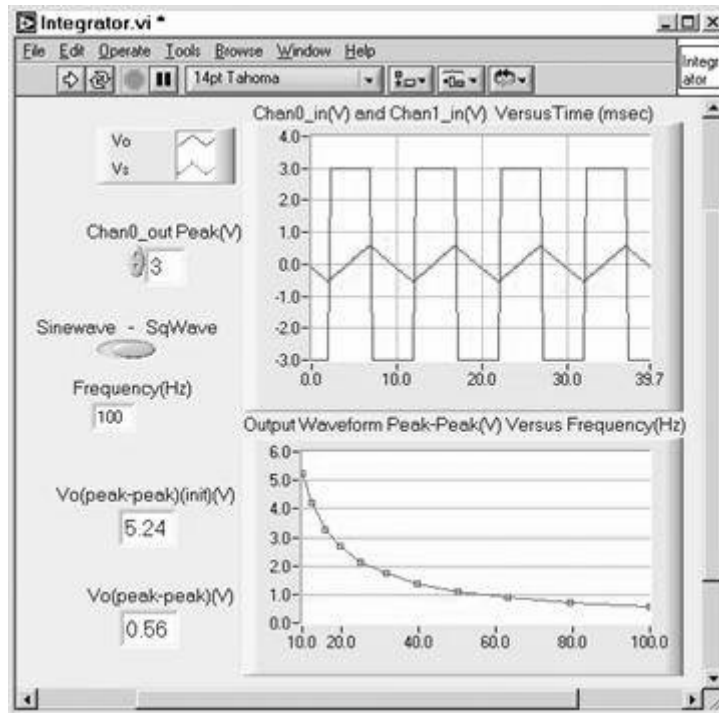


Procedure

- Connect the circuit, which includes adding the capacitor and moving Chan1_in to connect to Chan0_out.
- In the Front Panel of **Integrator.vi**, set V_s (Chan0_out Peak(V)) at 4 V to 10 V. This is adjusted to set the initial peak (lowest frequency) of the triangular wave. Also open subVI, **IterateVo.vi**, to observe the process of setting the output to near 0 V.
- **Run** the VI with the signal set to the SqWave mode. Re-adjust V_s for a value of V_o (Peak-to-Peak) (init) of between 5 V and 6 V. Verify that the initial and final peak-to-peak output voltage ratio is approximately 10. Note that it will be less than 10 due to the finite $R_f C_f$. The ideal integrator has infinite R_f . Note that only the last half of the output array is used in

the output voltage peak determination, to avoid initial transients. Also, the average value of the output voltage channel is subtracted from the waveform in the oscilloscope. *Obtain a log of the Front Panel. Default and save.*

- Now set the input waveform switch to Sine-wave and re-**run**. Note the lower peak value. *Obtain a log of the Front Panel.*



P12.3. Opamp Oscillator

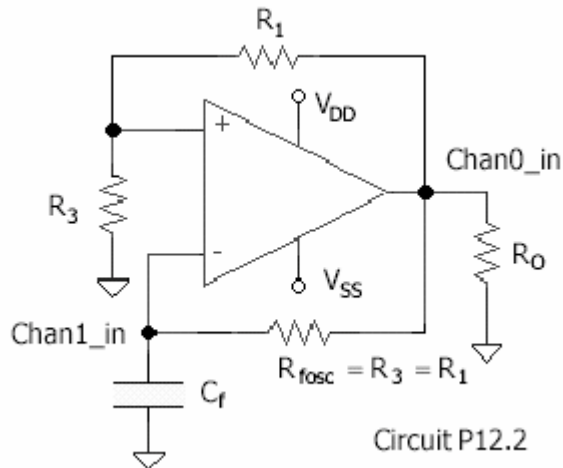
Components

Use $R_1 = R_3$ from the integrator.

Make $R_{fosc} = R_1 = R_3$ and use C_f from the integrator.

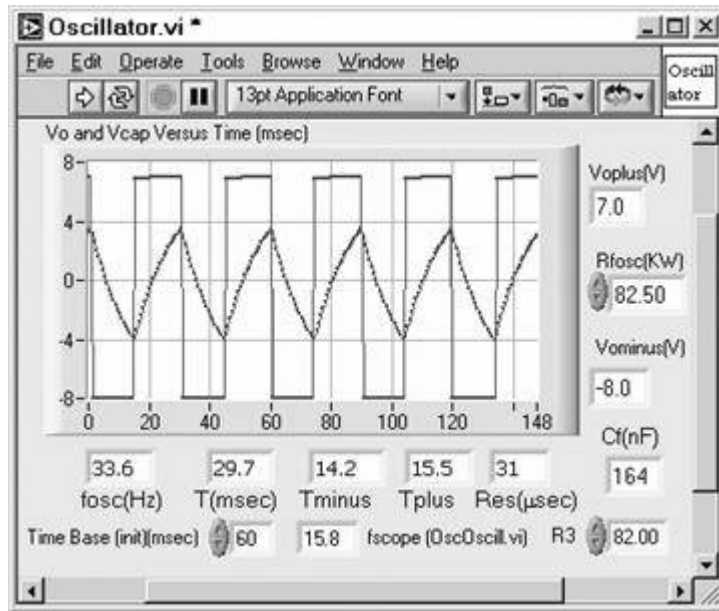
$$T_{osc} \approx 2.2R_{fosc}C_f \text{ (12.26)}$$

Note that T_{osc} is about $\frac{1}{2}$ of the design T_{max} from the integrator or about 50 ms.

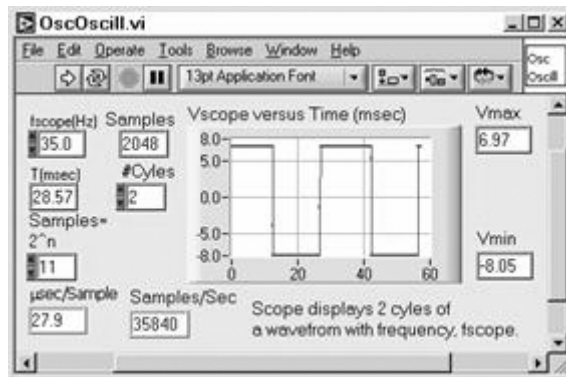


Procedure

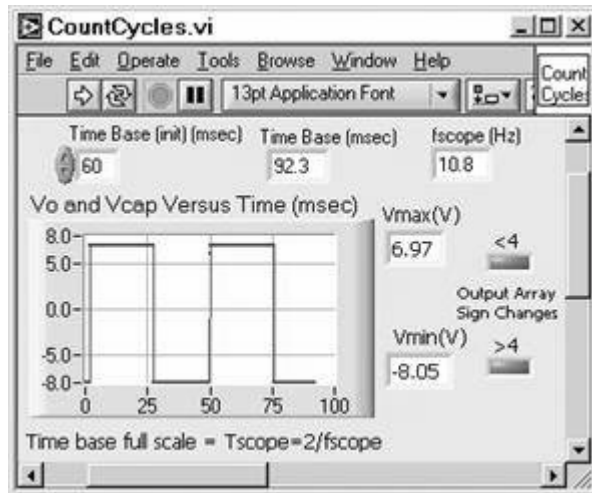
- Turn off the power supply to reconnect the circuit into the oscillator configuration. Note the connection of Chan1_in, which measures the capacitor voltage.
- The main VI, **Oscillator.vi**(below), contains subVI's **CountCycles.vi** and **T.vi**. **CountCycles.vi** determines the number of cycles in the oscillator output data array (Chan0_in) by checking for sign changes. It then increases or decreases the sample rate (for a fixed number of samples) until the number of sign changes in the array is 4. **T.vi** then checks the number of samples in the positive and negative pulses and multiplies the total by the sample rate to obtain the oscillator frequency. This frequency is used to set the time base of the oscilloscope, **OscOscill.vi**, to display five cycles of the oscillator waveform in the Front Panel of the main VI.



- Connect the circuit with the *power supply off* to avoid damage to the chip. Then turn on the power supply. Open the oscillator oscilloscope VI, **OscOscill.vi**. Set the oscilloscope frequency, **fscope**, to the expected frequency of the oscillator (e.g., about 50 Hz). This sets up the scope for a full sweep of $2/f_{scope} = 40$ msec (Note that the scope is set for 2048 samples per sweep. Thus the resolution is $2/(2048f_{scope}) = 19 \mu s$ for this example.)



- Run **OscOscill.vi** to verify that the circuit is oscillating. Reset **fscope** for a time base that includes about two cycles of oscillation and **rerun**. Note that this **fscope** setting is approximately the frequency of the oscillator.
- Open subVI **CountCycles.vi**. Set **TimeBase (init)(msec)** (full sweep time of scope) to include about two cycles of the oscillator waveform. (Use **Time Base = $2/f_{scope}$** .) **Run** the subVI to verify that the VI can find the final time base, which includes about two cycles.



- Open **Oscillator.vi**. Set **Time Base(init)(msec)** to the value obtained in **CountCycles.vi**. Set in the value of **Rfosc**. **Run** the VI to obtain a final plot of the oscillator waveform and the frequency and period of the oscillator. Note the value found for the capacitor. Compare this with the value expected. The precision of the value depends on the precision of the **Rfosc**. Note that if the **Time Base(init)** is not set close to the final value, the VI will find the correct value, after some searching. Set the value of **R3** for the record. It plays no role in the VI. *Save* the information in the Front Panel by obtaining a *log*.
- Replace R_{fosc} with $R_{fosc} \approx 10 R_{fosc}$ (very approximately). Set in the new value in **Oscillator.vi**. Use **Oscillator.vi** to measure the new oscillator frequency. Compare the capacitor measurement. Obtain a *log* for this case.
- Now replace R_3 with $R_3 \approx R_3/10$ (very approximately). Set in the new value of **R3** for the *log* record. **Run** the VI. Note that the capacitor measurement is not valid for this case. The capacitor now charges to about 10% of the maximum output voltages before switching. This increases the frequency of oscillation, due to the shorter transient time. Obtain a *log* and *save*.

Laboratory Project A. Communicating with the Circuit Board Using the DAQ

[PA.1 Sending and Receiving Voltages with the Sending and Receiving VIs](#)

[PA.2 Sending and Receiving Voltages from the Front Panel](#)

[PA.3 Plotting Measured Samples](#)

[PA.4 Using the Autoranging Voltmeter](#)

[PA.5 Observing the Oscilloscope Output Graph](#)

[PA.6 Discrete Output Voltage from the DAQ](#)

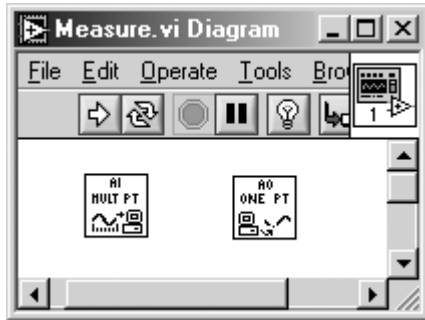
[PA.7 Discrete Input Voltage from the Circuit Board](#)

[PA.8 Using the Simultaneous Sending/Receiving Function](#)

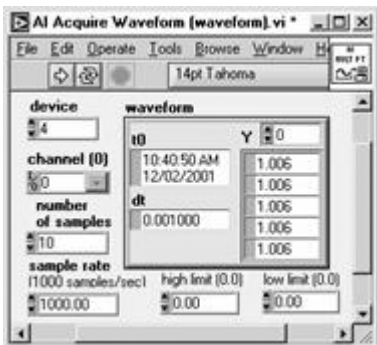
TABLE PA.1		
Diagram	Tools Palette	Shift/Right Click
	Functions Palette	Right Click–Stickpin to hold
	Menu Sequence:	Window>>Show Functions Palette
Front Panel	Tools Palette	Shift/Right Click
	Controls Palette	Right Click–Stickpin to hold
	Menu Sequence:	Windows>>Show Controls Palette

PA.1. Sending and Receiving Voltages with the Sending and Receiving VIs

- Connect Chan0_in directly to Chan0_out on the circuit board or connector block. *Double Click* on both of the icons in the Diagram of **Measure.vi** (initial version) to bring up the VI Front Panels. These are **AO Update Channel.vi** and **AI Acquire Waveform.vi**. Set both VIs to your device number for your DAQ card and to channel 0 (using the Operating Tool or finger). Set the number of samples to 10, for example. Set the high limit and low limit in **AI Acquire Waveform.vi** to default 0.0 V. Obtain a *default* status of the settings in both VIs. For this, go to menu Operate>>Make Current Values Default.



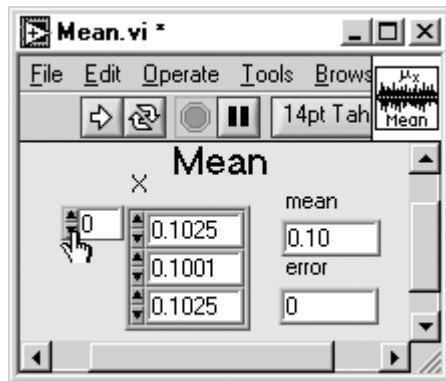
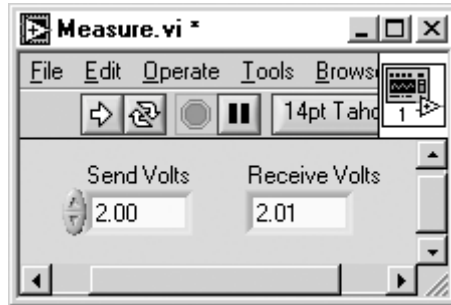
- In the Front Panel of **AO Update Channel.vi**, set in a voltage of 1 V or similar (using the Operating Tool). **Run** the VI, **AO Update Channel.vi**. To **run** the VI, use menu sequence Operate>> **Run** (or, preferred, Ctrl/R). Then **run AI Acquire Waveform.vi** and note the voltage read (waveform array Digital Indicator). Click on the up and down arrows by the Y to scroll through more data points.
- Verify that that send and receive are consistent. Try some other voltages. Note the maximum allowed is the highlimit, 10 V. Close the send and receive VIs. Leave **Measure.vi** open.



PA.2. Sending and Receiving Voltages from the Front Panel

- In the Front Panel of **Measure.vi** (version 2), **run** the VI (Ctrl/R) with a variety of voltages sent out and verify that the send (output) and read

(input) agree. From the Diagram of **Measure.vi**, *Double Click* on **Mean.vi** to open the Front Panel of this VI. Note the X Array Digital Control and the mean Digital Indicator. Now **run Measure.vi** with Send Volts = 0.1 V. Note that the measurement data appear in the Array Digital Control. These are the data points that are averaged.

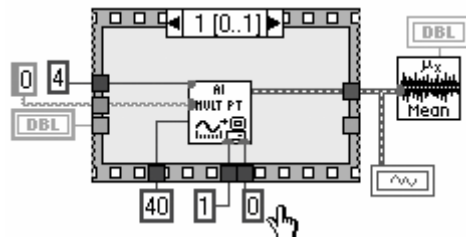


- Now go to **Mean.vi**, *Right Click* on the indicators, select Format and Precision, and type 4 for Digits of Precision. Then scroll through the array data (by *clicking* on the up and down arrows of the Array Digital Control as shown in the sample) and compare the numbers with the mean. There will probably be some scatter in the least significant digit, due to noise. Note that the number received and averaged is not exactly the same as that requested to be sent out. This is due to the discrete nature of ADC (analog-to-digital conversion) and DAC (digital-to-analog conversion) as discussed and demonstrated in a following section.

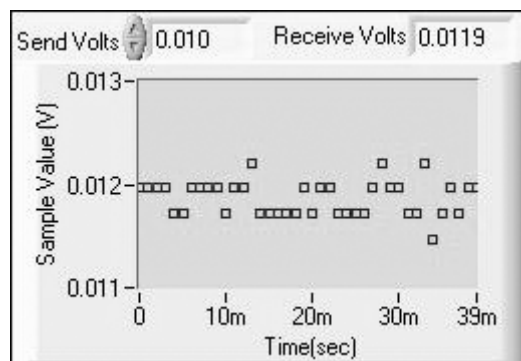
PA.3. Plotting Measured Samples

- Open **PlotSamples.vi**. Prepare the Diagram for displaying the samples. For high resolution (discussed later), set the high limit and low limit as shown in the example to 1 V and 0 V, respectively. The number of samples can be 10 to 100 and is not critical at this point. We only want to gain some experience with graphs and display the point scatter

expected under the conditions set up here. The number of samples is 40 in the example.



- Go to the Front Panel to **run** the VI with the voltage sent out similar to that shown in the example, 0.1 V. This must be lower than the high limit setting for the DAQ input, which is 1 V. At this voltage level, there will certainly be some data scatter. *Click* on the Y-axis numbers, go to Formatting..., and set the Digits of Precision to 3.



- Now **run** the VI repeatedly. Note that even though there is data scatter, the average value is relatively constant (in the Received Volts Indicator). Note also that the X-axis is in msec. The total time in the graph is 40 times 1 ms (including 0) and **AI Acquire Waveform.vi** is set for 1000 samples/sec.
- To set (or verify) the same sample rate as in the example, open the Front Panel of **AI Acquire Waveform.vi**. Type in the number 1000 for the sample rate and *default* the value. To *default*, *Right Click* on the indicator and go to Data Operations>>Make Current Value Default. *Save* the VI and close.
- Re-run** the VI with **Send Volts = 0.015**. Subtract the difference between the measured value of 0.1 V and 0.14 V sent out. Compare the difference with the resolution expected, as obtained from [Table PA.2](#). Note that the

high limit is 1V and the mode is Unipolar.

TABLE PA.2

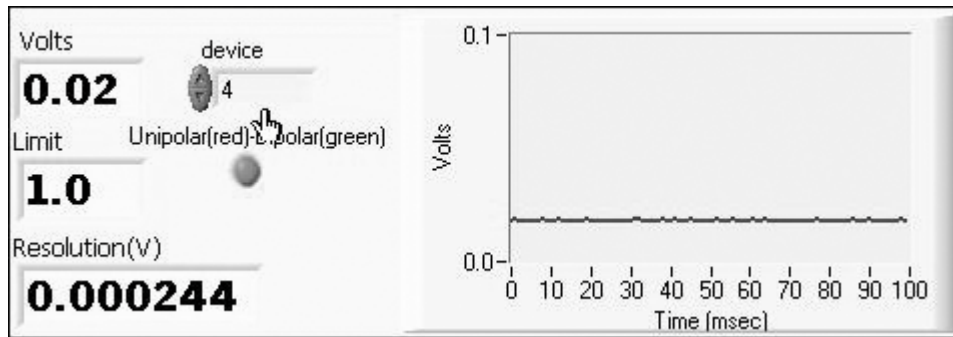
	Range	Resolution
Unipolar	0 to +10V	2.44 mV
	0 to +5V	1.22 mV
	0 to +2V	488 μ V
	0 to +1V	244 μ V
	0 to +500mV	122 μ V
	0 to +200mV	48.8 μ V
	0 to +100mV	24.4 μ V
Bipolar	-10 to +10V	4.88mV
	-5 to +5V	2.44mV
	-2 to +2V	1.22mV
	-1 to +1V	488 μ V
	-500 to +500mV	244 μ V
	-200 to +200mV	122 μ V
	-100 to +100mV	48.8 μ V
	-50 to +50mV	24.4 μ V

PA.4. Using the Autoranging Voltmeter

- Open a **new** VI in LabVIEW. Then *Right Click* on the Diagram, and go through Functions>>Select A VI.... and locate **ProjectA.llb**.



- Click on **Voltmeter.vi** and Click **OK**. Place the icon on the Diagram. *Double Click* on the icon to get the Front Panel.



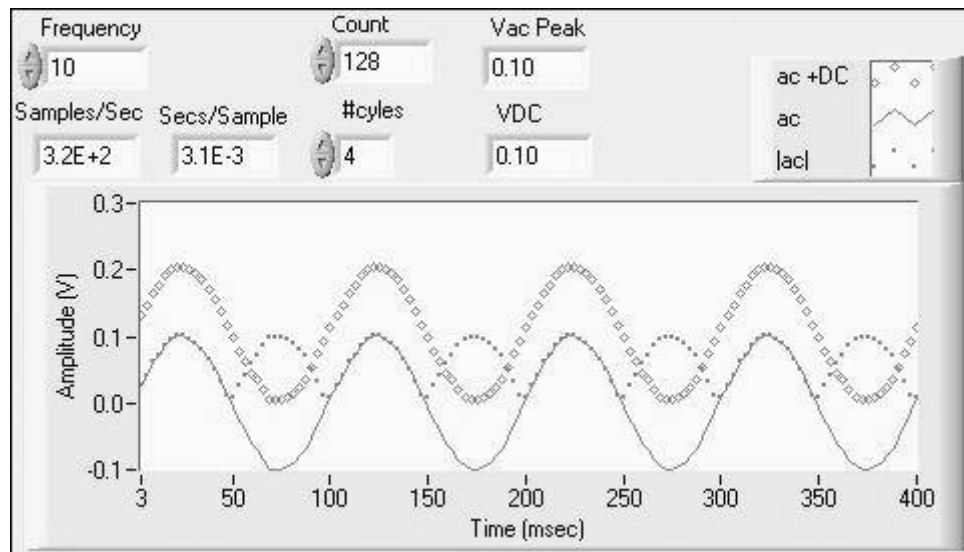
- Now place **AO Update Channel.vi** in the Diagram of your new VI. This is in Functions>>Data Acquisition>>Analog Output.



- *Double Click* on the icon to open the VI. Set the device numbers to match that of your DAQ. Verify that output channel 0 (Chan0_out) is connected directly to input channel 0 (Chan0_in) on the connector block or circuit board. Now **run AO Update Channel.vi** repeatedly to send out a series of voltages, such as 0.09 V, 0.9 V and 9 V and for each, **run Voltmeter.vi**. Observe the automatic changing of the limits setting for the various voltages sent out. Note that the voltage is always measured with good precision. Try a negative voltage out.

PA.5. Observing the Oscilloscope Output Graph

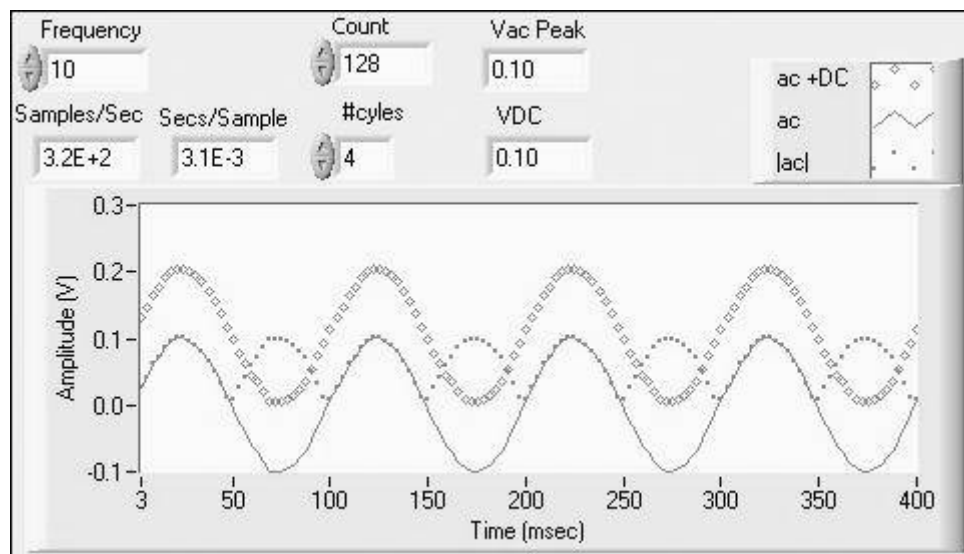
- To run the oscilloscope of [Fig. A.14](#), in the project library, **ProjectA.llb**, open **VI FG_A.vi**. (This VI can also conveniently be opened from the Diagram of a new LabVIEW VI.) Then under the Browse menu (from the Front Panel of **FG_A.vi**) go to Show VI Hierarchy. Locate the icon for **OscilloscopeA.vi** and *Double Click* to open the VI.



- Run **FG_A.vi** at various values of V_s (V_{ac} peak signal out) and V_{DCout} and observe the combination waveform output in **OscilloscopeA.vi**. The limit setting is 10 V. The function generator VI, **FG_A.vi**, is discussed in a following section.

PA.6. Discrete Output Voltage from the DAQ

- The VIs for this exercise are in **Project A.llb**. The first example is for DAC (digital-to-analog conversion), where the receiving resolution is much smaller (better) than the sending resolution. This therefore reveals the discrete nature of the output voltage.
- The VI for assessing the discrete nature of DAQ operation is **Discrete.vi**. This VI first sweeps the requested voltage to be sent out in a quasi-continuous manner. The programmed sweep increment size is Max Volts/100. For example, for a sweep range of Max Volts = 10 mV, the sweep increments are 100 μ V. However, the output voltage has a resolution of 4880 μ V (bipolar).

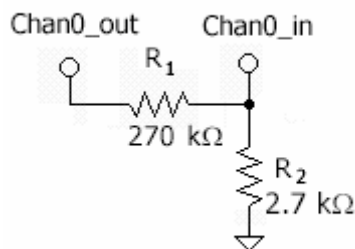


- The receiving function is set with high limit and low limit of magnitude 0.1 V. The resolution for receiving is thus $48.8 \mu\text{V}$, that is, much less than the the output voltage steps of magnitude $4880 \mu\text{V}$.
- We will examine, for example, the bipolar mode of the output channels. For this, we need to verify the DAQ configuration for bipolar output. Go to the Start Menu>>Programs>>National Instruments>>Measurements and Automation. (There may be a shortcut for this on the desktop.) Open Devices and Interfaces and then *Right Click* on the DAQ designator and get Properties. *Click* on the AO tab and set bipolar.
- Open **Discrete.vi**. Verify that Chan0_out is connected directly to Chan0_in on the connector block or circuit board. Verify that in the Front Panel of **Discrete.vi**, Max Vout is set to 10 mV (for a sweep step size of $100 \mu\text{V}$) and that the high limit is set for 0.1 V.
- **Run** the VI. The example of this section illustrates what to expect for the case of the DAQ configured for the bipolar mode. Note that the dc offset of the DAQ has been subtracted from the plot, such that the measured voltage appears to be zero volts for zero volts sent out.
- A number for the size of the measured steps is indicated [Vout Step Avg (mV)]. It is obtained by the VI as an average of all the steps. The number shown is expected, as it is twice the DAC reference voltage of $V_{\text{ref}} = 10\text{V}$ divided by 2^{12} or $2V_{\text{ref}}/2^{12} = 4.88\text{mV}$ where the exponent is the number of bits of the DAC. The steps are 2.44 mV for a unipolar output configuration.

- The graph on the right is a plot of input-channel samples versus sample index. The data illustrate the point scatter due to noise. The noise-free number in the plot at the end of the execution should be $2 \times 4.88 \text{ mV} = 9.76 \text{ mV}$. Note that the scattered sample points are separated by a discrete value of $48.8 \mu\text{V}$. An Array Digital Indicator displays the samples and a Digital Indicator shows the average.
- Try other full-range sweep values such as Max Out = 20 mV and 5 mV. With Max Out = 10 mV, go to the Diagram and reset the number of samples from 50 to 500. Re-run and note the increased precision, for example, in Chan0_in Array Avg.

PA.7. Discrete Input Voltage from the Circuit Board

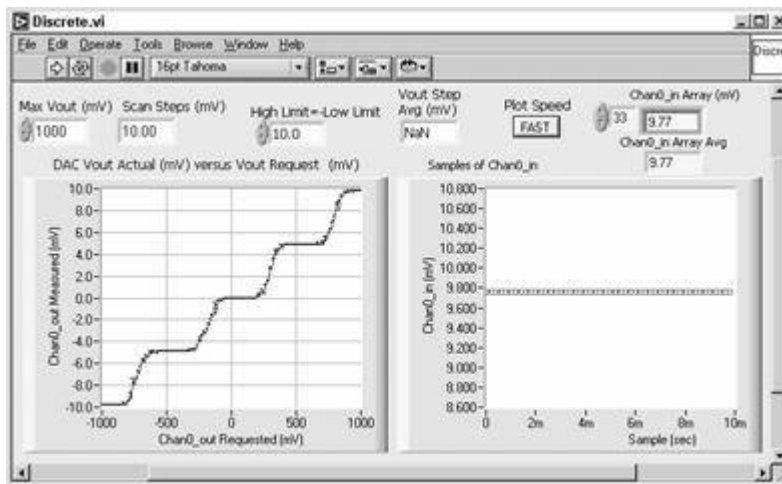
- For this measurement, we need a high-resolution output voltage source. If we are willing to give up voltage range, this can be accomplished with a simple resistor voltage divider. In the example here, the circuit forms an approximate 100:1 divider (270- and 2.7-k Ω resistors). This provides a source with a resolution of $49 \mu\text{V}$ (bipolar).



- **Discrete.vi** will be used to investigate the discrete nature of the input channels. Open the VI and set high limit = 0.1 (with low limit = -0.1), for a receiving resolution that is also about $49 \mu\text{V}$. Use Max Vout = 1 V (1000 mV). The output voltage *sweep range* is then $\text{Max Vout}/100 = 10 \text{ mV}$ with the voltage divider. The programmed output voltage *step size* is $\text{Max Vout}/100 = 10 \text{ mV}$. Thus, after the voltage divider, this is $100 \mu\text{V}$. The resolution of the output and input functions is similar.
- **Run Discrete.vi.** The plot is essentially a straight line with similar output and input resolution. Note that the voltage sent out (X-axis) is the actual channel voltage and that the voltage received (Y-axis) is after the voltage divider. Note also that the scatter of the input channel samples

(right graph) reflects the discrete nature of the receiving function and the points are separated by about 49 μV .

- Set high limit = 10 V. **Run** the VI with the same sweep range (example below). The output voltage (after the voltage divider) has a much higher resolution (better) than the receiving function. The steps in the sweep plot have a magnitude equal to the receiving resolution. Also, the sample points are scattered by the amount of the resolution of about 4.9 mV. An example of this case is shown in [Fig. A.18](#) (Unit A). To check the value of the differences between scattered points, **run** the VI on SLOW and halt the execution in a transition region. Then step through the Chan0_in Array(mV) Indicator.



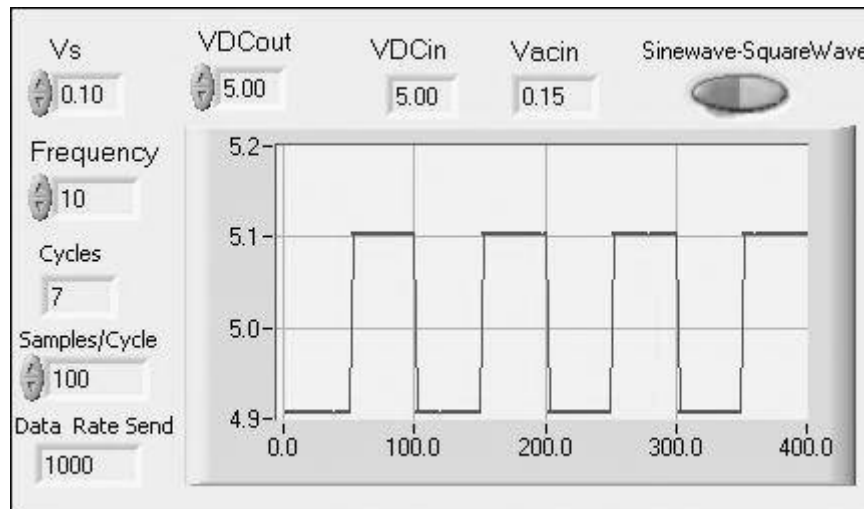
- The example here illustrates the transition range of voltage between steps; that is, the transition is not abrupt. This is due to noise and the bit uncertainty that exists in the transition regions. Again, the bit-uncertainty effect may be observed during the sweep with Plot Speed set on SLOW.
- For comparing the two cases with the two limit values and with smoothing of the data with averaging, use **AvgDiscrete.vi**. This VI **runs** the subVI **Discrete.vi** sequentially with the high limit set to 01 and 10 (with the low limit of -0.1 and -10). The two plots appear together in a single graph in the Front Panel of **AvgDiscrete.vi**.
- Open **AvgDiscrete.vi** and **Discrete.vi**. In **AvgDiscrete.vi**, verify that Max Out = 1000 mV and that Number Avgs is set at 5 to 10. Run **AvgDiscrete.vi** to compare the two plots with data smoothing.

PA.8. Using the Simultaneous Sending/Receiving Function

- With Chan0_out and Chan0_in connected on the circuit board, open

FG_A_2.vi. Also open **SR_A.vi** and **OscilloscopeA_2.vi**. Open these from Browse>>Show VI Hierarchy. **Run** the top VI for various V_s and VDCout values. **Run** the VI at various frequencies and note the change in Cycles and Sample Rate. The number of cycles is increased with frequency automatically to offset delay between sending and receiving. Note the Number of Samples in the array as indicated in the Front Panel of **SR_A.vi**, for the various changes. **Run** the VI with SquareWave selected.

- In the example, VDCout = 5 V and $V_s = 0.1$ V. There is a slight offset in this example. The oscilloscope's high limit and low limit are set for 10 V and -10 V. Therefore, the resolution is about 5% of the V_s value. There may also be some DAQ offset.



Laboratory Project B. Characterization of the Bipolar Junction Transistor for Circuit Simulation

[PB.1 SPICE Parameters and Transistor Diagram](#)

[PB.2 SPICE Equations](#)

[PB.3 Diode-Connected Transistor Measurements](#)

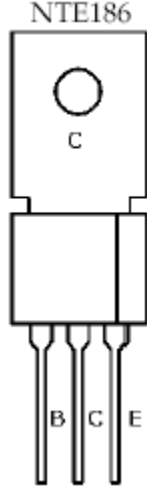
[PB.4 Measurement of \$\beta_{DC}\$ versus the Collector Current](#)

[PB.5 BJT Output Characteristic Measurement](#)

PB.6 Simulation of the Output Characteristic Measurement

Exercises and Analysis ExerciseB.mcd-ProjectB1.mcd-ProjectB2.mcd-ProjectB3.mcd.

PB.1. SPICE Parameters and Transistor Diagram

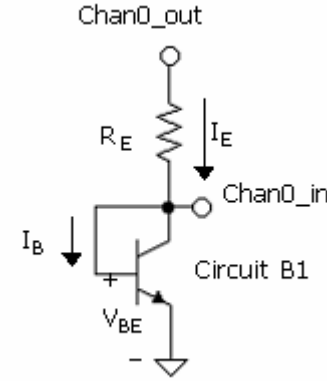
SPICE PARAMETERS			
SPICE Name	Math Symbol	Description	
IS	I_S	Saturation current.	
NF	n_F	Forward-current emission coefficient.	
ISE	I_{SE}	Base - emitter leakage saturation current.	
BF	β_F	Ideal maximum forward current gain.	
NE	n_E	Base - emitter leakage emission coefficient.	
VA	V_A	Forward Early voltage.	
BR	β_R	Reverse current gain.	

PB.2. SPICE Equations

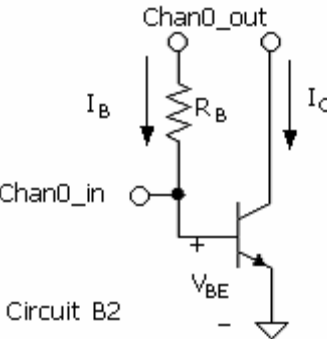
SPICE Equation		Description
$\beta_{DC} = \frac{I_C}{I_B}$	B.6	Dc ratio of collector current and total base current.
$I_C = I_S e^{V_{BE} / n_F V_T}$	From B.7	Collector current to base - emitter voltage relation for $V_{BC} = 0$ V.
$I_S = I_C e^{\frac{-V_{BE}}{V_T}}$	B.25	Form of (B.7) to calculate I_S .
$I_C = I_S e^{V_{BE} / n_F V_T} \left(1 - \frac{V_{BC}}{V_{AF}} \right)$	B.7	Active-mode collector

SPICE Equation		Description
		current to base - emitter voltage relation.
$I_B = \frac{I_S}{\beta_F} e^{V_{BE}/n_F V_T}$	B.10	Ideal base current.
$I_B \approx \frac{I_S}{\beta_F} e^{V_{BE}/n_F V_T} + I_{SE} e^{V_{BE}/n_E V_T}$	B.12	Total base current in the active mode.
$I_C = I_S \left[\left(e^{V_{BE}/V_T} - e^{(V_{BE}-V_{CE})/V_T} \right) \left(1 - \frac{V_{BE}-V_{CE}}{V_{AF}} \right) - \frac{1}{\beta_R} \left(e^{(V_{BE}-V_{CE})/V_T} - 1 \right) \right]$	B.23	General equation for collector current versus terminal voltages of the common emitter.
$I_C = I_S \left[\left(e^{V_{BE}/V_T} - e^{(V_{BE}-V_{CE})/V_T} \right) \left(1 - \frac{V_{BE}-V_{CE}}{V_{AF}} \right) - \frac{1}{\beta_{Rleak}} \left(e^{(V_{BE}-V_{CE})/n_C V_T} - 1 \right) \right]$	B.26	General equation for collector current with base leakage current and an artificial β_R .
$\beta_{DC} = \frac{\beta_F \left(1 - \frac{V_{BC}}{V_{AF}} \right)}{1 + X \left(\frac{I_{Co}}{I_S} \right)^{\left(\frac{1}{n_E} - 1 \right)}} \quad I_{Co} = I_C / \left(1 - \frac{V_{BC}}{V_{AF}} \right)$ $X = \frac{\beta_F I_{SE}}{I_S}$	B.38	Dc beta versus collector current.

PB.3. Diode-Connected Transistor Measurements



Circuit B1



Circuit B2

Components

$$R_E \approx \frac{\text{Chan0_out}_{\max} - \text{Chan0_in}_{\max}}{I_{E\max}}$$

$\text{Chan0_out}_{\max} = 10 \text{ V}$
 $\text{Chan1_in}_{\max} = 0.6 \text{ V}$
 $I_{E\max} \approx 1 \text{ mA}$
 $R_B \approx \beta_{DC\max} R_E \quad \beta_{DC\max} \approx 50$

Exercise care when forcing the transistor into the circuit board.

LabVIEW Computations

$$I_C = I_E - I_B$$

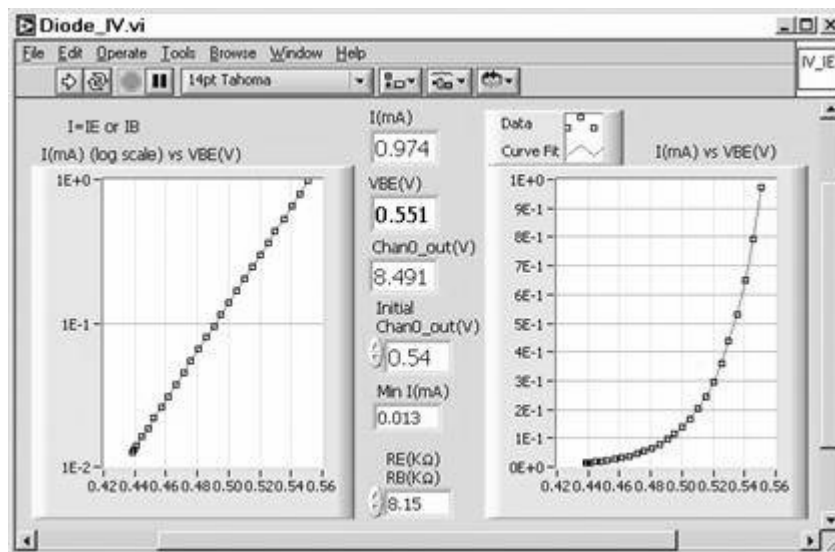
$$\beta_{DC} = \frac{I_C}{I_B}$$

$$n_F = 1 / (V_T \cdot \text{Slope})$$

Slope is from curve fit of $\ln(I_C)$ vs. V_{BE}

Procedure

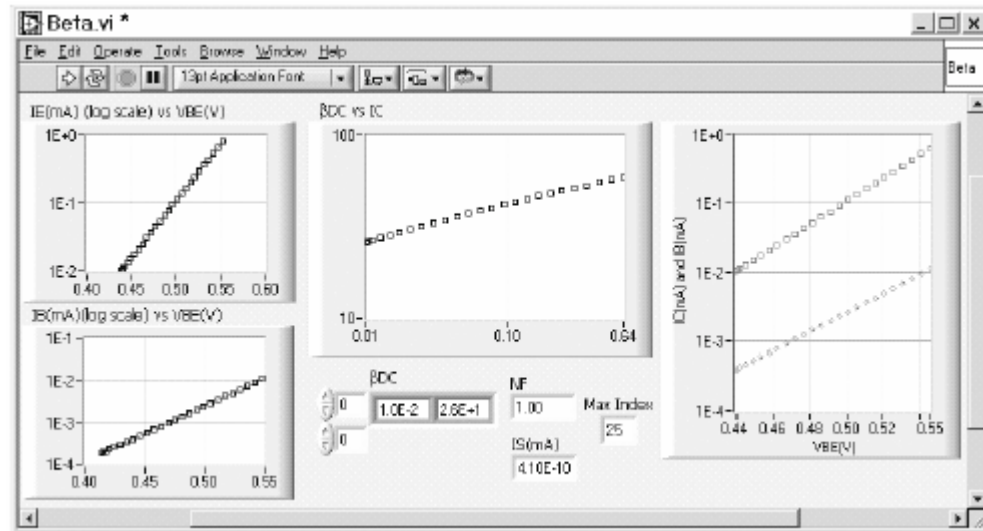
- **Diode_IV.vi** is used to obtain the current - voltage characteristic for Circuits, B1 and B2. The results will be installed into two Control Graphs of **Beta.vi** (below). Running **Beta.vi** then produces values for I_C , β_{DC} versus I_C , and parameters n_F and I_S .
- Connect Circuit B1. In the Front Panel of **Diode_IV.vi**, set the value of R_E . **Run Diode_IV.vi** and re-run while resetting Initial Chan 0_out for Min I_E of about 0.01 mA. The VI will halt for $I_E=1$ mA or Chan0_out=10 V. Obtain a log of the results. *Default* the Front Panel to save Initial Chan0_out. It is used in the next circuit.
- Connect Circuit B2. Set the value of R_B . **Run** the VI. It will halt for Chan0_out at about 10 V. Obtain a *log* of the results.



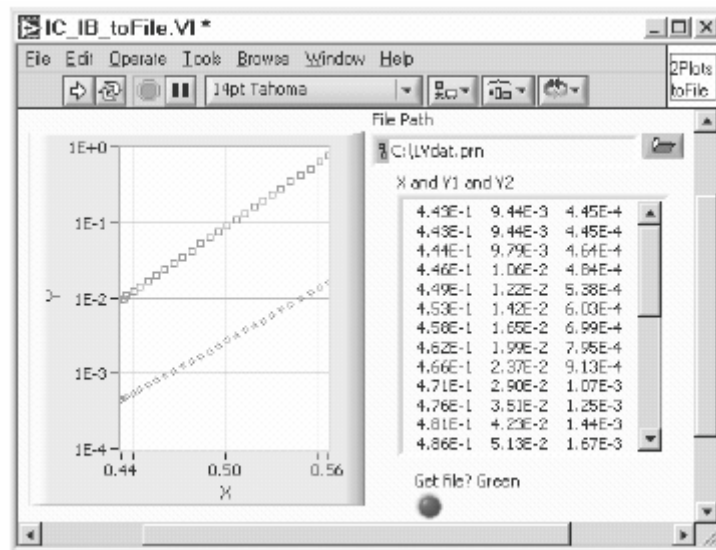
- Open **Beta.vi**. Copy the I_E and I_B data from the graph (log scale) of **Diode_IV.vi** and paste the data into the Control Graphs I_E (log scale) versus

VBE and IB versus VBE, respectively. These data are saved in the log of the Front Panel of **Diode_IV.vi**. (Reminder: For data copy and paste, *Right Click* on graph face and Data Operations/Copy or Paste.) *Default and save Beta.vi*.

- **Run** the computation VI, **Beta.vi**. **Beta.vi** will compute and plot β_{DC} versus I_C and indicate parameters n_F and I_S . Note that the plot uses the values of VBE from the data of IE. An interpolation of the VBE values of IB then is made to find the corresponding IB values.

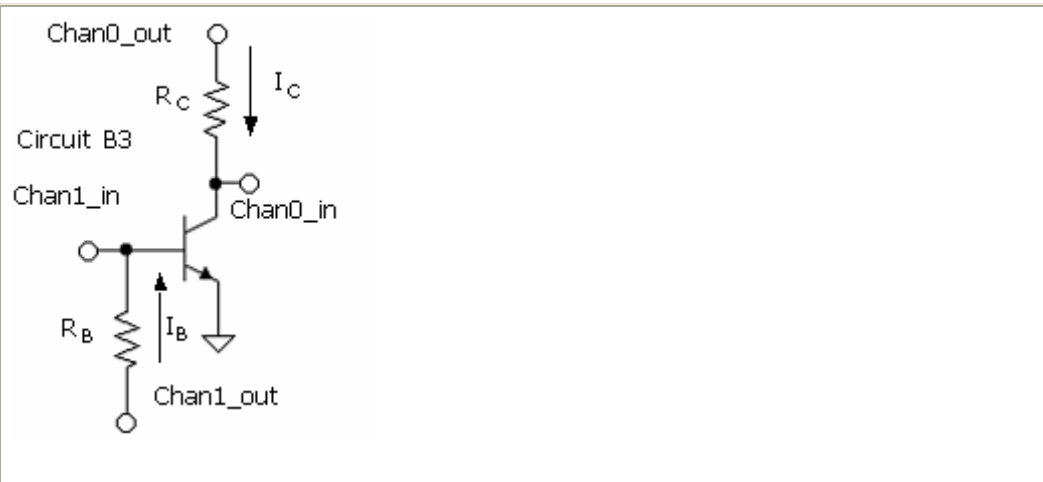


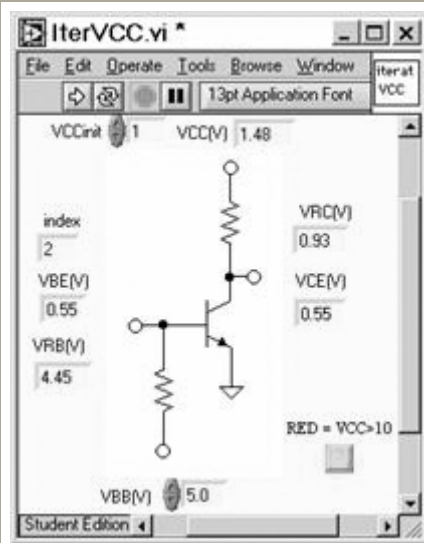
- **Beta.vi** finds the highest VBE on the low end (for IE or IB) and the smallest VBE on the high end, such that the two plots coincide over the full range of VBE in the combined-plot graph. Note the VBE scale in the right graph of **Beta.vi**. The computation of n_F uses thermal voltage, V_T , computed for $T = 27^\circ\text{C}$. At the low currents of the measurements, the device heating is probably negligible and the transistor temperature is probably about 27°C .
- Obtain a *data* file of measured I_B and I_C for use in a [Project B](#) Mathcad file, ProjectB1.mcd. Open **IC_IBtoFile.vi** (below) from the **ProjectB.llb**. Copy and paste the plots in the graph on the right side of **Beta.vi** into the Control Graph of **IC_IBtoFile.vi**. Write your file path and file name. The file must appear in the folder with the ProjectB1.mcd.



- For *simulation* only, copy and paste the data stored in the log of the Front Panel of **Diode_IV.vi**. Follow the procedure for **Beta.vi** above.

PB.4. Measurement of β_{DC} versus the Collector Current



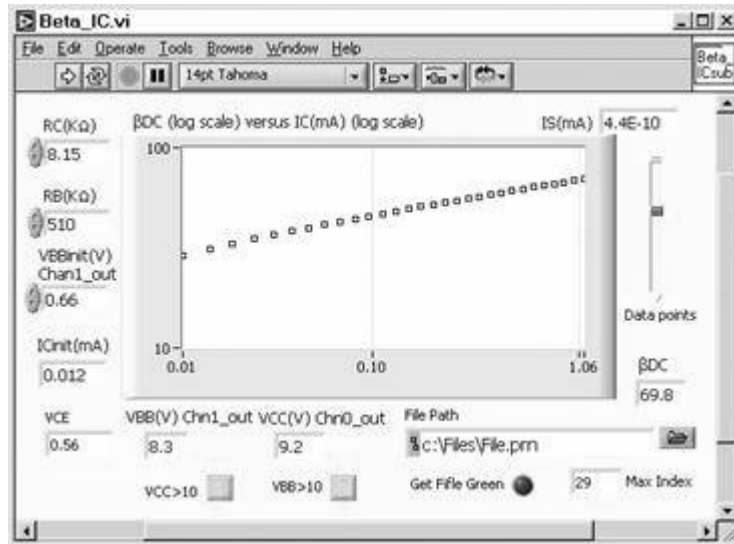


Components

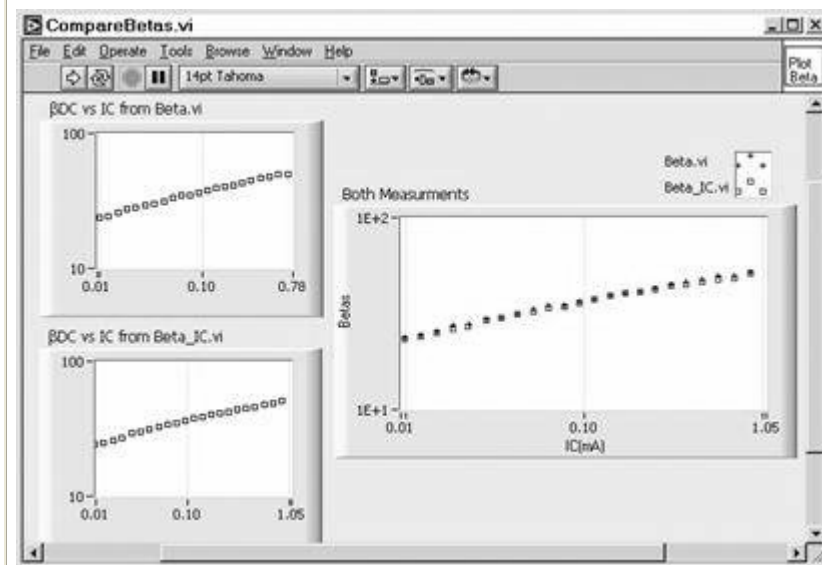
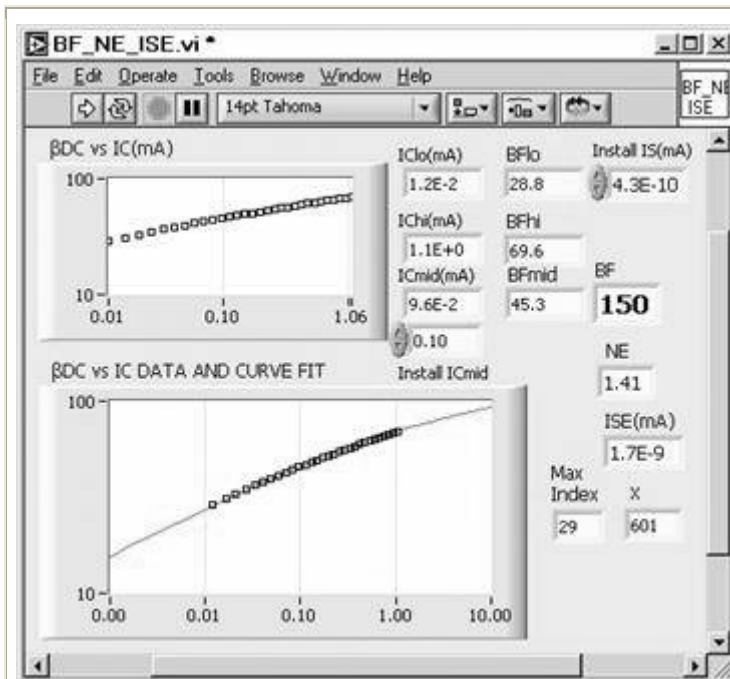
Use R_B and R_C from [PB.3](#).

Procedure

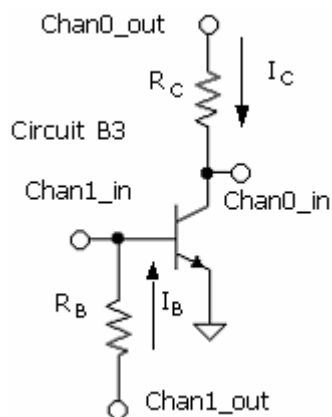
- Connect Circuit B3 and **run** subVI, **IterVCC.vi**. The VI adjusts V_{CC} to set $V_{CE} = V_{BE}$. Try various values of V_{BB} at $V_{CCinit} = 1$ V. Verify that the VI can adjust V_{CC} to make $V_{BE} = V_{CE}$ ($V_{CB} = 0$). With a proper choice of resistors, as verified in [PB.3](#), $V_{CC} \approx 10$ V for $V_{BB} \approx 10$ V. **Beta_IC.vi** sweeps V_{BB} until $I_{Cmax} \approx 1$ mA (very approximately) or $Chan0_out = 10$ V.
- Open **Beta_IC.vi**. Set values for R_C and R_B in the Digital Controls. **Run** the VI and re-**run** while resetting V_{BBinit} to obtain an initial value of I_C of about 0.01 mA. This should match the lowest current in the measurement of the emitter current of Circuit B1. Note that if V_{CC} or V_{BB} attempts to become greater than 10 V, the VI will halt with $I_C < 1$ mA. A red V_{BB} or V_{CC} button will indicate this condition. If the maximum current in the plot is near this value, this is sufficient. *Default and save* the Front Panel. Note that for the case of $V_{CC} > 10$ V for the last measured current, the last (invalid) data point set will be eliminated from the Mathcad data file.



- Obtain a data file of the graph. Set Get File Green (logic 1) to green and write the file path and file name. **Run** the VI to obtain the data file. The data file is used in ProjectB2.mcd.
- For obtaining a data set at a later time, such as for simulation only, open **Beta_IC.vi**; retrieve the data log from the Front Panel. Then go to the Diagram of **Beta_IC.vi** and open **XYtoDataFile2.vi** (or use Browse>>Show VI Hierarchy). Copy and paste data and **run** the data file VI to obtain the file.
- Open **BF_NE_ISE.vi** (below). Install your plot data and value of IS from **Beta_IC.vi** into the Control Graph and Digital Indicator, respectively. **Run** the VI to obtain IS, NE, and ISE. The VI performs a curve fit using (B.38). Three data-point sets used by the VI are nearest to 0.01, 0.1 and 1 mA.
- The VI, **CompareBetas.vi** (below), is for comparing the results from the two measurements from PB.3 and PB.4. Open the VI and paste data from **Beta.vi** and **Beta_IC.vi** in the two Control Graphs. **Run** the VI to compare the plots. The results are expected to be very similar. This is experimental proof of the fact that I_B is independent of V_{BC} , unlike the collector current. Note that $V_{BC} = 0$ V in the measurement with **Beta_IC.vi**, but that $V_{CB} = I_B R_B$ in the base current measurement of **Diode_IV.vi**. The latter is as high as about 9 V.



PB.5. BJT Output Characteristic Measurement



Components

Same circuit as in [PB.4](#)

LabVIEW Computations

SPICE I_C versus V_{CE} . This is performed with subVI **SPICE.vi**.

Straight-line curve fit from active-region data:

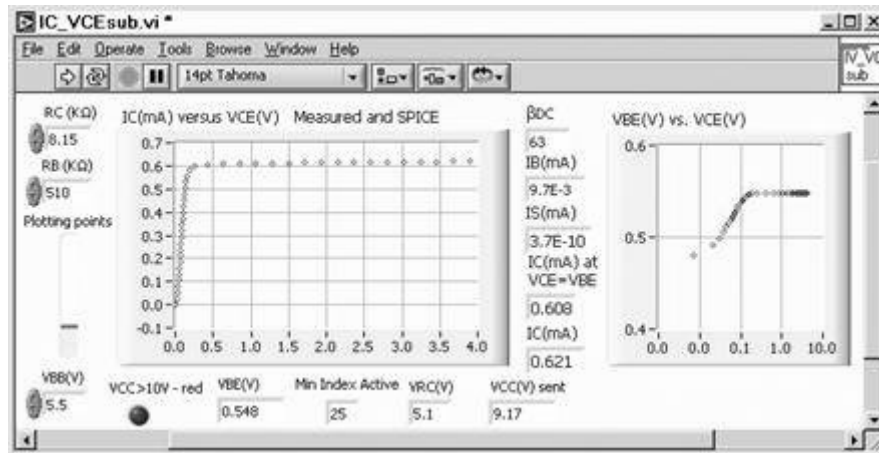
$$I_C \approx \frac{I_{C0}}{V_{AF}} V_{CE} + I_{C0} = \text{Slope} \times V_{CE} + I_{C0}$$

$I_{C0} \equiv I_C(V_{CE} = 0)$ (active-region extrapolation)

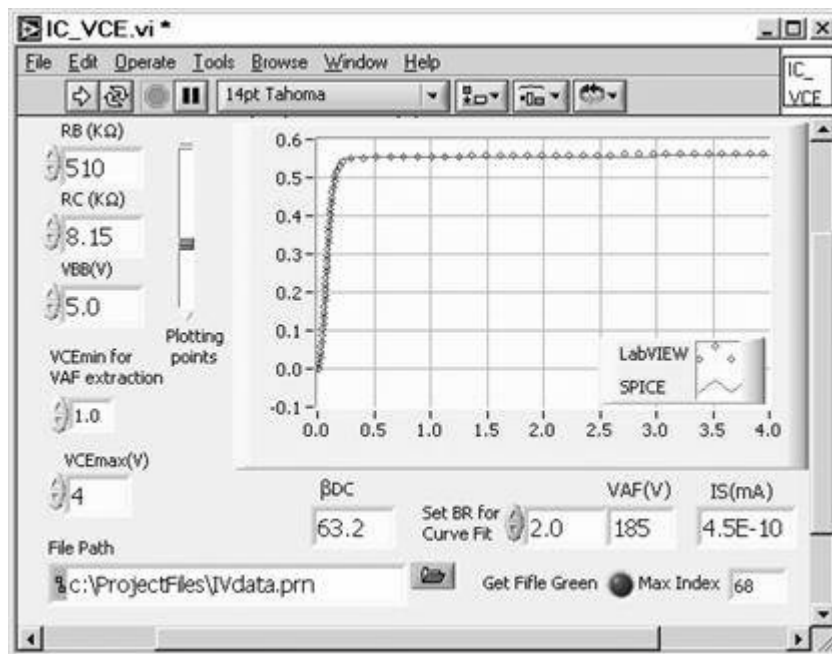
$$VAF = \text{Slope}/I_{C0}$$

Procedure

- Open subVI, **IC_VCEsub.vi**. This VI will send out a value of V_{BB} and sweeps V_{CC} to sweep V_{CE} . The maximum V_{CE} is set for 4 V. Set the value of the resistors. **Run** and re-run while setting V_{BB} for an active-region collector current of about 0.5 mA. Note that at this collector current, a maximum V_{CE} of 4 V should be attainable as the circuit was designed ([PB.3](#), [PB.4](#)) for $I_{Cmax} \approx 1$ mA for $V_{Rc} \approx 9$ V. *Default and save.*



- Now open VI, **IC_VCE.vi**. Set the value of the resistors. Set VBB from the subVI, **IC_VCEsub.vi**. Set VCEmax at 1 V. **Run** the VI and adjust BR for a best fit. Then re-**run** the VI with VCEmax set at 4 V to obtain a good value for VAF. VAF is computed from data points in the range $1 < VCE < VCE_{max}$. *Default and save* the Front Panel. *Double check* to be sure that the subVI is *defaulted and saved*. The data are used in the output characteristic measurement simulation.



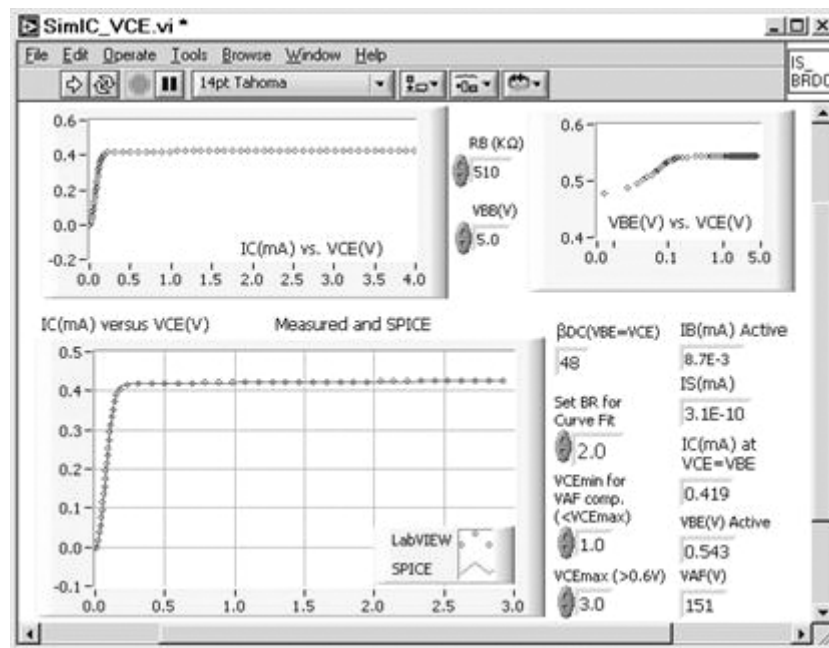
PB.6. Simulation of the Output Characteristic Measurement

Procedure

- SimIC_VCE.vi** simulates the output characteristic measurement and associated curve fitting. Data are supplied from an actual measurement

from any sample **IC_VCEsub.vi**. The data include I_C versus V_{CE} and V_{BE} versus V_{CE} .

- Copy and paste the two data sets from the sample (saved Front Panel or log data) **IC_VCEsub.vi**. Set up all values in the Digital Controls. The information will be used to compute and display the output characteristic, measured and SPICE, and various parameters and dc variables in the Digital Indicators. Parameters β_{DC} and I_S are computed for $V_{CE} = V_{BE}$, according to the definitions. VAF is computed from a selected range of β_{DC} from the active region. (Continued below.)



- **Run** the VI and re-run while adjusting R_B for the best fit. Use a small V_{CEmax} for this. Verify that $V_{CEmin} < V_{CEmax}$ for this simulation. Note that for the best fit, the SPICE curve goes from slightly greater to slightly less than the measured data with increasing V_{CE} in the saturation region. Re-run for computing VAF in various ranges (e.g., 1 V to 3 V, 2 V to 4 V, etc). Note that VAF varies, and depends on the voltage range, unlike the ideal SPICE basic model. Of primary interest is the evaluation range of 1 V to 3 V, as this brackets the bias voltage of 2 V of the amplifier project, [Project C1](#). Default this case.
- Note also that the curves will fit in the active region for any combination of V_{BB} and R_B . However, the β_{DC} value will only be correct for the actual circuit values. For clarity on this point and others, the computations will be repeated in the Mathcad project file. Save the

results.

Laboratory Project C1. NPN Common-Emitter Amplifier

[PC.1 SPICE Equations and Pin Diagram](#)

[PC.2 DC Circuit Setup and Parameter Determination](#)

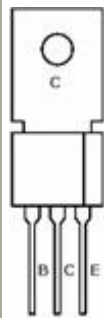
[PC.3 Amplifier Gain at One Bias Current](#)

[PC.4 Amplifier Gain versus Bias Current](#)

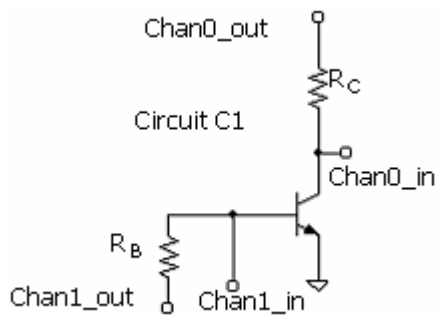
[PC.5 Gain-Measurement Frequency Response](#)

Exercises and Analysis ExerciseC1.mcd - ProjectC1.mcd

PC.1. SPICE Equations and Pin Diagram

SPICE Equation		Description	
Common Emitter Amplifier with R_c (bias) Load			
$a_{vb} = -g_m R_C$ $g_m = I_C/V_T$	C.17	Base-to-collector gain.	
$a_v = -\beta_{ac} \frac{R_C}{R_s}$	C.22	Circuit gain with large source resistor.	
$a_{vb} = -g_m \frac{R_C r_o}{R_C + r_o}$	C.28	Precision base-to-collector gain.	
$r_\pi = \beta_{ac} \frac{V_T}{I_C}$	C.15	Transistor input resistance.	
$r_{on} = \frac{V_{AFn}}{I_C}$	C.40	Transistor output resistance.	

PC.2. DC Circuit Setup and Parameter Determination



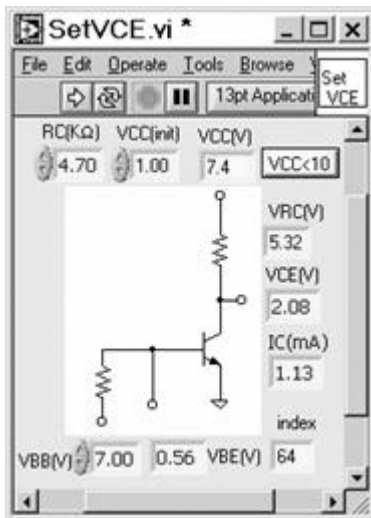
Components

$$R_B = (V_{BB} - V_{BE}) \beta_{DC} / I_{Cmax} \quad V_{BE} \approx 0.55 \text{ V}$$

$$R_C = (V_{CC} - V_{CE}) / I_{Cmax} \quad V_{CE} \approx 2 \text{ V}$$

$$I_{Cmax} \approx 1 \text{ mA} \quad V_{CC} = V_{BB} = 9 \text{ V}$$

Use ProjectC1.mcd for calculations. Verify bias of Circuit C3 (i.e. $V_{CC} < 10 \text{ V}$).



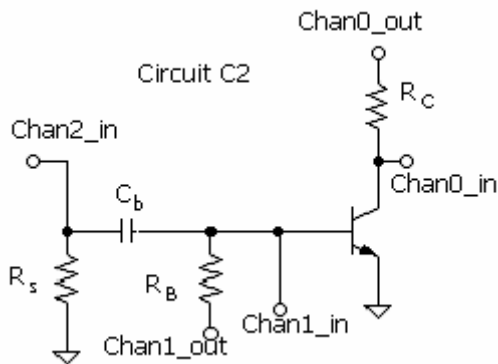
Procedure

- Type in R_C . Run **SetVCE.vi** while re-adjusting V_{BB} to verify that $0.1 \text{ mA} < I_C < 1 \text{ mA}$ can be obtained without $V_{CC} > 10 \text{ V}$. V_{CC} is ramped up to obtain $V_{CE} = 2 \text{ V}$. This will be the operating point (bias) voltage in the gain measurement. Determine V_{BB} for $I_C = 0.1 \text{ mA}$, as in the example.

Default and save this in the Front Panel, for use in the next two parts. Now re-run and determine the value of V_{BB} corresponding to $I_C=0.5$ mA. This is used in the next step.

- Circuit C1 is used to obtain **dc** parameters as in [Project B](#). To obtain a fresh set of parameter values, open subVI **IC_VCEsubC.vi**. Set in R_C and R_B and V_{BB} as determined above for $I_C=0.5$ mA.
- **Run** the VI. When it has been verified that the VI operates properly, open **IC_VCE_C.vi** and set in R_C and R_B and V_{BB} for $I_C=0.5$ mA in the Front Panel of this VI. **Run** the VI to obtain values for V_{AF} and I_S . Get V_{AF} for $1\text{ V} < V_{CE} < 3\text{ V}$. *Default and save* the Front Panel to preserve these numbers. These parameters are also available for [Project C2](#) on the npn – pnp amplifier.

PC.3. Amplifier Gain at One Bias Current



Components

Compute C_b for Circuit C3. (Use ProjectC1.mcd.) It will be more than adequate for Circuit C2.

$$f_{3dB} \approx \frac{1}{2\pi C_b r_\pi} \quad f_{3dB} \approx 30\text{Hz}$$

$$R_s = 10r_\pi$$

SubVIs

SetVSD.vi (Set Bias)

FG_10.vi (Function Generator)

FG_12.vi (Function Generator)

SR_10.vi (Send – Receive Function)

SR_12.vi (Send – Receive Function)

Oscilloscope_10.vi (Chan0_in)

Oscilloscope_12.vi (Chan2_in)

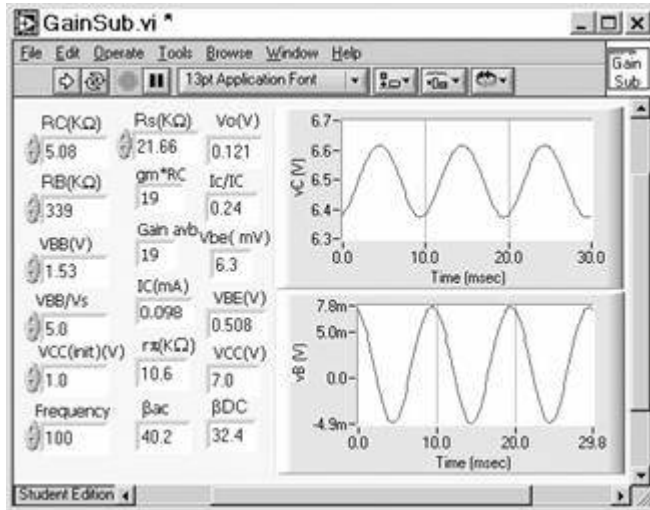
LabVIEW Computations

$$\beta_{DC} = \frac{(\text{Chan0_out} - \text{Chan0_in})/R_C}{(\text{Chan1_out} - \text{Chan1_in})/R_B} \quad \beta_{DC} = \frac{(\text{Chan0_in}_{acPeak})/R_C}{(\text{Chan2_in}_{acPeak})/R_B}$$

$$r_{\pi} = g_m V_T / I_C$$

Procedure

- Connect the additional components in Circuit C2, R_s , and C_b . Connect Chan2_in for reading for reading the base signal voltage.
- **GainSub.vi** sends out bias and signal voltage (Chan1_out). (The signal is superimposed on the bias; the bias is modulated with signal.) Chan0_in reads the response (output of amplifier). The VI then resends the input signal (Chan1_out) and reads the response at the base (Chan2_in). These results are displayed in the two graphs. The ac peak values are used to calculate the gain. The VI also computes and displays r_{π} , β_{DC} , and β_{ac} .



- Open **GainSub.vi**. Set the Digital Controls for collector resistor, R_C , base resistor, R_B , and R_s . R_s is used to compute the signal current through R_s . Set Frequency at between 10 and 1000 Hz. Set V_{BB} for 0.1 mA as determined with **SetVCE.vi**. $V_{CC}(\text{init})$ must be less than or equal to 2 V. *Default* the Front Panel to hold these values for when **GainSub.vi** is used as a subVI.
- Ratio V_{BB}/V_s is set at 5. The VI uses this to set $V_s = V_{BB}/5$, producing, roughly, $I_B/I_b = 5$. This results from $I_B = (V_{BB} - V_{BE})/R_B$ and $I_b = V_s/R_B$. This current ratio provides for operation in the linear mode.
- **Run GainSub.vi** and verify $I_C \approx 0.1 \text{ mA}$. Verify that signals for $V_{be} = V_{in}$ and $V_{ce} = V_o$ appear to be undistorted (maintain sine-wave integrity). Assess all numbers in the Digital Indicators. At the low current of the measurement, Gain should be very close to $g_m \cdot R_C$. *Default and save* the Front Panel to save settings for the next VI.

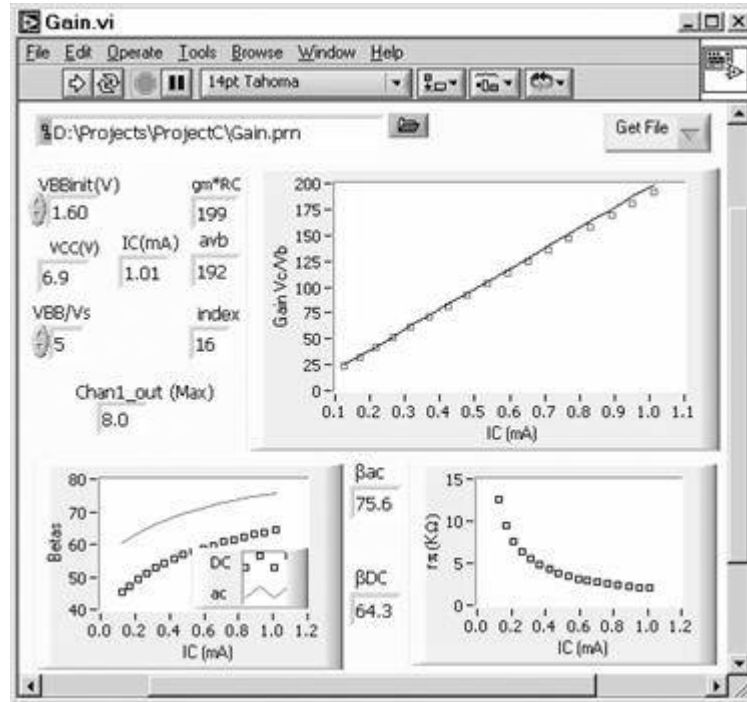
PC.4. Amplifier Gain versus Bias Current

Procedure

- **Gain.vi** is the main VI that obtains the voltage gain as a function of collector bias current. Open **Gain.vi** and verify that $V_{BB}(\text{init})$ is set to your value for $I_C = 0.1 \text{ mA}$ and $V_{BB}/V_s = 5$. $V_{CC}(\text{init})$ will be forced to 1 V in the execution of **Gain.vi**. Keep subVI **GainSub.vi** open to help observe the measurement in progress and to obtain Front Panel information for the maximum current. Verify that the correct data are defaulted in the Front Panel of **GainSub.vi**. The resistors values are not

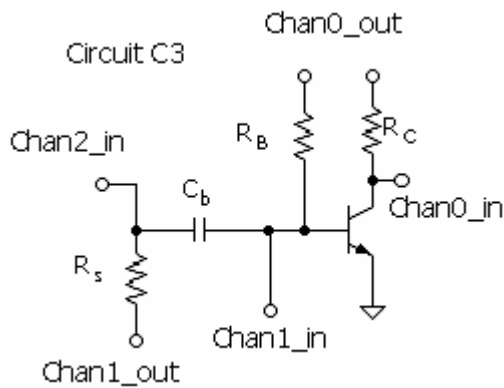
reset in the top VI.

- Open **5COLDATA.vi**. Open VIs using Browse>>Show VI Hierarchy from the main VI.



- **Run Gain.vi** to obtain the gain for a range of collector current with maximum $I_C \approx 1\text{mA}$. Note that there is a 500 ms delay between bias steps to allow the capacitor to charge to steady state. Verify that Chan1_out(Max) (bias plus signal) does not exceed 10 V. Set the file mode switch to Get File. **Run** the VI to obtain a data file.
- The data file includes results for I_C , a_{vb} , β_{DC} , and β_{ac} and r_{π} . The data provide for making a comparison between measured data and SPICE computations in the Mathcad project file. *Default and save* the Front Panel of **5COLDATA.vi** for a data file backup. *Default* the Front Panel of Gain.vi to save the value of VBB for the maximum current. *Default and save* the Front Panel of **GainSub.vi** to save the value of VBE at the maximum current.
- For obtaining a file from the saved data in **Gain.vi**, use **GainData.vi**. Transfer the data from the three Indicator Graphs of **Gain.vi** to the three equivalent Control Graphs in **GainData.vi**. **Run GainData.vi** to obtain the data file.

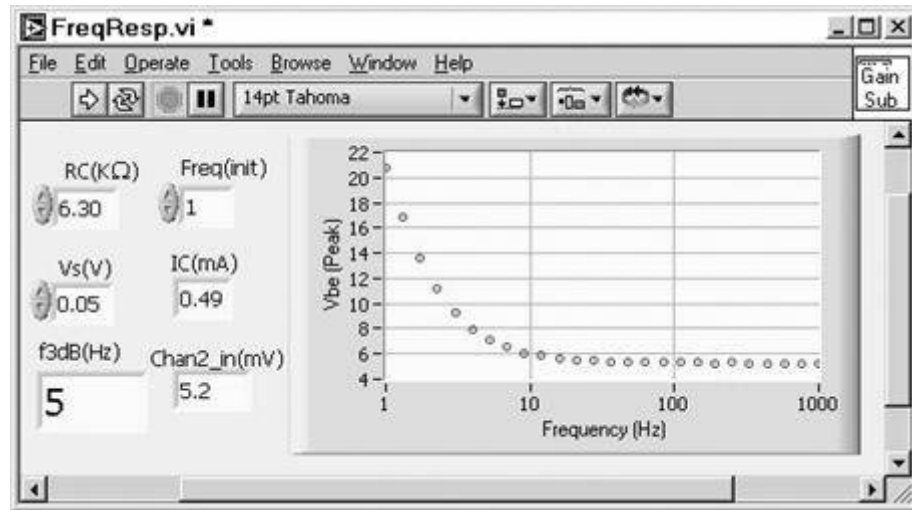
PC.5. Gain-Measurement Frequency Response



Procedure

- Circuit C3 has the input signal applied through the coupling capacitor. Chan0_out V_{CC} provides the input and output circuit bias. The base signal with **dc** removed is measured again with Chan2_in.
- Reconnect the circuit according to Circuit C3. Connect the top of R_B to Chan2_out and the bottom of R_s to **Chan2_out**.
- Open **SetVCEC3.vi** and **run** to determine circuit bias I_C and V_{CC} . Note that if $V_{CC} > 10$ with $V_{CE} < 2$ V, a smaller value of R_C or larger value of R_B is required; that is, there is not a valid solution. (If $V_{CE} \approx 2$ V for $V_{CC} = 10$ V, this is satisfactory.) This situation could occur if the value of β_{DC} used in the resistor design calculation was not representative of the value for your transistor. When the circuit operates properly, *default* the Front Panel to save the values. This saves a set of variable values for the Mathcad project file.
- The VI for the frequency-response measurement is **FreqResp.vi** (below). The VI sends out a signal through out Chan1_out over the frequency range of) Freq(init) to 1 kHz. The response at the base input node on the source side of the capacitor is measured with in Chan2_in as in Circuit C2.
- Open **FreqResp.vi** and install your value of R_C in the Front Panel. Select V_s such as to obtain $V_{be} \approx 5$ mV based on $V_{be} \approx (r_\pi/R_s)V_s$. Recall that r_π versus current is contained in the data file obtained from **Gain.vi** and in the Front Panel of **Gain.vi**. The information should also be saved in the Front Panel of **5ColData.vi**.
- Open subVI **FG1Chan.vi** to monitor the waveform of the signal from in **Chan2_in**. Keep **SetVCEC3.vi** open to verify that the **dc** setup functions properly. Note that the **dc** setup process is configured to be

slow to allow the capacitor transient to be completed.



- **Run FreqResp.vi.** Verify that in Chan2_in (ac peak) is about 5 to 10 mV at the highest frequency. Adjust Vs accordingly. Verify that a flat response is obtained at the higher frequencies. If not, a larger capacitor is needed. *Default and save* the Front Panel to preserve the f3dB information.
- From the **ProjectC1.llb**, open **XYToDataFile.vi**. Copy and paste the graph data to obtain a frequency-response data set.

Laboratory Project C2. NPN – PNP Common-Emitter Amplifier with Current-Source Load

[PC.6 SPICE Equations and Pin Diagram](#)

[PC.7 Measurement of the PNP Parameters](#)

[PC.8 DC Circuit Setup](#)

[PC.9 Measurement of the Amplifier Gain](#)

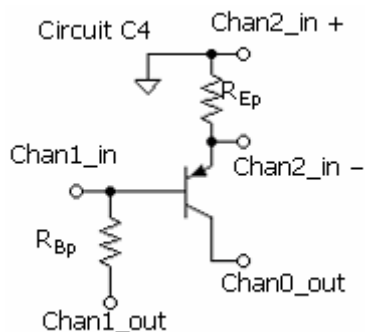
Exercises and Analysis ExerciseC2.mcd - ProjectC2.mcd

PC.6. SPICE Equations and Pin Diagram

SPICE Equation		Description	
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SPICE Equation		Description
$a_v = -\beta_{acn} \frac{R_{onp}}{R_{Bn}}$	C.52	Amplifier gain.
$a_{vb} = -g_m \frac{r_{on} R_{op}}{r_{on} + R_{op}}$	C.44	Base-collector gain.
$R_{onp} = \frac{r_{on} R_{op}}{r_{on} + R_{op}}$	C.53	Amplifier load.
$r_{on} = \frac{V_{AFn}}{I_C}$, $r_{op} = \frac{V_{AFp}}{I_C}$	C.40 , C.41	NPN and PNP output resistance.
$R_{Op} = \left(1 + g_m \frac{R_{Ep}}{r_{\pi p} + R_{Bp} + R_{Ep}} r_{\pi p} \right) r_{Op}$	C.47	Transistor output resistance.
$f_{3dB} = \frac{1}{2\pi\beta_{acp} R_{Ep} C_b} \sqrt{\left(\frac{r_{on} + r_{op}}{r_{op}} \right)^2 - 2}$	C.67	Design frequency of the amplifier.

PC.7. Measurement of the PNP Parameters



Components, PNP

Calculate R_{Ep}

$$0.5 < I_{Ep} R_{Ep} < 1 \text{ V} \quad V_{REp} < 1 \text{ V at } I_{Ep} = 1 \text{ mA}$$

$$I_{Ep} = \left[(1 + \beta_{DCp}) / \beta_{DCp} \right] I_C \quad \beta_{DCp} \approx 50$$

Calculate R_{Bp}

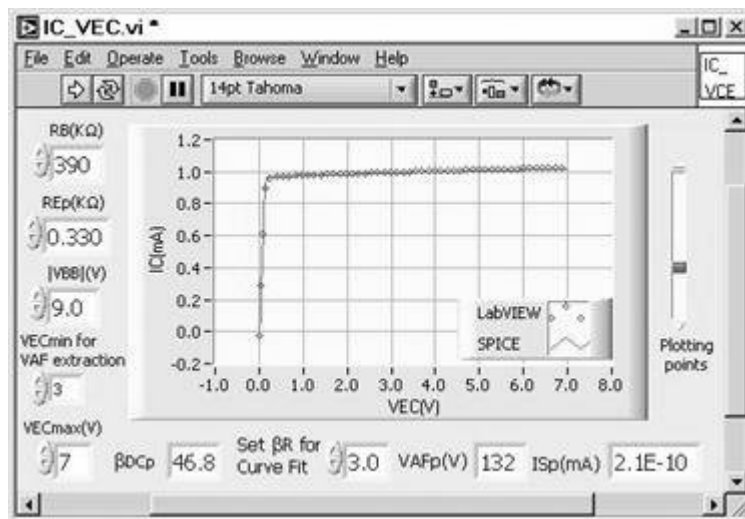
$$I_C \approx I_E \approx 1.5 \text{ mA} \quad |V_{BBp}| = 10 \text{ V}$$

$$V_{EB} \approx 0.6 \text{ V} \quad \beta_{DCp} \approx 50$$

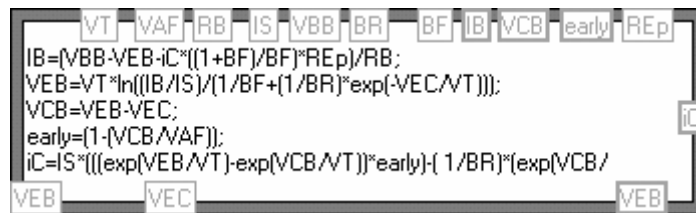
$$R_{Bp} \approx \frac{|V_{BBp}| - V_{EB} - V_{REp}}{I_C} \beta_{DCp} \quad V_{CC} \equiv \text{Chan0_out} \quad V_{BBp} \equiv \text{Chan1_out}$$

Procedure

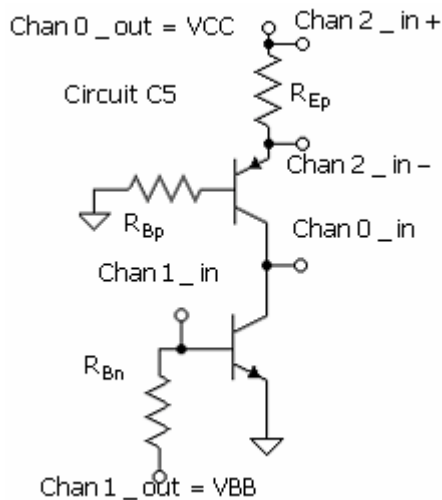
- Circuit C4 is for obtaining the relevant SPICE parameters for the pnp. Of interest are V_{AFp} , β_{DCp} , and I_{Sp} . To obtain the parameters, connect Circuit C4 using the resistors calculated for this part (pnp). These will also be used in the complete amplifier. The VI for obtaining the parameters is **IC_VEC.vi**.
- Open **IC_VEC.vi**. Install, in their Digital Indicators, R_{Bp} and R_{Ep} . **Run** the VI **IC_VEC.vi** with subVI **IC_VECsub.vi** open to observe the plot in progress. Reset V_{BB} as necessary to obtain an active-region I_C of about 1 mA. Verify that $I_C = 1 \text{ mA}$ with $|V_{BB}| < 10 \text{ V}$ can be obtained. $|V_{BB}|$ should be roughly 10 V at $I_C = 1 \text{ mA}$ for maximum measurement precision. Adjust parameter β_R for a curve fit using $V_{CEmax} = 1 \text{ V}$. Note that V_{CEmin} is automatically set to 0.5 V for small V_{CEmax} .



- Determine the value of VAF specifically for the range that includes the bias value of VCE for the amplifier, which is $V_{CE} \approx 5$ V. Set $V_{CEmin} = 4$ V and $V_{CEmax} = 6$ V. Run the VI to obtain VAF. Compare the result with $V_{CEmin} = 3$ V and $V_{CEmax} = 7$ V. Note that in both cases the range brackets the operating point $V_{CE} = 5$ V. *Default and save the Front Panel to preserve the parameter values.*
- The computational subVI, **SPICE_IV_Rep.vi**, computes the SPICE plot, which is plotted along with the measured data. The Formula Node from the Diagram of **SPICE_IV_Rep.vi** is shown below. A solution for I_C as a function of V_{EC} using the circuit and device equations is obtained. This is also explored in the project Mathcad file.



PC.8. DC Circuit Setup



Components NPN

$$R_{Bp} \approx R_{Bn}.$$

If slightly different, use $R_{Bp} > R_{Bn}$.

Design $I_C = 1\text{ mA}$.

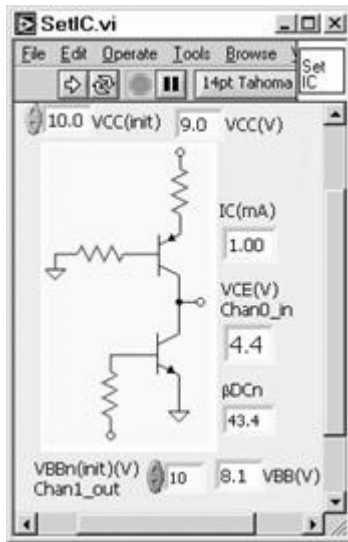
$$I_C = \frac{V_{BB} - V_{BE}}{R_{Bn}} \beta_{DCn}$$

$$V_{CC} = \text{Chan0_out} \quad V_{BB} = \text{Chan1_out}$$

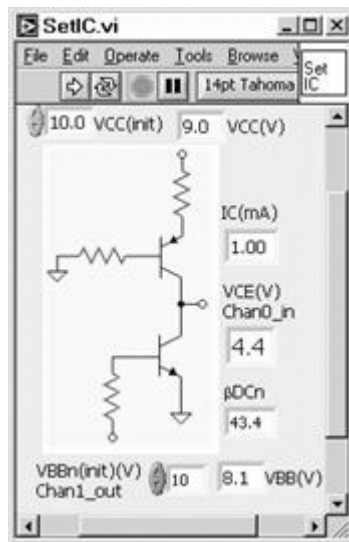
$$\beta_{DCn} \approx \beta_{DCp} \text{ and } V_{BE} \approx V_{EB} \approx 0.6 \text{ V}$$

Procedure

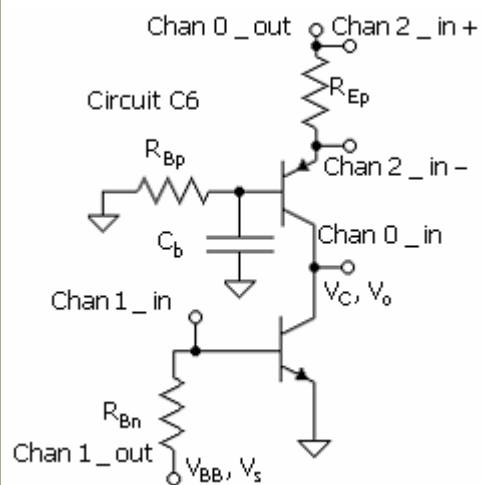
- **SetVCE2.vi** ramps V_{BB} downward from the maximum, $V_{BB}(\text{init}) = V_{CC}(\text{init})$. Thus, initially, the npn will be in saturation and V_{CE} will be very approximately 100 mV. The first check is for one point, maximum $V_{BB} = V_{CC} = 10 \text{ V}$. The initial goal is verify that the npn is in saturation and that the current is greater than 1 mA for $V_{CC} = 10 \text{ V}$.
- Connect Circuit C5. Note that the bottom of R_{Bp} is now ground and Chan1_out moves to the bottom of R_{Bn} . Also, the collectors are connected together and Chan1_in moves to the base of the npn. Move the plus side of Chan2_in and the top of R_{Ep} to Chan0_out .
- Set in resistor values in **SetVCE2.vi**. Verify that $V_{BB}(\text{init}) = 10 \text{ V}$ and $V_{CC}(\text{init}) = 10 \text{ V}$. Set the Run Mode switch to One VCC (logic 1). **Run** the VI. Verify that $I_C > 1 \text{ mA}$ and that V_{CE} is 50 to 300 mV (in saturation).
- **SetVCE2.vi** calculates the V_{CE} operating point (bias voltage) as $(V_{CC} - V_{REp})/2$ for any V_{CC} (optimum signal swing magnitude). When the Run Mode is set to Set VCE, the execution of the VI halts when this level of V_{CE} is reached.



- Set the Run Mode switch to Set VCE. Run the VI. Verify that the execution halts when VCE is about equal to VCE(V)Op.Pt. Note that in the example, $I_C = 1.14 \text{ mA}$ for $V_{CC} = 10 \text{ V}$ and $V_{BB} = 9.02 \text{ V}$. Verify that the final VCE is close to VCE(V)Op.Pt. *Default* the Front Panel to *save* the values of RBn and REp.
- **SetIC.vi** runs **SetVCE2.vi** as a subVI. It ramps VCC downward to find the VCC corresponding to the design $I_C = 1 \text{ mA}$. Open **SetIC.vi** and have **SetVCE2.vi** open as well. **SetIC.vi** will be a subVI in the gain measurement VI. **Run** the VI to verify that the VI can determine the VCC corresponding to $I_C = 1 \text{ mA}$. In the example, VCC has been ramped down to 9.0 V and VBB is now down to 8.1 V. The routine will occur automatically in the execution of the gain measurement VI.



PC.9. Measurement of the Amplifier Gain



Component Computation

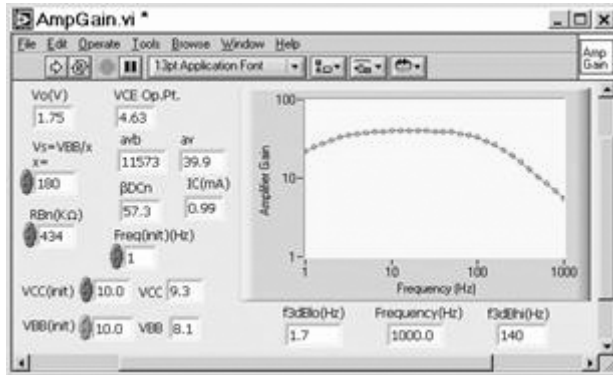
$$f_{3dB} = \frac{1}{2\pi\beta_{acp}R_{Ep}C_b} \sqrt{\left(\frac{r_{on} + r_{op}}{r_{op}}\right)^2 - 2}$$

$$C_b = \frac{1}{2\pi\beta_{DCp}R_{Ep}f_{3dB}} \cdot \sqrt{\left(\frac{r_{op} + r_{on}}{r_{op}}\right)^2 - 2}$$

$$f_{3dB} \approx f_{av} = 1 \text{ Hz} \quad \beta_{acp} \approx \beta_{DCp}$$

Procedure

- **AmpGain.vi** sweeps the signal frequency over the range $f_{init} < f_s < 1000$ Hz. Note that on the low end of the sweep, this is a fairly slow process. The function generator VI sends several cycles at each frequency, which is, e.g., 1 sec/ cycle at $f_s = 1$ Hz. The amplifier voltage gain is calculated at each frequency step. This is the signal output voltage, V_o , divided by the signal input voltage, V_s . V_s is corrected for digital-to-analog error. The correction is performed by subVI, **DAC.vi**.
- The first measurement is made *without* C_b . Enter the value of R_{Bn} in the Front Panel of **AmpGain.vi**. Although it is defaulted in **SetVCE2.vi**, it is needed in the Top VI in the calculation of a_{vb} . Also open subVI's **SetIC.vi** and **FG1Chan.vi** to observe the program in progress. At least temporarily open **SetVCE2.vi** to verify that the component values in the Front Panel are correct and have been made the default values.
- Set Freq(init) to 10 Hz in the Front Panel of **AmpGain.vi**. Verify that VCC(init) and VBB(init) are set to 10 V. **Run** the VI. Upon completion of the execution there should be a low-frequency plateau in the response curve. The VI uses the maximum in the plot for the amplifier (circuit) gain. The roll-off at high frequencies is due to circuit board and transistor capacitance. Note that the example is with the capacitor in place. There will be no low-frequency roll-off without the capacitor.
- Now reset VCC(init) and VBB(init) to match the values in the Digital Indicators (in lieu of 10 V as in the example). The dc set up will be considerably faster. Re-run **AmpGain.vi** to verify that it functions properly.



- If the gain curve appears already to be in the high-frequency roll-off at the lowest frequencies, re-run the VI with $Freq(init)$ set to 1 Hz.
- Re-run the VI and adjust X (downward for increasing V_s) to obtain a V_o (signal sine-wave peak) of about 1 V or about one-third of V_{CE} (operating point). Note that the DAQ limit must be set at 10 V to accommodate the dc + signal measured with Chan0_out. Therefore, the output signal voltage must be relatively large to have sufficient measurement resolution. Log the Front Panel.
- Add the capacitor, C_b , to Circuit C6. Set $Freq(init)$ to 1 Hz. Run **AmpGain.vi**. If a proper selection of C_b was made, the response curve should have a flat portion as in the example. The flat segment contains the gain value. Note that if f_{3dBlo} indicates 1 Hz, then it is less than 1 Hz. In this case, lower $Freq(init)$ to obtain a curve that includes the f_{3dB} frequency. It may be necessary to increase X (decrease V_s) for a satisfactory value of V_o , as the gain is higher with the capacitor. Log the Front Panel when a satisfactory result has been obtained.